Question 1:

For First instance: $x_1 = 1, x_2 = 1, y = 0$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$
$$= 0.5 + (-1)(1) + (1)(1)$$
$$= 0.5 - 1 + 1$$
$$= 0.5$$
$$P = \frac{1}{1 + e^{-y}}$$
$$= \frac{1}{1 + e^{-0.5}}$$
$$= 0.62 > 0.5 \text{ [using the numpy's exp function]}$$

Considering a threshold of 0.5, instance 1 is classified as 1.

For 2nd instance: $x_1 = 0, x_2 = 2, y = 1$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$
$$= 0.5 + (-1)(0) + (1)(2)$$
$$= 0.5 - 0 + 2$$
$$= 2.5$$
$$P = \frac{1}{1 + e^{-y}}$$
$$= \frac{1}{1 + e^{-2.5}}$$
$$= 0.92 > 0.5 \text{ [using the numpy's exp function]}$$

Considering a threshold of 0.5, instance 2 is classified as 1.

For 3rd instance: $x_1 = 1, x_2 = 1.5, y = 1$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$
$$= 0.5 + (-1)(1) + (1)(1.5)$$
$$= 0.5 - 1 + 1.5$$
$$= 1$$

$$P = \frac{1}{1 + e^{-y}}$$
$$= \frac{1}{1 + e^{-1}}$$
$$= 0.73 > 0.5$$ [using the numpy's exp function]

Considering a threshold of 0.5, instance 3 is classified as 1.

Question 2:

y = 0, 1, 1

Probability = 0.62, 0.92, 0.73

TP = 2, TN = 0, FP = 1, FN = 0

Accuracy = $\frac{TP+TN}{TP+TN+FP+FN}$

= $\frac{2+0}{3}$

= $\frac{2}{3}$

= 0.67

Precision = $\frac{TP}{TP+FP}$

= $\frac{2}{2+1}$

= $\frac{2}{3}$

= 0.67

Recall = $\frac{TP}{TP+FN}$

= $\frac{2}{2+0}$

= 1

F1 = $\frac{Precision*Recall}{Precision+Recall}$

= $\frac{0.67*1}{0.67+1}$

= $\frac{0.67}{1.67}$

= .40

Question 3:

If a threshold equal 0.72 is considered, the instance 1, 2, and 3 are classified as 0, 1, 1 which matches the actual values in the validation data set.

Hence, we get, TP = 2, TN = 1, FP = 0, FN = 0.

And, The Accuracy $= \frac{TP+TN}{TP+TN+FN+FP}$

$$= \frac{2+1}{2+1}$$

$$= \frac{3}{3}$$

$$= 1$$

Question 4:

For First instance: $x_1 = 2, x_2 = 0, y = 0$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

= 0.5 + (-1) (2) + (1)(0)

= 0.5 -2 + 1

= -1.5

$$P = \frac{1}{1+e^{-y}}$$

$$= \frac{1}{1+e^{1.5}}$$

$$= 0.18$$

Considering a threshold of 0.5, instance 1 is classified as 0 (0.18< 0.5).

Considering a threshold of 0.72, instance 1 is classified as 0 (0.18<0.72).

For 2nd instance: $x_1 = 1.2, x_2 = 1, y = 0$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$= 0.5 + (-1)(1.2) + (1)(1)$$

$$= 0.5 - 1.2 + 1$$

$$= 0.3$$

$$P = \frac{1}{1 + e^{-y}}$$

$$= \frac{1}{1 + e^{-0.3}}$$

$$= 0.57$$

Considering a threshold of 0.5, instance 2 is classified as 1 (0.57> 0.5).

Considering a threshold of 0.72, instance 2 is classified as 0 (0.57< 0.72).

For 3rd instance: $x_1 = 1, x_2 = 0.6, y = 0$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$= 0.5 + (-1)(1) + (1)(0.6)$$

$$= 0.5 - 1 + 0.6$$

$$= 0.1$$

$$P = \frac{1}{1 + e^{-y}}$$

$$= \frac{1}{1 + e^{-0.1}}$$

$$= 0.52$$

Considering a threshold of 0.5, instance 3 is classified as 1 (0.52 > 0.5).

Considering a threshold of 0.72, instance 2 is classified as 0 (0.52<0.72).

For 4th instance: $x_1 = 1, x_2 = 1.5, y = 1$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$
$$= 0.5 + (-1)(1) + (1)(1.5)$$
$$= 0.5 - 1 + 1.5$$
$$= 1$$

$$PP = \frac{1}{1 + e^{-y}}$$
$$= \frac{1}{1 + e^{-1}}$$
$$= 0.73$$

Considering a threshold of 0.5, instance 4 is classified as 1 (0.73> 0.5).

Considering a threshold of 0.72, instance 4 is classified as 1 (0.73> 0.72).

<u>Accuracy for threshold 0.5:</u>

Probabilities are: 0.18, 0.57, 0.52, 0.73 (0, 1, 1, 1)

Actual Values of y are: 0, 0, 0, 1

Accuracy $= \frac{TP+TN}{TP+TN+FT+FN}$

$= \frac{1+1}{1+1+0+2}$

$= \frac{2}{4}$

$= 0.5$

<u>Accuracy for threshold 0.72:</u>

Probabilities are: 0.18, 0.57, 0.52, 0.73 (0, 0, 0, 1)

Actual Values of y are: 0, 0, 0, 1

$$\text{Accuracy} = \frac{TP+TN}{TP+TN+FT+FN}$$

$$= \frac{1+3}{1+3+0+0}$$

$$= \frac{4}{4}$$

$$= 1$$