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Lista Matemática Aplicada – Transformada Z

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1) Determine as transformadas z das seguintes funções:

a) $x(k) = -2u(k) + 0{,}7^k\, u(k)$

2.	$u[n]$	$\frac{1}{1-z^{-1}}$
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$

$$x[k] = -2 * \frac{1}{1-z^{-1}} + \frac{1}{1-0{,}7z^{-1}}$$

$$x[k] = -\frac{2}{1-z^{-1}} + \frac{1}{1-0{,}7z^{-1}}$$

b) $x(k) = u(k-2) + \delta(k-1)$

2.	$u[n]$	$\frac{1}{1-z^{-1}}$

$$x[n-k] \overset{Z}{\longleftrightarrow} z^{-k}X(z).$$

$$x\,[k] = z^{-2} * = -\frac{1}{1-z^{-1}} + z^{-k}$$

$$x\,[k] = \frac{1}{z^2} * \frac{1}{1-z^{-1}} + z$$

$$x\,[k] = z + \frac{1}{z(z-1)}$$

c) $x(k) = \left(1 - 0{,}5^k\right)u(k)$

2.	$u[n]$	$\frac{1}{1-z^{-1}}$
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$

$$X[k] = 1u(k) - 0{,}5^k\, u(k)$$

$$x[k] = 1 * \frac{1}{1-z^{-1}} - \frac{1}{1-0{,}5z^{-1}}$$

$$x[k] = \frac{1}{1-z^{-1}} - \frac{1}{1-0{,}5z^{-1}}$$

d) $x(k) = 2\delta\,(k) - 3\left(0{,}5^k\, u(k)\right)$

1.	$\delta[n]$	1
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$

$$x[k] = 2 - 3 * \frac{1}{1-0{,}5z^{-1}}$$

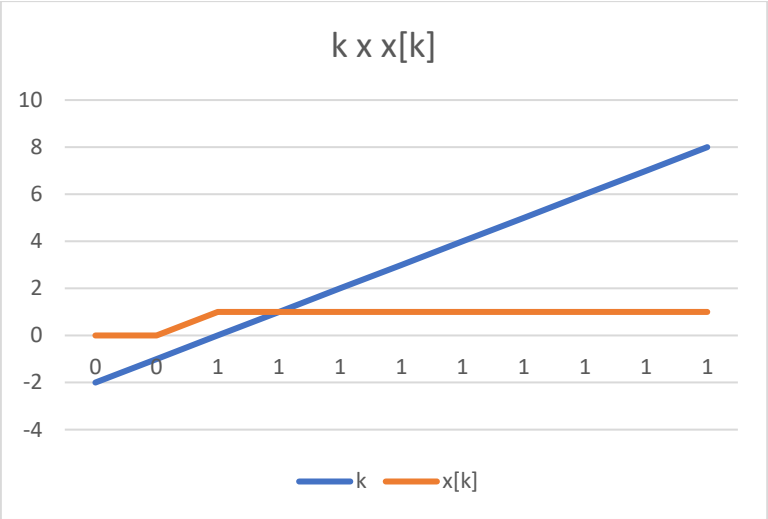
2)

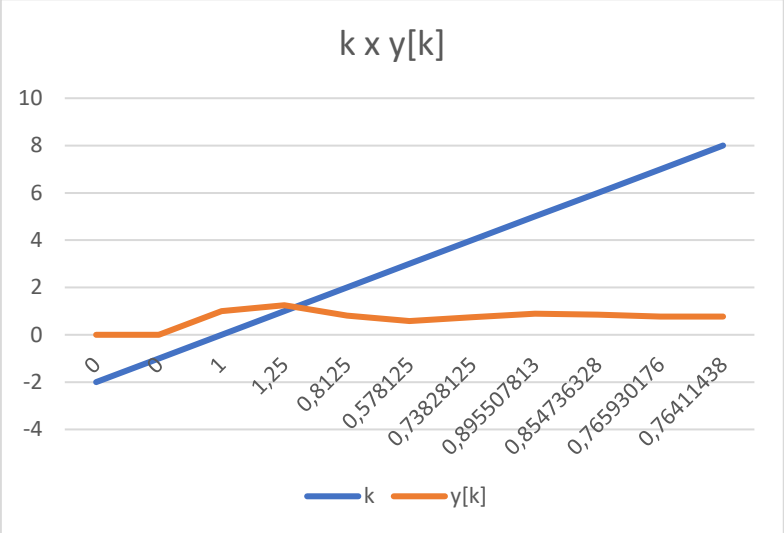
Considere um sistema discreto descrito pela seguinte equação diferença: $y(k)-\frac{1}{4}y(k-1)+\frac{1}{2}y(k-2)=x(k)$. Calcule a saída $y(k)$ para uma entrada $x(k)=u(k)$ (degrau unitário), para $-2 \le k \le 8$. Apresente os gráficos de $y(k)$ e $x(k)$ em função de k.

$$y(k) - \frac{1}{4} * y(k-1) + \frac{1}{2} * y(k-2) = x(k)$$

$$x(k) + \frac{1}{4} * y(k-1) - \frac{1}{2} * y(k-2) = y(k)$$

k	x[k]	y[k]	y[k-1]	y[k-2]
-2	0	0	0	0
-1	0	0	0	0
0	1	1	0	0
1	1	1,25	1	0
2	1	0,8125	1,25	1
3	1	0,578125	0,8125	1,25
4	1	0,738281	0,578125	0,8125
5	1	0,895508	0,738281	0,578125
6	1	0,854736	0,895508	0,738281
7	1	0,76593	0,854736	0,895508
8	1	0,764114	0,76593	0,854736





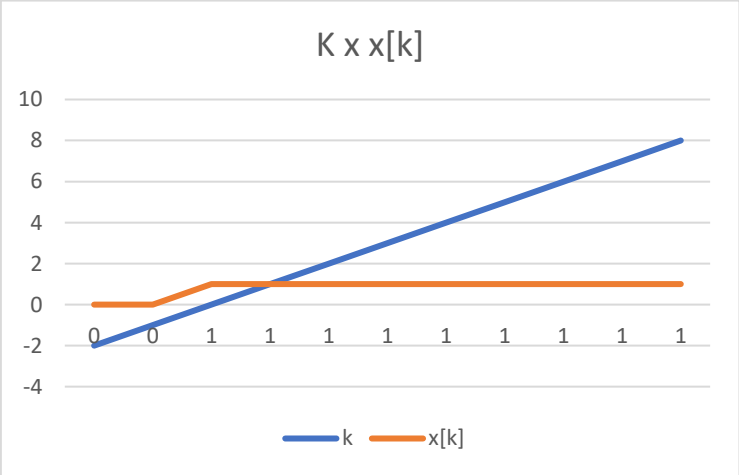
3) Repita o exercício 2 para as seguintes equações diferença:

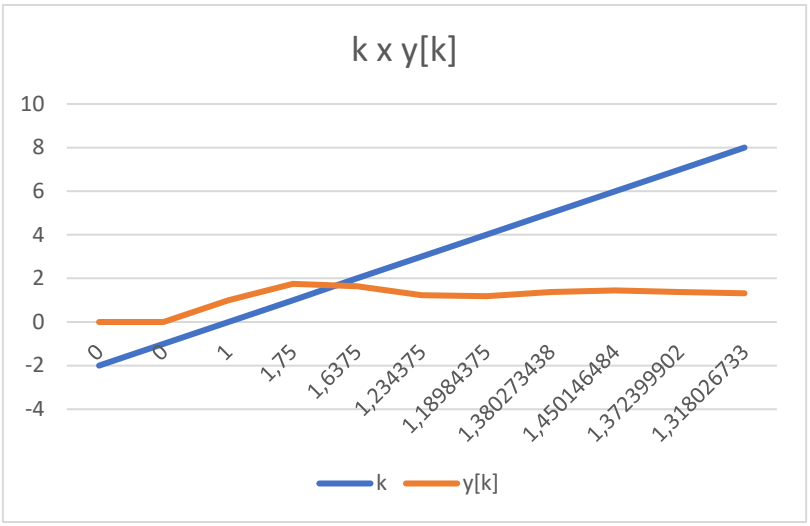
a) $y(k)-\frac{1}{4}y(k-1)+\frac{1}{2}y(k-2)=x(k)+\frac{1}{2}x(k-1)+\frac{1}{5}x(k-2)$.

b) $y(k)=0,2.x(k)+0,3.x(k-1)+0,3.x(k-2)+0,2.x(k-3)$

a) $y(k) = x(k) + \frac{1}{2}\,x(k-1) + \frac{1}{5}x(k-2) + \frac{1}{4}\,y(k-1) - \frac{1}{2}\,y(k-2)$

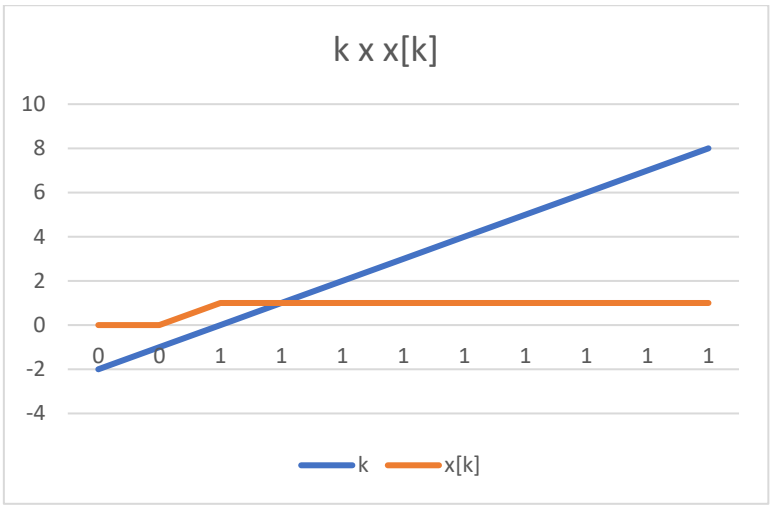
k	x[k]	x[k-1]	x[k]-2	y[k]	y[k-1]	y[k-2]
-2	0	0	0	0	0	0
-1	0	0	0	0	0	0
0	1	0	0	1	0	0
1	1	1	0	1,75	1	0
2	1	1	1	1,6375	1,75	1
3	1	1	1	1,234375	1,6375	1,75
4	1	1	1	1,189844	1,234375	1,6375
5	1	1	1	1,380273	1,189844	1,234375
6	1	1	1	1,450146	1,380273	1,189844
7	1	1	1	1,3724	1,450146	1,380273
8	1	1	1	1,318027	1,3724	1,450146

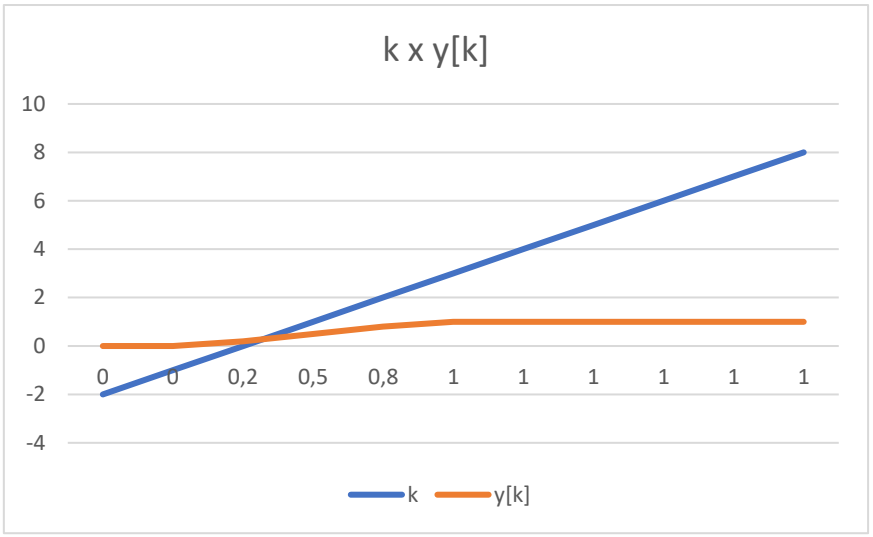




b) $y(k) = 0{,}2x(k) + 0{,}3x(k-1) + 0{,}3x(k-2) + 0{,}2x(k-3)$

k	x[k]	x[k-1]	x[k]-2	x[k-3]	y[k]
-2	0	0	0	0	0
-1	0	0	0	0	0
0	1	0	0	0	0,2
1	1	1	0	0	0,5
2	1	1	1	0	0,8
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	1	1	1	1	1
8	1	1	1	1	1





4) Determine a função de transferência e os pólos/zeros dos sistemas discretos modelados pelas seguintes equações diferença:

a) $y(k)+\frac{1}{4}y(k-1)=x(k)-\frac{1}{2}x(k-1)$

b) $y(k)+\frac{4}{3}y(k-1)-\frac{1}{2}y(k-2)=-2x(k)$

a)

$$y(k)+\frac{1}{4}*y(k-1)=x(k)-\frac{1}{2}*x\,(k-1)$$

$$Y(Z)+\frac{1}{4}*Z^{-1}*Y(Z)=Y(Z)-\frac{1}{2}*Z^{-1}*X(Z)$$

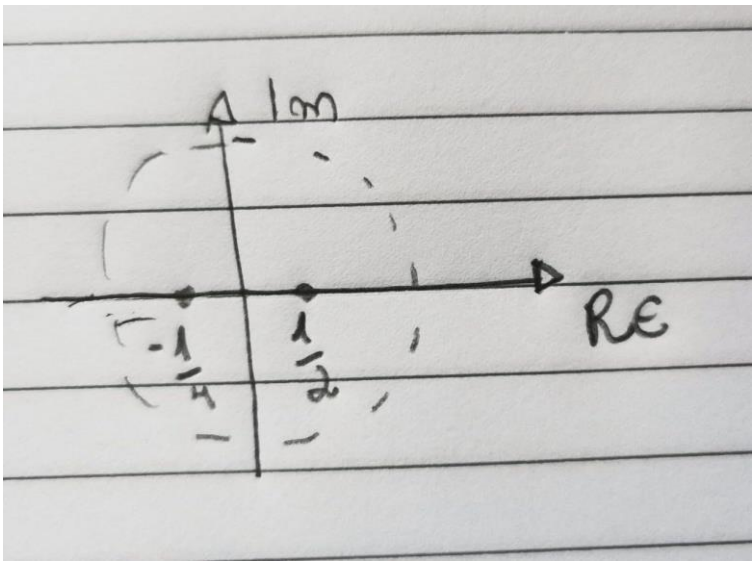
$$1+\frac{1}{4}*Z^{-1}*Y(Z)=1-\frac{1}{2}*Z^{-1}*X(Z)$$

$$\frac{Y(Z)}{X(Z)}=\frac{1-\frac{1}{2}*Z^{-1}}{1+\frac{1}{4}*Z^{-1}}$$

$$H(z)=\frac{Y(Z)}{X(Z)}=\frac{Z-\frac{1}{2}}{Z+\frac{1}{4}}$$

Zero: $\frac{1}{2}$

Polo: $-\frac{1}{4}$



b)

$$y(k) + \frac{4}{3}\, y\,(k-1) - \frac{1}{2}\, y(k-2) = -2x(k)$$

$$Y(Z) + \tfrac{4}{3}\, Z^{-1} - \tfrac{1}{2}\, Z^{-2} * Y(Z) = -\,2\mathrm{X(Z)}$$

$$(1 + \tfrac{4}{3}\, Z^{-1} - \tfrac{1}{2}\, Z^{-2}) * Y(Z) = -2\, X(Z)$$

$$\frac{Y(Z)}{X(Z)} = \frac{-2}{1 + \frac{4}{3}\, Z^{-1} - \frac{1}{2}\, Z^{-2}}$$

$$H(Z) = \frac{-2\, z^2}{z^2 + \frac{4}{3}\, Z - \frac{1}{2}}$$

Zeros:

$$-2Z^2 = 0$$

$$-2 = Z^2$$

$$2 = -Z^2$$

$$Z^2 = 1{,}41$$

Pólos:

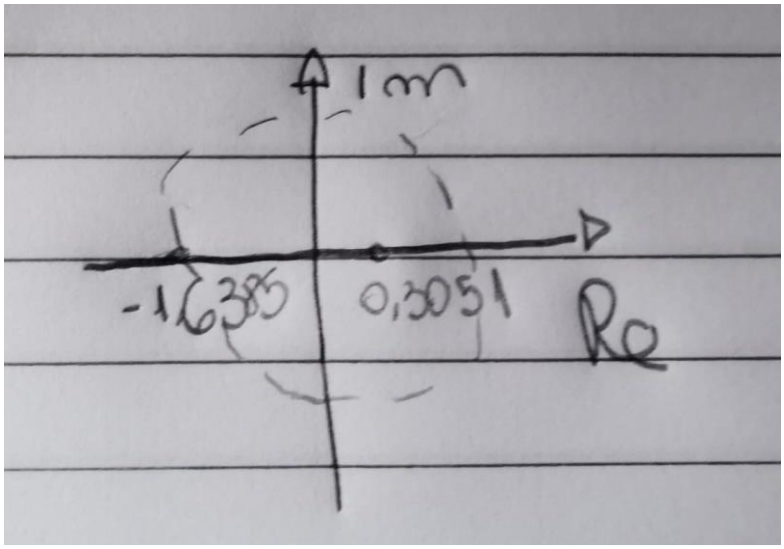
$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$

$$x = \frac{-\frac{4}{3} \pm \sqrt{\frac{4^2}{3}} - 4 * 1 * -\frac{1}{2}}{2 * 1}$$

$$x = 0{,}3051$$

$$x' = -1{,}6385$$



5) Repita o exercício 4 para as equações diferença do exercício 3

a) $y(k) - \frac{1}{4}y(k-1) + \frac{1}{2}y(k-2) = x(k) + \frac{1}{2}x(k-1) + \frac{1}{5}x(k-2)$.

b) $y(k) = 0{,}2.x(k) + 0{,}3.x(k-1) + 0{,}3.x(k-2) + 0{,}2.x(k-3)$

a) $Y(Z) - \frac{1}{4}Z^{-1} + \frac{1}{2}Z^{-2} * Y(Z) = X(Z) + \frac{1}{2}Z^{-1} + \frac{1}{5}Z^{-2} * \mathrm{X(Z)}$

$$1 - \frac{1}{4}Z^{-1} + \frac{1}{2}Z^{-2} * Y(Z) = 1 + \frac{1}{2}Z^{-1} + \frac{1}{5}Z^{-2} * X(Z)$$

$$\frac{Y(Z)}{X(Z)} = \frac{1 + \frac{1}{2}Z^{-1} + \frac{1}{5}Z^{-2}}{1 - -\frac{1}{4}Z^{-1} + \frac{1}{2}Z^{-2}}$$

$$H(Z) = \frac{Z^2 + \frac{1}{2}Z + \frac{1}{5}}{Z^2 - \frac{1}{4}Z + \frac{1}{2}}$$

Zeros:

$$z^2 + \frac{1}{2}Z + \frac{1}{5}$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$

$$x = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{2}^2 - 4 * 1 * + \frac{1}{5}}}{2 * 1}$$

$$x = \frac{-\frac{1}{2} \pm \sqrt{-0{,}55}}{2 * 1}$$

$$x = -0{,}25 \ + \ 0{,}3780\, i$$

$$X = -0{,}25 \ - \ 0{,}3780\, i$$

Pólos:

$$z^2 - \frac{1}{4} Z + \frac{1}{2}$$

$$x = \frac{-b \ \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$

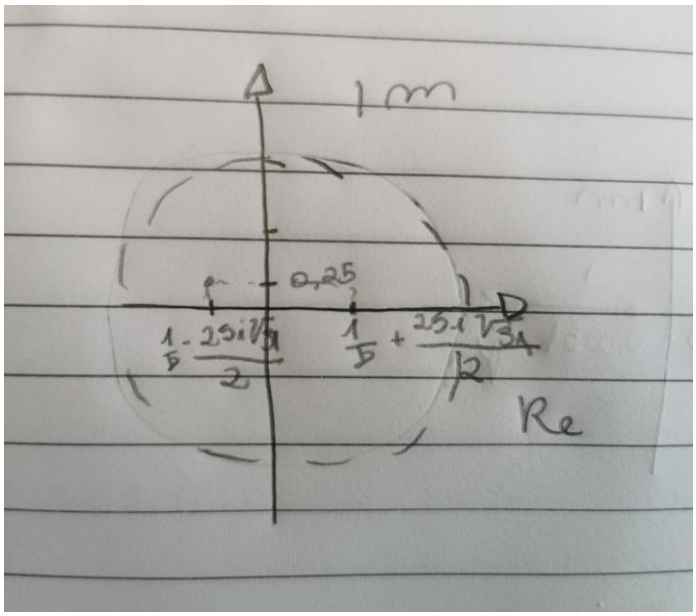
$$x = \frac{\frac{1}{4} \pm \sqrt{\frac{1}{4}^2 - 4 * 1 * + \frac{1}{2}}}{2 * 1}$$

$$x = \frac{\frac{1}{4} \pm \sqrt{-1{,}9375}}{2 * 1}$$

$$x \ = \frac{\frac{1}{4} \pm \ -\sqrt{1{,}9375}}{2 * 1}$$

$$x \ = \frac{1}{8} - \frac{i\, 25 * \sqrt{31}}{2}$$

$$x \ = \frac{1}{8} + \frac{i\, 25 * \sqrt{31}}{2}$$



a) $y(k)-\frac{1}{4}y(k-1)+\frac{1}{2}y(k-2)=x(k)+\frac{1}{2}x(k-1)+\frac{1}{5}x(k-2)$.

b) b) $y(k)=0,2.x(k)+0,3.x(k-1)+0,3.x(k-2)+0,2.x(k-3)$

$$Y(Z)=0,2\,Z+0,3Z^{-1}+0,3Z^{-2}+0,2Z^{-3}\,X(Z)$$

$$H(Z)=\frac{0,2\,Z+0,3Z^{-1}+0,3Z^{-2}+0,2Z^{-3}}{1}$$

$$H(Z)=\frac{0,2\,Z^3+0,3Z^2+0,3Z+0,2}{Z^3}$$

Zeros:

Pólos:

6) Determine e esboce no plano complexo z (desenhe também o círculo de raio unitário!) os pólos e zeros das seguintes funções de transferência:

a) $H(z)=\frac{z+0,6}{(z^2+0,6z+0,2)(z-1)}$;

b) $H(z)=\frac{z^{-1}+0,8z^{-2}}{1+z^{-1}+0,41z^{-2}}$.

a) Zeros: 0,6
Pólos:

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$

$$x = \frac{-0{,}6 \pm \sqrt{0{,}6^2 - 4 * 1 * 0{,}2}}{2 * 1}$$

$$x = \frac{-0{,}6 \pm \sqrt{-0{,}44}}{2}$$

$$X = -0{,}3 - 0{,}33166\, i$$

$$X = -0{,}3 + 0{,}33166\, i$$

b) Zeros: 0,8

Pólos:

6) Determine e esboce no plano complexo z (desenhe também o círculo de raio unitário!) os pólos e zeros das seguintes funções de transferência:

a) $H(z) = \frac{z + 0{,}6}{(z^2 + 0{,}6z + 0{,}2)(z - 1)}$;

b) $H(z) = \frac{z^{-1} + 0{,}8z^{-2}}{1 + z^{-1} + 0{,}41z^{-2}}$.

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 * 1 * 0{,}41}}{2 * 1}$$

$$x = \frac{-1 \pm \sqrt{0,64}}{2}$$

$$x = -0,5 + 0,4i$$

$$x = -0,5 - 0,4i$$