$$A_1 = \begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix} \quad x^T A x > 0 \quad \forall x \neq 0, \; x \in \mathbb{R}^2$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2x_1 + 3x_2 \\ 3x_1 + 9x_2 \end{bmatrix}$$

$$= 2x_1^2 + \underbrace{3x_1x_2 + 3x_1x_2}_{=6x_1x_2} + 9x_2^2$$

$$= \underbrace{(x_1 + 3x_2)^2}_{x_1^2 + 9x_2^2 + 6x_1x_2} + x_1^2 > 0 \Rightarrow A_1 \text{ es def } (+)$$

$$A_2 = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} = 2x_1^2 + 6x_1x_2 + 6x_2^2$$

$$= (x_1 + 3x_2)^2 + x_1^2 - 3x_2^2 \Rightarrow \text{no es def } (+)$$

$P_2(\mathbb{R})$ = polinomios de grado 2 (coef. reales)

$B = \{1, 1+x, x + x^2\}$

$\cdot B_1 = \{1, x, x^2\}$

$\langle p, q \rangle = p(0)q(0) + p(1)q(1) + p(-1)q(-1)$

elementos de la base

$a_0 + a_1 x + a_2 x^2 = p$

$\hat{p} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$ $\qquad p = \sum_{i=1}^{3} \cancel{\alpha_i} a_i$

los coef de $\hat{p}$

$\mathbb{R}^n \quad (2, 0 \ldots), (0, 2, 0 \ldots 0)$

$\hat{x} = x/2$

• $P_1(\mathbb{R})$ el esp de los polinomios de grado 1

$B = \{1, x\}$

$$\langle p, q\rangle = p(0)q(0) + p(3)q(3)$$

$p = a_0 + a_1 x$

$q = b_0 + b_1 x$

$$= a_0 b_0 + (a_0 + a_1 3)(b_0 + b_1 3)$$

$$= 2a_0 b_0 + a_0 b_1 3 + 3 b_0 a_1 + a_1 b_1 3^2$$

$$\begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix}$$

$$\langle p, q\rangle = \underbrace{\begin{bmatrix} a_0 & a_1 \end{bmatrix}}_{\hat{p}^T} \underbrace{\begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} b_0 \\ b_1 \end{bmatrix}}_{\hat{q}} = \hat{p}^T A_1 \hat{q}$$

Ejemplo distancia de Mahalanobis.

x, y v.a. con = distrib.

$$d(x,y) = \sqrt{(x-y)^T S^{-1} (x-y)}$$

$S > 0 \Rightarrow S^{-1} > 0$

$S \; cov(x,y)$

$\downarrow$

$$\langle x, y \rangle = x^T S^{-1} y$$

$\leftarrow$ def +

P.i.: $x^T I y$

Ej de transf.

• $\Phi : \mathbb{R} \to \mathbb{R}$ $\quad \Phi(x) = x^2$

-1 y $1 \to 1$
no inyect

No sobreyect (solo obtengo nº ≥ 0)

• $\Phi : \mathbb{R} \to \mathbb{R}$ $\quad \Phi(x) = e^x$

Es inyectiva
no sobreyectiva ($e^x \geq 0$)

• $\Phi : \mathbb{R} \to \mathbb{R}$, $\quad \Phi(x) = x^3 - 3x = x(x^2 - 3)$

No inyectiva
Es sobreyectiva

• $\Phi : \mathbb{R} \to \mathbb{R}$, $\Phi(x) = x^3$ $\Rightarrow$ $\left.\begin{array}{l}\text{Es iny.}\\ \text{es sobre}\end{array}\right\} \Rightarrow$ biyectiva

- $\Phi: \mathbb{R}^3 \to \mathbb{R}^2 \quad \Phi(x) = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} x$

  Es isolineal $\quad x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3k \\ 0 \\ -k \end{bmatrix} = \begin{bmatrix} 3k - 3k \\ 3k - 3k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

- $\Phi: P_2(\mathbb{R}) \to \mathbb{R}^3 \quad \Phi(p) = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$ → isomorfismo

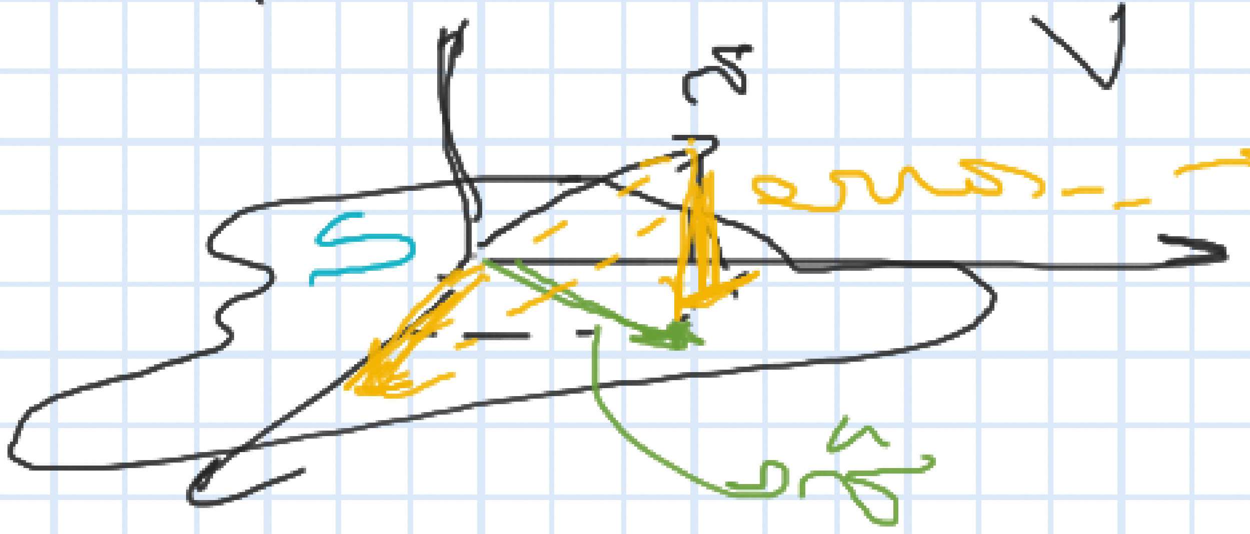
  $(p = a_0 + a_1 x + a_2 x^2)$

  $\Phi(\lambda p + \alpha q) = \lambda \Phi(p) + \alpha \Phi(q)$

- $\Phi: \mathbb{R}^3 \to \mathbb{R}^3 \quad \Phi(x) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} x$ endomorfismo

- $\Phi: \mathbb{R}^3 \to \mathbb{R}^3 \quad \Phi(x) = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}}_{\text{es inversible}} x \to$ automorfismo

$\pi^2 = \pi \circ \pi$



$x - \hat{x} \perp S$

EV de var. aleatorios $\mathbb{E}[X^2] < \infty$

↳ Es un EV con P.I (cuerpo en $\mathbb{R}$)

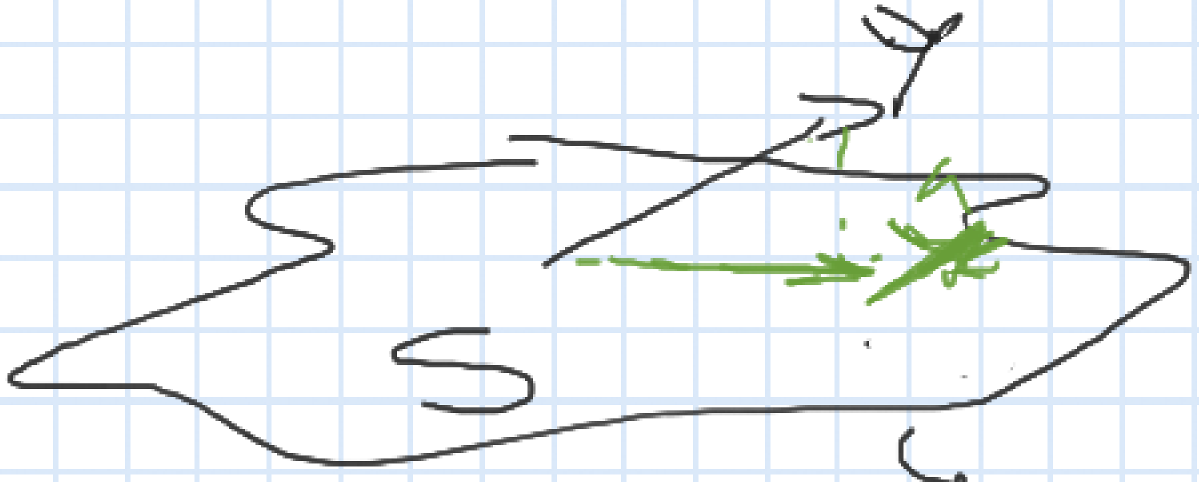
$\langle X, Y\rangle = \mathbb{E}[X \cdot Y]$

$\|(Y-\hat{Y})\|^2 = \langle (Y-\hat{Y}), (Y-\hat{Y})\rangle$

dist $= \mathbb{E}[(Y-\hat{Y})^2]$

↳ error cuadrático medio

$\hat{Y}$ va a ser la proy $\perp$



↳ $aX+b$

$\langle Y-\hat{Y}, s\rangle = 0 \ \forall s \in S$

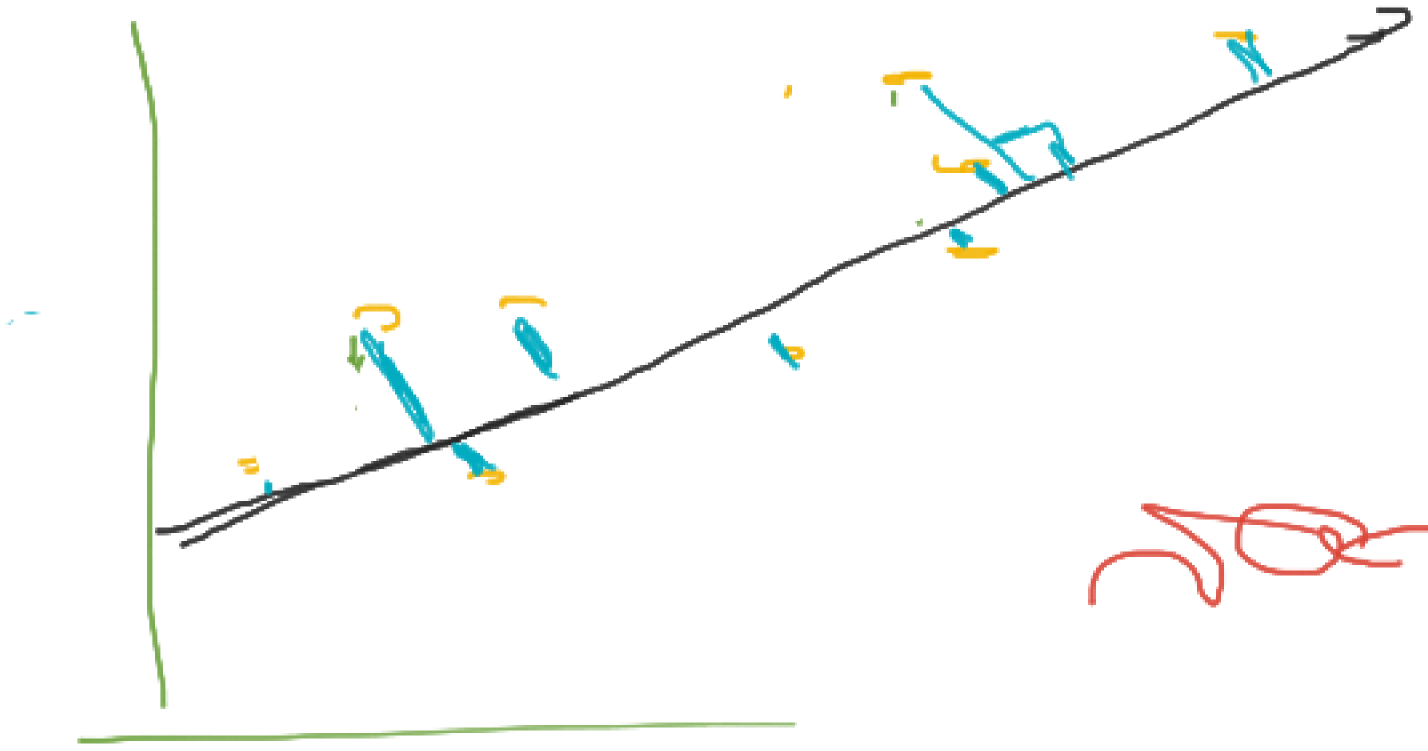
$S = \{1, X\}$

$\langle Y-\hat{Y}, 1\rangle = 0 = \langle Y-(aX+b), 1\rangle = \mathbb{E}[Y - aX - b] = \mathbb{E}[Y] - a\mathbb{E}[X] - b$

↳ $b = \mathbb{E}[Y] - a\mathbb{E}[X]$

$\langle Y-\hat{Y}, X\rangle = 0 = \langle Y-(aX+b), X\rangle = \mathbb{E}[YX] - a\mathbb{E}[X^2] - b\mathbb{E}[X]$

$\mathbb{E}[YX] - a\mathbb{E}[X^2] - \mathbb{E}[Y]\mathbb{E}[X] + a\mathbb{E}[X]^2 =$



$$\mathrm{var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\mathrm{cov}(X,Y) - a\,\mathrm{var}(X) = 0$$

$$\hookrightarrow a = \frac{\mathrm{cov}(X,Y)}{\mathrm{var}(X)}$$ Recta de regresión

$$b = \mathbb{E}[Y] - \frac{\mathrm{cov}(X,Y)}{\mathrm{var}(X)}\,\mathbb{E}[X]$$

$$Y = \frac{\mathrm{cov}(X,Y)}{\mathrm{var}(X)}\left(X - \mathbb{E}[X]\right) + \mathbb{E}[Y]$$

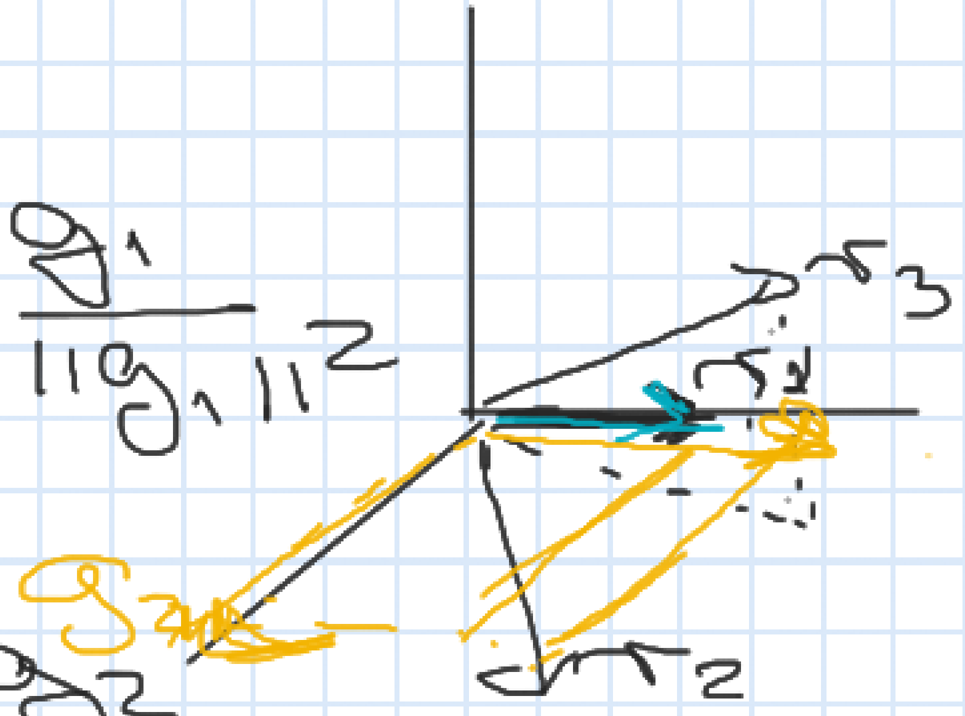
Proceso de Gram-Schmidt

Sea V e.v. con P.I. $\langle \cdot , \cdot \rangle$

base $B = \{v_1, \dots, v_n\} \rightarrow \tilde{B} = \{\tilde{v}_1, \dots, \tilde{v}_n\}$ que sea BON.

$$g_1 = v_1 \rightarrow \tilde{v}_1 = \frac{g_1}{\|g_1\|}$$

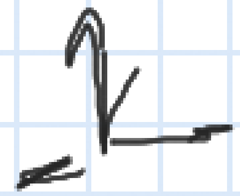
$$g_2 = v_2 - \langle v_2, \underbrace{\frac{g_1}{\|g_1\|}}_{\tilde{v}_1} \rangle \underbrace{\frac{g_1}{\|g_1\|}}_{\tilde{v}_1} = v_2 - \langle v_2, g_1 \rangle \frac{g_1}{\|g_1\|^2}$$



$$g_3 = v_3 - \langle v_3, g_1 \rangle \frac{g_1}{\|g_1\|^2} - \langle v_3, g_2 \rangle \frac{g_2}{\|g_2\|^2}$$

$\vdots$

$$g_n = v_n - \sum_{i=1}^{n-1} \langle v_n, g_i \rangle \frac{g_i}{\|g_i\|^2}$$



$$\tilde{v}_i = \frac{g_i}{\|g_i\|^2}$$