Random variables

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Flip a coin n times. let X be the number of heads

X is a random variable.

X can take specific values called x

$x = \{0, 1, \ldots n\}$

ω represent an event,

for example $\{HTHHT\ldots\}$,

Claim: $|x| = n+1$

Is this claim reasonable?

Assume we flip a coin n times ($n \in \mathbb{Z}$)

let Ω the event space and $\omega \in \Omega$, a given combination of HT.

$\exists\ \omega \in \Omega \ni \omega$ is all head.

then $\exists\ x = n$

Also $\exists\ \omega \ni \omega$ is all tails.

So $\exists\ x = 0$

between 0 and n we can get any number of head, inclusive

therefore we can get n+1 values of x.

$|x| = n+1$ //

that being said, we now need to find their corresponding probability.

Claim $|\omega| = 2^n$

from the counting tecniques

when we want to find how many times we can choose n long string using only H and T

this implies, each ω has probability $\frac{1}{2^n}$.

Now we only need to find out how often each ω repeat.

this can also be obtained by counting techniques.

Claim: ω is counted $\binom{x}{n}$ times

Is this claim reasonable?

we want to count ω with h without taking into account the order.

So it is reasonable to use a combination formula.

Now we only have to justify the choices of sample and "Population".

the choice of x is straight forward, we want to count x h's

to justify n as the population, think of each ω as a string of HT so $\omega =$ HTHHHTTTH.

so for n flips, ω is a string of size n.

So for ωs with 1 H, we choose 1 in n

for ωs with 2 Hs, we choose 2 in n, and so on

Another way to look at it is how many ωs can we count with x hs w/o accounting for the order.

$\binom{x}{n}$ ωs.

So we get the following Random variable distribution

x	$P(X=x)$
0	$\frac{1}{2^n}\cdot\binom{0}{n}$
1	$\frac{1}{2^n}\cdot\binom{1}{n}$
2	$\frac{1}{2^n}\cdot\binom{2}{n}$
3	$\frac{1}{2^n}\binom{3}{n}$
$\vdots$	$\vdots$
n	$\frac{1}{2^n}\cdot\binom{x}{n}$

result

Let's try to implement this in a Computer system! C++