The Comparison Test

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Evaluating a Limit

1) $\sum_{n=1}^{\infty}\frac{1}{n^3}$ $\sum_{n=1}^{\infty}\frac{3n^2}{n^5+n^3+1}$

```
syms n
limit(((1/n^3)/((3*n^2)/(n^5+n^3+1))),n,inf)
```

ans =

$$\frac{1}{3}$$

2) $\sum_{n=1}^{\infty}\frac{1}{n^4}$ $\sum_{n=1}^{\infty}\frac{4n+37}{7n^5+6n+2}$

```
limit((1/n^4)/((4*n+37)/(7*n^5+6*n+2)),n,inf)
```

ans =

$$\frac{7}{4}$$

3) $\sum_{n=0}^{\infty}\frac{3^n}{4^n}$ $\sum_{n=0}^{\infty}\frac{3^n-1}{4^n+1}$

```
limit((3^n/4^n)/((3^n-1)/(4^n+1)),n,inf)
```

ans = 1

4) $\sum_{n=0}^{\infty}\frac{1}{9^n}$ $\sum_{n=0}^{\infty}\frac{2^n-3}{18^n+3}$

```
limit((1/9^n)/((2^n-3)/(18^n+3)),n,inf)
```

ans = 1

5) $\sum_{n=0}^{\infty} \frac{n^2}{e^n}$ $\sum_{n=0}^{\infty} \frac{3n^2+n+1}{e^n+1}$

```
limit((n^2/exp(n))/((3*n^2+n+1)/(exp(n+1))),n,inf)
```

```
ans =
```

$$\frac{e}{3}$$

Finding The Sum of a Series

1) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ $\sum_{n=1}^{\infty} \frac{3n^2}{n^5+n^3+1}$

I found that their sums were not 3 times (or $\frac{1}{3}$) the value of each other.

```
a1 = double(symsum(1/n^3,n,1,inf))
```

```
a1 = 1.2021
```

```
a2 = double(symsum(3*n^2/(n^5+n^3+1),n,1,inf))
```

```
a2 =
   1.5079 + 0.0000i
```

```
a1/a2
```

```
ans =
   0.7972 - 0.0000i
```

2) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $\sum_{n=1}^{\infty} \frac{4n+37}{7n^5+6n+2}$

I found that their sums did not have a ratio of $\frac{7}{4}$

```
b1 = double(symsum(1/n^4,n,1,inf))
```

```
b1 = 1.0823
```

```
b2 = double(symsum(((4*n+37)/(7*n^5+6*n+2)),n,1,inf))
```

```
b2 =
   2.9634 - 0.0000i
```

```
b1/b2
```

```
ans =
   0.3652 + 0.0000i
```

3) $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$ $\sum_{n=0}^{\infty} \frac{3^n - 1}{4^n + 1}$

I found that their sums did not have a ratio of 1

```
c1 = double(symsum((3^n/4^n),n,1,inf))
```

```
c1 = 3
```

```
c2 = double(symsum(((3^n-1)/(4^n+1)),n,1,inf))
```

```
c2 = 2.5295
```

```
c1/c2
```

```
ans = 1.1860
```

4) $\sum_{n=0}^{\infty} \frac{1}{9^n}$ $\sum_{n=0}^{\infty} \frac{2^n - 3}{18^n + 3}$

I found that their sums did not have a ratio of 1

```
d1 = double(symsum((1/9^n),n,1,inf))
```

```
d1 = 0.1250
```

```
d2 = double(symsum(((2^n-3)/(18^n+3)),n,1,inf))
```

```
d2 = -0.0436
```

```
d1/d2
```

```
ans = -2.8694
```

5) $\sum_{n=0}^{\infty} \frac{n^2}{e^n}$ $\sum_{n=0}^{\infty} \frac{3n^2+n+1}{e^n+1}$

I foudn that their sums did not have ratio of $\frac{e}{3}$

```
e1 = double(symsum((n^2/exp(n)),n,1,inf))
```

```
e1 = 1.9923
```

```
e2 = double(symsum(((3*n^2+n+1)/(exp(n+1))),n,1,inf))
```

```
e2 = 2.7516
```

```
e1/e2
```

```
ans = 0.7241
```

Tail Ratios

1) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ $\sum_{n=1}^{\infty} \frac{3n^2}{n^5+n^3+1}$

I found that the ratio of their sums were not $\frac{1}{3}$ exactly but was a very very close estimation..

```
a1 = double(symsum(1/n^3,n,100,inf))
```

```
a1 = 5.0502e-05
```

```
a2 = double(symsum(3*n^2/(n^5+n^3+1),n,100,inf))
```

```
a2 = 1.5150e-04
```

```
a1/a2
```

```
ans = 0.3334
```

2) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $\sum_{n=1}^{\infty} \frac{4n+37}{7n^5+6n+2}$

I found that their sums did not have a ratio of $\frac{7}{4}$ exactly, but the ratio which they produced was very close to the actual some of the series.

```
b1 = double(symsum(1/n^4,n,100,inf))
```

```
b1 = 3.3837e-07
```

```
b2 = double(symsum(((4*n+37)/(7*n^5+6*n+2)),n,100,inf))
```

```
b2 = 2.0683e-07
```

```
b1/b2
```

```
ans = 1.6359
```

3) $\sum_{n=0}^{\infty} \frac{3^n}{4^n} \qquad \sum_{n=0}^{\infty} \frac{3^n - 1}{4^n + 1}$

I found that their sums did have a ratio of 1.

```
c1 = double(symsum((3^n/4^n),n,100,inf))
```

```
c1 = 1.2829e-12
```

```
c2 = double(symsum(((3^n-1)/(4^n+1)),n,100,inf))
```

```
c2 = 1.2829e-12
```

```
c1/c2
```

```
ans = 1
```

4) $\sum_{n=0}^{\infty} \frac{1}{9^n} \qquad \sum_{n=0}^{\infty} \frac{2^n - 3}{18^n + 3}$

I found that their sums did have a ratio of 1.

```
d1 = double(symsum((1/9^n),n,100,inf))
```

```
d1 = 4.2355e-96
```

```
d2 = double(symsum(((2^n-3)/(18^n+3)),n,100,inf))
```

```
d2 = 4.2355e-96
```

```
d1/d2
```

```
ans = 1
```

5) $\sum_{n=0}^{\infty} \frac{n^2}{e^n}$ $\sum_{n=0}^{\infty} \frac{3n^2+n+1}{e^n+1}$

I found that their tail sums did not have ratio of $\frac{e}{3}$ exactly, but it was far closer than the standard method of finding the ratio.

```
e1 = double(symsum((n^2/exp(n)),n,100,inf))
```

```
e1 = 5.9543e-40
```

```
e2 = double(symsum(((3*n^2+n+1)/(exp(n+1))),n,100,inf))
```

```
e2 = 6.5934e-40
```

```
e1/e2
```

```
ans = 0.9031
```

Summary

Overall, I found that when evaluating a limit only works for certain series' and will more than likely become more complicated than putting together ikea furniture. After finding the instructions , they are mildely nice. I observed that when evaluating a limit, we were able to get a far more accurate if not 100% accurate as to what was going on in that function. Whereas when we look at the data recieved by us using the "sum of a series" service, we found that because of the low starting number, the resulting amount was massively thrown off by it. We then noticed that when taking the sum of the tail end of a series, we were able to skip the more complex topics and "larger" things might effect the function in the long run.