Practice problems (don't turn in):

1. [DPV] Problem 7.10 (max-flow = min-cut example)
2. [DPV] Problem 7.17 (bottleneck edges)
   (Part (e) where you devise an algorithm to find bottleneck edges is more challenging but a nice problem to try.)
3. [DPV] Problem 7.19 (verifying max-flow)
4. For a bipartite graph $G = (V_1 \cup V_2, E)$ where $|V_1| = |V_2| = n$ a *perfect matching* is a subset S of edges where each vertex is incident exactly 1 edge in S. In other words, it's a matching of size n. Given a bipartite graph G show how to determine if G has a perfect matching by a reduction to the max-flow problem. So given G define an input to the max-flow problem, and given a max-flow for this input how do you determine if the original graph G has a perfect matching or not? What is the running time of your algorithm?
   (For hints see [DPV] Chapter 7.3 (Bipartite matching) and the beginning of Problem 7.24.)

Problem 1 [DPV] Problem 7.18 (a) (b)

Solve the following problems by reducing to the original max-flow problem. In other words, explain how to solve the new flow variant problem using an algorithm for solving max-flow as a black-box. Explain how to take an input for the new problem and define an input for the original max-flow problem. Then given a max-flow f^* to this input you just defined, explain how to get the solution to the new problem.

(a) There are many sources and many sinks, and we wish to maximize the total flow from all sources to all sinks.

(b) Each vertex also has a capacity on the maximum flow that can enter it.

Answer:

Problem 2 [DPV] Problem 5.22 (a)

Prove the following property carefully:
Pick any cycle in the graph (denote it by C), and let e^* be the heaviest edge in that cycle C. Thus, $w(e^*) \geq w(e')$ for all $e' \in C$. Then there is a minimum spanning tree T' that does not contain e^*.

Hint: Take a MST T which contains e^*. Construct a new tree T' which does not contain e^* and $w(T') \leq w(T)$.
Answer:

Problem 3 [DPV] Problem 5.9 (d)

If the lightest edge in a graph is unique, then it must be part of every MST.

Is the above statement True or False?

If True, then prove that is correct (explain why it always holds). Or if it is False then give a counterexample (show a graph where it does not hold).

Answer: