$y_1 = r$

$y_2 = r$

$y_3 = r$

x_k^-

$$\begin{bmatrix} p_x(k) \\ p_y(k) \\ p_z(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \\ a_x(k) \\ a_y(k) \\ a_z(k) \end{bmatrix}$$

np.random.normal

(0, 10, 3)

mediciones

$$y_k = \underbrace{C}_{[I \ 0 \ 0]} x_k + r_k = \begin{bmatrix} p_x(k) \\ p_y(k) \\ p_z(k) \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

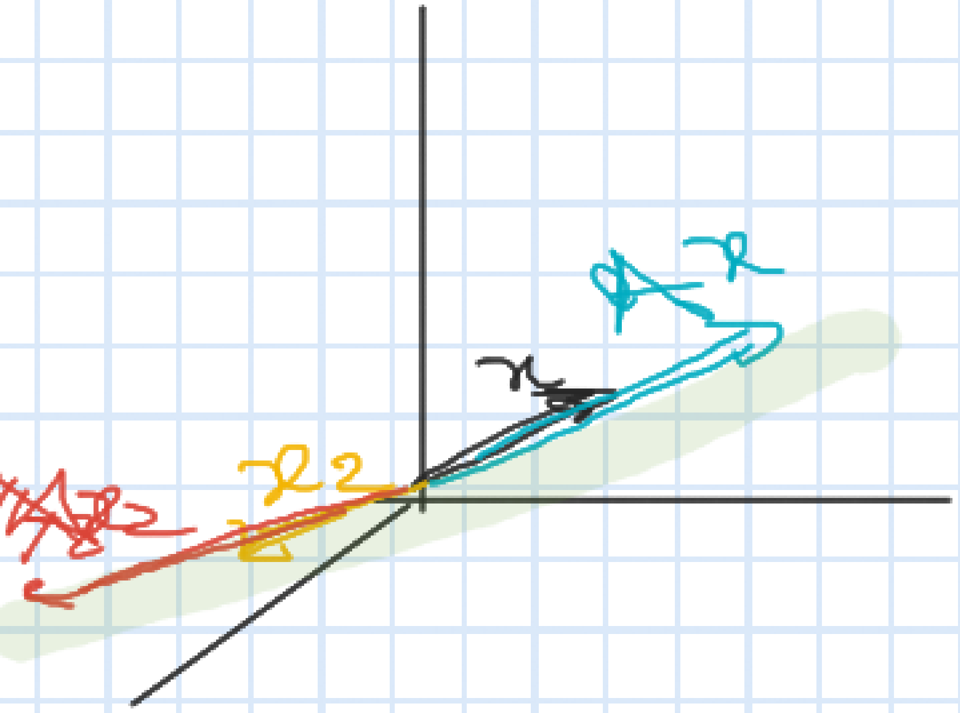
$$\begin{cases} \bar{x} = \begin{bmatrix} \text{Pos} \\ \text{vel} \\ \text{Acc} \end{bmatrix} \in \mathbb{R}^{9 \times m} \\ y = \underbrace{\begin{bmatrix} I & 0 & 0 \end{bmatrix}}_{3 \times 9} \underbrace{x}_{9 \times 3} \in \mathbb{R}^{3 \times m} \\ y = y + \text{np.random.normal}(0, 10, y.\text{shape}) \end{cases}$$

$\sim$ vidéo réel.

$$\underbrace{A\begin{bmatrix}1/2\\1/3\\1\end{bmatrix}}_{x} = 3\begin{bmatrix}1/2\\1/3\\1\end{bmatrix}$$

λ = autovalor

$$v = k\begin{bmatrix}1/2\\1/3\\1\end{bmatrix} \to Av = k\Bigg(\underbrace{A\begin{bmatrix}1/2\\1/3\\1\end{bmatrix}}_{3\begin{bmatrix}1/2\\1/3\\1\end{bmatrix}}\Bigg) = 3\Bigg(\underbrace{k\begin{bmatrix}1/2\\1/3\\1\end{bmatrix}}_{v}\Bigg)$$



$S = \text{gen}\{x_1\}$ é subespaço de $\mathbb{R}^3$

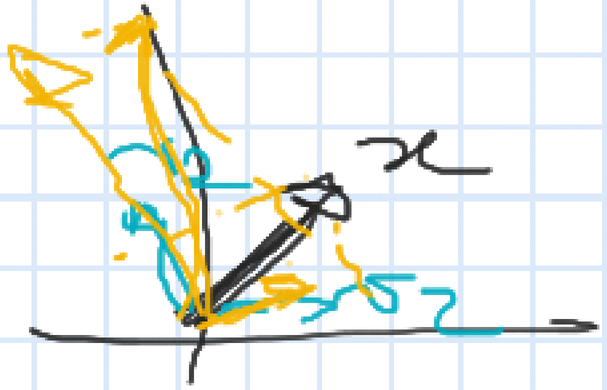
$$A x_1 = \lambda_1 x_1$$

λ_1: autovalor — x_1: autovetor

$A \in \mathbb{R}^{2\times 2}$ $\quad (\lambda_1, v_1)$ (supongo v_1, v_2 l.i.)

(λ_2, v_2)

$x \in \mathbb{R}^2 \quad x = \alpha_1 v_1 + \alpha_2 v_2$



$Ax = \alpha_1 \lambda_1 v_1 + \alpha_2 \lambda_2 v_2$

simétricos

semidefinida positiva

$AA^T \rightsquigarrow (\lambda_i, u_i)$

$A^TA \rightsquigarrow (\lambda_i, v_i)$

$$\underbrace{\begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix}}_{\text{BON}} \begin{bmatrix} \sqrt{\lambda_1} & \\ & \sqrt{\lambda_2} \\ & & \ddots \end{bmatrix} \begin{bmatrix} - & v_1 & - \\ - & v_2 & - \end{bmatrix}$$

Las columnas de U y V son ortonormales.

$$Ax = y, \quad A \in \mathbb{R}^{m \times n}, \quad m > n$$

$$U^T = U^{-1}, \quad V^T = V^{-1}$$ } porque son matrices ortonormales

$$\underbrace{U_r \Sigma_r V_r^T}_{A} x = y$$

$$U_r \in \mathbb{R}^{r \times r}$$

$$\underbrace{U_r^T U_r}_{I} \Sigma_r V_r^T x = U_r^T y$$

$$\Sigma_r = \begin{bmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_r \end{bmatrix} \quad \sigma_i > 0$$

$$\underbrace{\Sigma_r^{-1} \Sigma_r}_{I} V_r^T x = \Sigma_r^{-1} U_r^T y$$

$$V_r \in \mathbb{R}^{r \times r}$$

$$U_r^{-1} = U_r^T \qquad V_r^{-1} = V_r^T$$

$$\underbrace{V_r V_r^T}_{I} x = V_r \Sigma_r^{-1} U_r^T y$$

$$UU^T = I$$

$$VV^T = I$$

SPOK