

1. Number Systems

There are infinite ways to represent a number. The four commonly associated with modern computers and digital electronics are: Decimal, Binary, Octal, and Hexadecimal.

Decimal (base 10) is the way most human beings represent numbers.

Decimal counting goes:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 and so on.

Binary (base 2) is the natural way most digital circuits represent and manipulate numbers.

Binary counting goes:

0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, and so on.

Octal (base 8) was previously a popular choice for representing digital circuit numbers in a form that is more compact than binary. Octal is sometimes abbreviated as **oct**.

Octal counting goes:

0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, and so on.

Hexadecimal (base 16) is currently the most popular choice for representing digital circuit numbers in a form that is more compact than binary. Hexadecimal numbers are sometimes represented by preceding the value with '0x', or '0X', as in 0x1B84. Hexadecimal is sometimes abbreviated as **hex**.

Hexadecimal counting goes:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21 and so on. Note: A~F can be substituted with a~f.

All four number systems are equally capable of representing any number. Furthermore, a number can be perfectly converted between the various number systems without loss of numeric value.

Two observations: For a number system of base b , where b is 10 (decimal), 2 (binary), 8 (oct) or 16 (hex), and others

- Starting from the right most position, the 1st position has base value b^0 , the 2nd position has base value b^1 , the 3rd position has base value b^2 and so on, i.e., ... $b^3 b^2 b^1 b^0$. Thus a decimal literal 134 represents one 10^2 and three 10^1 and four 10^0 . That is, $1 \times 10^2 + 3 \times 10^1 + 4 \times 10^0 = 100 + 30 + 4 = 134$.
- For each position, there are b possible values, ranging from 0 to $b-1$. So each position of decimal has 10 possible values (0-9), binary has 2 values (0-1), oct has 8 values (0-7), and hex has 16 values (0-9 and then A-F -- we cannot use 10-15 as each position can have only one letter). In general, for a number system of base b , there are b digit symbols.

2. Conversion from Binary, Oct, Hex to Decimal

2.1 Binary to Decimal

They say there are only 10 people in this world: those that understand binary and those that don't. Ha ha.

If you don't get that joke, you'll need a method to convert from binary to decimal.

[Required] The conversion can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 2.

$$11100001 = (1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ = 225 \text{ decimal}$$

[Optional] One another method involves addition and multiplication.

1. Start the decimal result at 0.
2. Remove the most significant binary digit (leftmost) and add it to the result.
3. If all binary digits have been removed, you're done. Stop.
4. Otherwise, **multiply the result by 2.**
5. Go to step 2.

Here is an example of converting 11100001 binary to decimal:

Binary Digits	Operation	Decimal Result	Operation	Decimal Result
11100001	+1	1	$\times 2$	2
1100001	+1	3	$\times 2$	6
100001	+1	7	$\times 2$	14
00001	+0	14	$\times 2$	28
0001	+0	28	$\times 2$	56
001	+0	56	$\times 2$	112
01	+0	112	$\times 2$	224
1	+1	225	done	

2.2 Octal to Decimal

[Required] The conversion can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 8.

$$345_{\text{octal}} = (3 \times 8^2) + (4 \times 8^1) + (5 \times 8^0)$$

$$= (3 \times 64) + (4 \times 8) + (5 \times 1) = 229_{\text{decimal}}$$

[Optional] Converting octal to decimal can be done with addition and multiplication.

1. Start the decimal result at 0.
2. Remove the most significant octal digit (leftmost) and add it to the result.
3. If all octal digits have been removed, you're done. Stop.
4. Otherwise, **multiply the result by 8.**
5. Go to step 2.

Octal Digits	Operation	Decimal Result	Operation	Decimal Result
345	+3	3	$\times 8$	24
45	+4	28	$\times 8$	224
5	+5	229	done.	

2.3 Hexadecimal to Decimal

[Required] Converting hexadecimal to decimal can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 16. Of course, hexadecimal letter values need to be converted to decimal values before performing the math.

Hexadecimal:	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

34F hexadecimal:

$$= (3 \times 16^2) + (4 \times 16^1) + (F \times 16^0)$$

$$= (3 \times 256) + (4 \times 16) + (15 \times 1) = 847_{\text{decimal}}$$

[Optional] Converting hex to decimal can also be done with addition and multiplication.

1. Start the decimal result at 0.
2. Remove the most significant hex digit (leftmost) and add it to the result.
3. If all hex digits have been removed, you're done. Stop.
4. Otherwise, **multiply the result by 16.**
5. Go to step 2.

Of course, hexadecimal letter values (i.e., A - F) need to be converted to decimal values before performing the math.

Hex Digits	Operation	Decimal Result	Operation	Decimal Result
34F	+3	3	$\times 16$	48
4F	+4	52	$\times 16$	832
F	+15	847	done.	

3. Conversion from Decimal to Binary, Oct and Hex

A repeated division and remainder algorithm can convert decimal to binary, octal, or hexadecimal.

1. Divide the decimal number by the desired target radix (2, 8, or 16).
2. Append the remainder as the next most significant digit.
3. Repeat until the decimal number has reached zero.

3.1 Decimal to Binary [optional]

Here is an example of using repeated division to convert 1792 decimal to binary:

Decimal Number	Operation	Quotient	Remainder	Binary Result
1792	$\div 2 =$	896	0	0
896	$\div 2 =$	448	0	00
448	$\div 2 =$	224	0	000
224	$\div 2 =$	112	0	0000
112	$\div 2 =$	56	0	00000
56	$\div 2 =$	28	0	000000
28	$\div 2 =$	14	0	0000000
14	$\div 2 =$	7	0	00000000
7	$\div 2 =$	3	1	100000000
3	$\div 2 =$	1	1	1100000000
1	$\div 2 =$	0	1	11100000000
0	done.			

3.2 Decimal to Octal [optional]

Here is an example of using repeated division to convert 1792 decimal to octal:

Decimal Number	Operation	Quotient	Remainder	Octal Result
1792	$\div 8 =$	224	0	0
224	$\div 8 =$	28	0	00
28	$\div 8 =$	3	4	400
3	$\div 8 =$	0	3	3400
0	done.			

3.3 Decimal to Hexadecimal [optional]

Here is an example of using repeated division to convert 1792 decimal to hexadecimal:

Decimal Number	Operation	Quotient	Remainder	Hexadecimal Result
1792	$\div 16 =$	112	0	0
112	$\div 16 =$	7	0	00
7	$\div 16 =$	0	7	700
0	done.			

Another example, where letters are involved.

Decimal Number	Operation	Quotient	Remainder	Hexadecimal Result
48879	$\div 16 =$	3054	15	F
3054	$\div 16 =$	190	14	EF
190	$\div 16 =$	11	14	EEF
11	$\div 16 =$	0	11	BEEF
0	done.			

4. Conversion from Binary to Oct and Hex

4.1 Binary to Octal

[Required] An easy way to convert from binary to octal is to group binary digits into sets of **three**, starting with the least significant (rightmost) digits.

Binary: 11100101 = 11 100 101

011 100 101 Pad the most significant digits with zeros if necessary to complete a group of three.

Then, look up the decimal value of each group, as shown in the table:

Binary:	000	001	010	011	100	101	110	111
Octal:	0	1	2	3	4	5	6	7

Binary = 011 100 101

Octal = 3 4 5 = 345

4.2 Binary to Hexadecimal

[Required] An equally easy way to convert from binary to hexadecimal is to group binary digits into sets of **four**, starting with the least significant (rightmost) digits.

Binary: 11100101 = 1110 0101

Then, look up the decimal value of each group, as shown in the table:

Binary:	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal:	0	1	2	3	4	5	6	7
Binary:	1000	1001	1010	1011	1100	1101	1110	1111
Hexadecimal:	8	9	A	B	C	D	E	F

Binary = 1110 0101

Hexadecimal = E 5 = E5

5. Conversion from Oct and Hex to Binary

5.1 Octal to Binary

[Required] Converting from octal to binary is as easy as converting from binary to octal (just the opposite operation). Simply look up each octal digit to obtain the equivalent group of three binary digits.

Octal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Octal = 3 4 5

Binary = 011 100 101 = 011100101 binary

5.2 Hexadecimal to Binary

[Required] Converting from hexadecimal to binary is as easy as converting from binary to hexadecimal (just the opposite operation). Simply look up each hexadecimal digit to obtain the equivalent group of four binary digits.

Hexadecimal:	0	1	2	3	4	5	6	7
Binary:	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal:	8	9	A	B	C	D	E	F
Binary:	1000	1001	1010	1011	1100	1101	1110	1111

Hexadecimal = A 2 D E

Binary = 1010 0010 1101 1110 = 1010001011011110 binary

6. What else? Conversion between Oct and Hex

6.1 Octal to Hexadecimal [optional]

When converting from octal to hexadecimal, it is often easier to first convert the octal number into binary and then from binary into hexadecimal. For example, to convert 345 octal into hex:

Octal = 3 4 5

Binary = 011 100 101 = 011100101 binary

Binary = 1110 0101

Hexadecimal = E 5 = E5 hex

Therefore, through a two-step conversion process, octal 345 equals binary 011100101 equals hexadecimal E5.

6.2 Hexadecimal to Octal [optional]

When converting from hexadecimal to octal, it is often easier to first convert the hexadecimal number into binary and then from binary into octal. For example, to convert A2DE hex into octal:

Hexadecimal = A 2 D E

Binary = 1010 0010 1101 1110 = 1010001011011110 binary

Add leading zeros or remove leading zeros to group into sets of three binary digits.

Binary: 1010001011011110 = 001 010 001 011 011 110

Binary = 001 010 001 011 011 110

Octal = 1 2 1 3 3 6 = 121336 octal

Therefore, through a two-step conversion process, hexadecimal A2DE equals binary 1010001011011110 equals octal 121336.

This simplified tutorial contains the basics of different number bases.

You are responsible for the **[Required]** sections. That is,

- 1) conversions from Binary, Octal and Hex → Decimal, and
- 2) from Binary → Octal and Hex, and
- 3) from Octal and Hex → Binary.

You are not responsible for the **[Optional]** sections, including

- From Decimal to Binary, Hex, Oct
- Conversion between Hex and Oct.