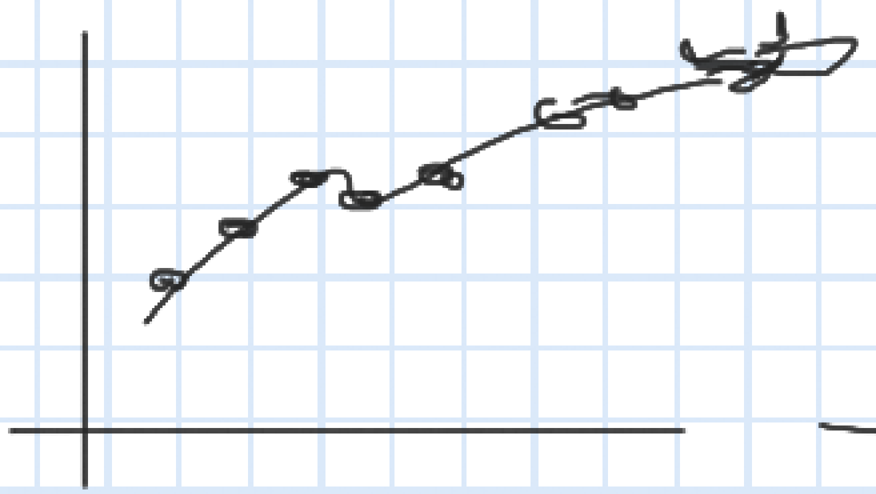


Supongo aceleración constante

$$v(t) = \dot{p}(t) \qquad a(t) = \dot{v}(t)$$

Taylor: $$f(t) = f(t_0) + \frac{f'(t_0)}{1!}(t-t_0) + \frac{f''(t_0)}{2!}(t-t_0)^2 + O(3)$$

frec de muestreo

$$\begin{cases} p(t_k) = p(t_{k-1}) + v(t_{k-1})\,T + \dfrac{a(t_{k-1})}{2}T^2 \\ v(t_k) = v(t_{k-1}) + a(t_{k-1})\,T \\ a(t_k) = a(t_{k-1}) \end{cases}$$

$$\underbrace{\begin{bmatrix} p(t_k) \\ v(t_k) \\ a(t_k) \end{bmatrix}}_{x_k} = \underbrace{\begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}}_{A_k} \underbrace{\begin{bmatrix} p(t_{k-1}) \\ v(t_{k-1}) \\ a(t_{k-1}) \end{bmatrix}}_{x_{k-1}} + w_{k-1}$$

$$P(t_k) : \begin{bmatrix} P_{t_k}(\check{x}) \\ P_{t_k}\check{y} \\ P_{t_k}\check{z} \end{bmatrix}$$

$$w_k \sim N(0, \Sigma)$$

$y_{t_k} = p(t_k) + v_k$ (ruido posición)

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p(t_k) \\ v(t_k) \\ a(t_k) \end{bmatrix} + v_k$$

(ruido posición y velocidad)

$$y(t_k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p(t_k) \\ v(t_k) \\ a(t_k) \end{bmatrix} + v_k$$

$$a x_1 + b = y_1 \to (x_i, y_i) \quad i = 1 \dots m$$

$$\vdots$$

$$a x_m + b = y_m$$

$$\underbrace{\begin{bmatrix} x_1 & 1 \\ \vdots & \\ x_m & 1 \end{bmatrix}}_{\substack{X \\ m \times 2}} \begin{bmatrix} a \\ b \end{bmatrix} = \overbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}}^{y}$$

$$X^T X \begin{bmatrix} a \\ b \end{bmatrix} = X^T \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$$

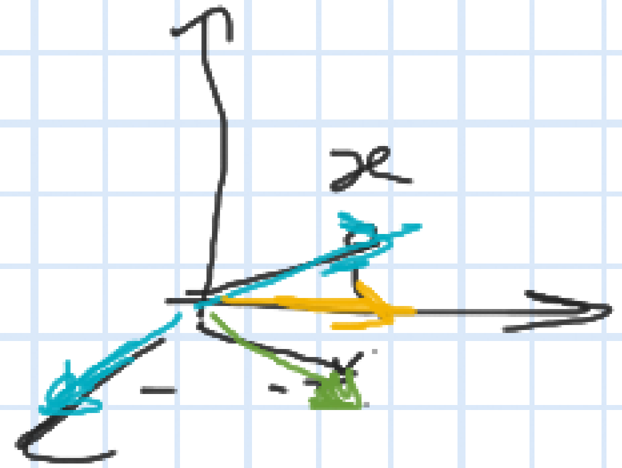
$$\to \hat{y} = \underbrace{X \overbrace{(X^T X)^{-1} X^T y}^{\begin{bmatrix} a \\ b \end{bmatrix}}}_{\text{proj.}} \to y$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = (X^T X)^{-1} X^T \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$$

$$\hat{x}_{k|k} = \sum_{i=1}^{k} \alpha_i \, y_i$$

$S_k = \{y_0, y_1, \dots y_k\}$

busco proy de x_k sobre S_k



$(1, 0, 0)$

$(0, 1, 0)$

$\langle x, (1,0,0) \rangle \, (1,0,0)$

Será cierto que $\hat{x}_{k+1|k} = A_k \hat{x}_{k|k}$ es la

$\hookrightarrow$ proy $\perp$ de x_{k+1} sobre S_k

proy $\perp$ de x_{k+1} sobre $S_k = \text{gen}\{y_0 \dots y_k\}$

def. de proy

$$\langle x_{k+1} - A_k \hat{x}_{k|k}, y_m \rangle = 0, \quad m = 0, \dots, k$$

$$\langle \underline{A_k} x_k + B_k v_k - \underline{A_k} \hat{x}_k, y_m \rangle$$

$$\langle A_k (x_k - \hat{x}_{k|k}), y_m \rangle + \underbrace{\langle B_k v_k, y_m \rangle}_{0} = 0$$

$\perp$

$\Rightarrow \hat{x}_{k+1|k} = A_k \hat{x}_{k|k}$ es la proy $\perp$ de x_{k+1} sobre S_k

$$P_{k|k} \| x_k - \hat{x}_{k|k} \|^2 = \langle (x_k - \hat{x}_{k|k}), (x_k - \hat{x}_{k|k}) \rangle$$

$$P_{k+1|k} = \langle (\underset{\text{r.a}}{x_{k+1}} - \underset{\text{r.a}}{\hat{x}_{k+1|k}}), (x_{k+1} - \hat{x}_{k+1|k}) \rangle$$

$$= \mathbb{E}\left[(\underset{A_k x_k + B_k v_k}{x_{k+1}} - \underbrace{\hat{x}_{k+1|k}}_{A_k \hat{x}_{k|k}})(x_{k+1} - \hat{x}_{k+1|k})^T \right]$$

$$= \mathbb{E}\left[A_k (x_k - \hat{x}_{k|k})(A_k (x_k - \hat{x}_{k|k}))^T \right] +$$

$$+ \mathbb{E}\left[\underbrace{B_k v_k}\,\underbrace{(A_k (x_k - \hat{x}_{k|k}))^T}_{=0} \right) + 0 + \ldots$$

$$+ \mathbb{E}\left[B v_k (B v_k)^T \right]$$

$$P_{k+1|k} = A_k \underbrace{\mathbb{E}\left[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T\right]}_{P_{k|k}} A_k^T + B_k \underbrace{\mathbb{E}\left[v_k v_k^T\right]}_{Q_k} B_k^T$$

$$= A_k P_k A_k^T + B_k Q_k B_k^T$$

Immersiones $B\{y_0, \dots, y_k\} \to B' = \{\tilde{e}_1, \dots, \tilde{e}_k\}$

$e_0 = y_1$

$e_1 = y_1 - \langle y_1, e_0 \rangle \frac{e_0}{\|e_0\|^2}$ (mismo punto que G-S)

$\vdots$

$e_k = y_k - \sum_{m=0}^{k-1} \langle y_k, e_m \rangle \frac{e_m}{\|e_m\|^2}$

$\tilde{e}_0 = e_0 / \|e_0\|, \dots, \tilde{e}_k = e_k / \|e_k\| \to B'$ es BON.

$S_k = \mathrm{gen}\{y_0, \dots, y_k\} = \mathrm{gen}\{\tilde{e}_0, \dots, \tilde{e}_k\}$

$y_0 \to \hat{x}_{1|1} \to \hat{x}_{2|1} = A_k \hat{x}_{1|1}$

$y_2 \to \hat{x}_{2|2} \to \hat{x}_{3|2} = A_k \hat{x}_{2|2}$

$$\hat{x}_{k+1|k+1} = \sum_{m=0}^{k+1} \langle x_{k+1}, e_m \rangle \frac{e_m}{\|e_m\|^2} = \underbrace{\sum_{m=0}^{k} \langle x_{k+1}, e_m \rangle \frac{e_m}{\|e_m\|^2}}_{\hat{x}_{k+1|k}} + \langle x_{k+1}, e_{k+1} \rangle \frac{e_{k+1}}{\|e_{k+1}\|^2}$$

$x_k \quad \tilde{e}_k$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + \langle x_{k+1}, e_{k+1} \rangle \frac{e_{k+1}}{\|e_{k+1}\|^2}$$

$$\langle e_m, e_m \rangle = \langle e_m, y_k$$

$$e_m = y_m - \underbrace{\sum_{e=1}^{m-1} \langle y_m, e_e \rangle \frac{e_e}{\|e_e\|^2}}$$

$$\underbrace{C_m \hat{x}_{m|m-1}} \in S_{m-1}$$

"lo que esperaba recibir en el instante m dado las medic. hasta el instante m-1."

$$\langle e_m, e_m \rangle = \langle \overbrace{y_m}^{C_m x_m + v_m} - C_m \hat{x}_{m|m-1}, y_m - C_m \hat{x}_{m|m-1} \rangle$$

$$= \langle C_m (x_m - \hat{x}_{m|m-1}) + v_m, C_m (x_m - \hat{x}_{m|m-1}) + v_m \rangle$$

$$\langle e_m, e_m\rangle = C_m \underbrace{\langle x_m - \hat{x}_{m|m-1}, x_m - \hat{x}_{m|m-1}\rangle}_{P_{m|m-1}} C_m^T +$$

$$+ \underbrace{\langle v_m, v_m\rangle}_{R_k} + C_m \cancel{\langle x_m - \hat{x}_{m|m-1}, v_m\rangle}^{0}$$

$$\cancel{\langle v_m, x_m - \hat{x}_{m|m-1}\rangle}^{0} C_m^T$$

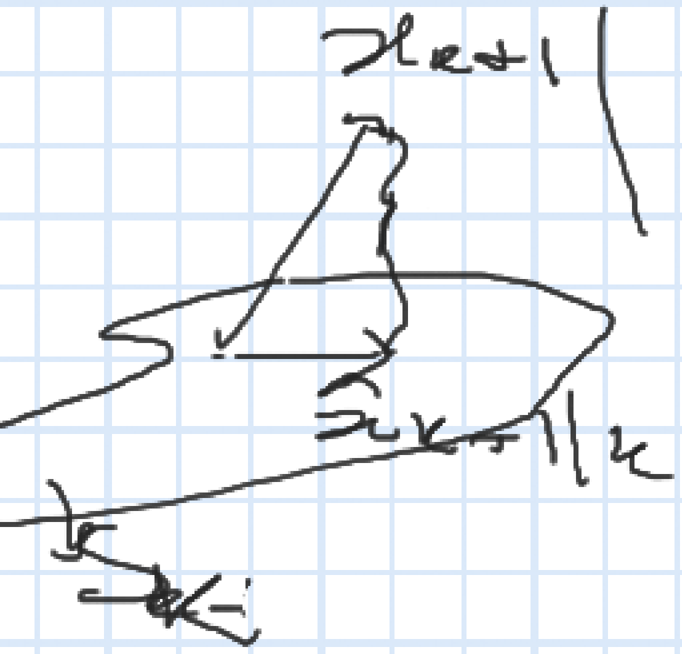
$$\langle e_m, e_m\rangle = C_m P_{m|m-1} C_m^T + R_k$$

(ganancia de Kalman) K_{k+1}

$$\Rightarrow \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + \overbrace{\langle x_{k+1}, e_{k+1}\rangle (C_{k+1} P_{k+1|k} C_{k+1}^T + R)^{-1}}^{K_{k+1}} \underbrace{(y_{k+1} - C_{k+1}\hat{x}_{k+1|k})}_{e_{k+1}}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_{k+1} - C_k \hat{x}_{k+1|k})$$

$$\langle x_{k+1}, e_{k+1} \rangle = \langle x_{k+1}, \underbrace{y_{k+1}}_{C_{k+1}x_{k+1}+v_{k+1}} - C_{k+1}\hat{x}_{k+1|k})$$



$$= \langle x_{k+1}, C_{k+1}(x_{k+1} - \hat{x}_{k+1|k}) + v_{k+1} \rangle$$

$$= \langle x_{k+1}, (x_{k+1} - \hat{x}_{k+1|k}) \rangle C_{k+1}^T + \underbrace{\langle x_{k+1}, v_{k+1} \rangle}_{=0}$$

$$\langle \underbrace{\hat{x}_{k+1|k}}_{\in S_k}, \underbrace{x_k - \hat{x}_{k|k-1}}_{\in S_k^\perp} \rangle = 0 \quad + (*)$$

$$(*) \quad \langle x_{k+1}, e_{k+1} \rangle = \langle x_{k+1}, (x_{k+1} - \hat{x}_{k+1|k}) \rangle C_{k+1}^T + \langle \hat{x}_{k+1|k}, x_{k+1} - \hat{x}_{k+1|k} \rangle C_{k+1}^T$$

$$= \overbrace{\langle x_{k+1} - \hat{x}_{k+1|k}, x_{k+1} - \hat{x}_{k+1|k} \rangle}^{P_{k+1|k+1}} C_{k+1}^T$$

$$\langle x_{k+1}, e_{k+1} \rangle = \overline{P_{k+1|k+1}}\, C_{k+1}^T$$

$$\Rightarrow h_m = \overline{P_{k+1|k+1}}\, \overline{C_{k+1}^T} \left(C_m \overline{P_{k+1|k}}\, C_m^T + R_k \right)^{-1}$$