Classification

Machine Learning for Behavioral Data March 13, 2023



Today's Topic

Week	Lecture/Lab	
1	Introduction	
2	Data Exploration	
3	Regression	
4	Classification	
5	Model Evaluation	
6	Time Series Prediction	
6 7	Time Series Prediction Time Series Prediction	

Complete pipeline for one use case:

- Data exploration
- Prediction
- Model evaluation

Getting ready for today's lecture...

- If not done yet: clone the repository containing the Jupyter notebook and data for today's lecture into your Noto workspace
- SpeakUp room for today's lecture:

https://go.epfl.ch/speakup-mlbd

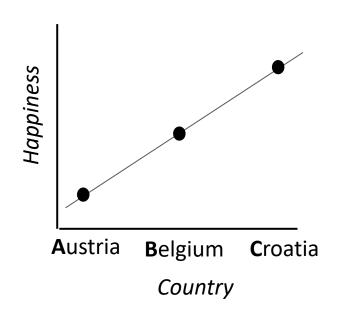


Short quiz about the past...

 [Exploration] Based on the provided graph, what can you say about the relationship between country and happiness?



- a) Happiness increases with country
- b) Happiness decreases with country
- c) It is not possible to compute a correlation between country and happiness



Short quiz about the past...

[Regression] Which GLM family should you use when the output variable is continuous?

- a) Binomial
- b) Poisson
- c) Gaussian

```
SpeakUp Poll!

Outline

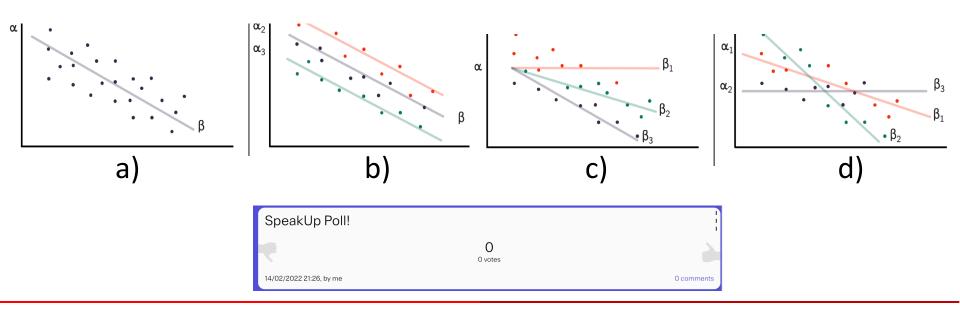
Outline

14/02/2022 21:26, by me

Ocomments
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Short quiz about the past...

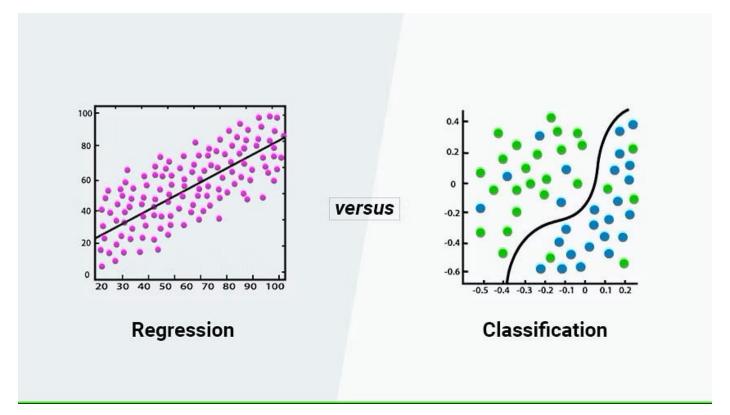
[Regression] Which of the following are examples of models with **only** fixed effects? In the plots, α denotes intercept, β denotes slope.



Idea

- In classification, a single aspect of the data (predicted variable) is modeled by some combination of other aspects of the data (features)
 - The predicted variable is categorical (set of classes)
 - Examples:
 - Prediction of dropout in massive open online courses (binary)
 - Exploration of user categories in a simulation (multiclass)

Classification



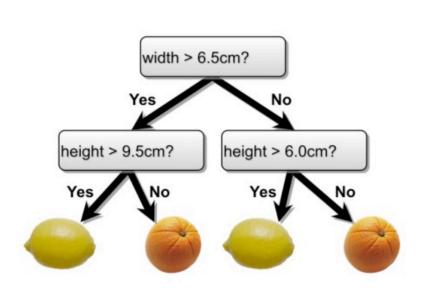
Today's Use Case: Flipped Classroom Course

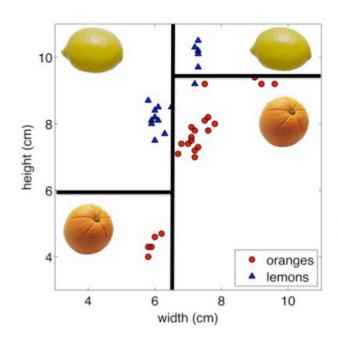
- Participants: 288 EPFL students of a course taught in *flipped* classroom mode with a duration of 10 weeks
- Structure:
 - Preparation: watch videos (and solve simple quizzes) on new
 content at home as a preparation for the lecture
 - Lecture: discuss open questions and solve more complex tasks
 - Lab session: solve paper-an-pen assignments
- Data: clickstream data (all interactions of the student)
- Classes to predict: Pass, Fail

Agenda

- Traditional Classification Methods:
 - Decision Trees
 - Random Forest
 - K-Nearest Neighbor
 - Logistic Regression
- Performance Metrics
- Classification of Time Series

Decision Trees - Example

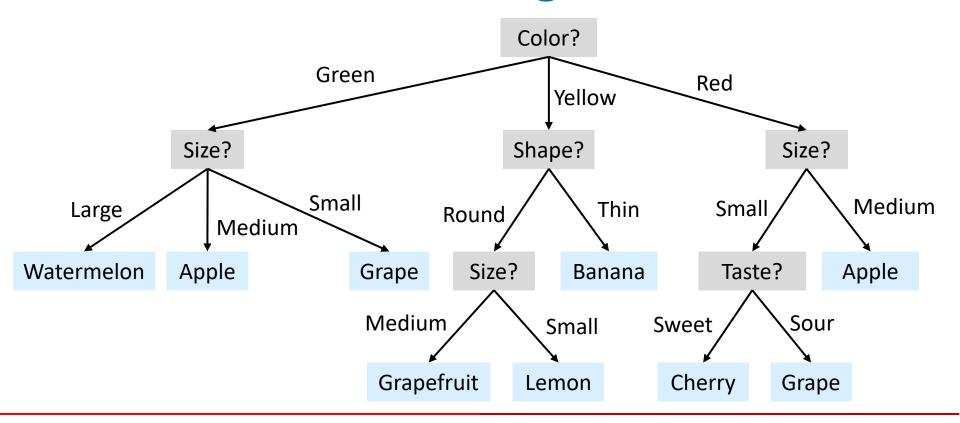




Decision Trees - Idea

- 1. Pick an attribute/feature, do a simple yes/no test
- 2. Conditioned on a choice, pick another attribute, do another test
- 3. In the leaves, assign a class with majority vote
- 4. Do other branches as well

Decision Trees - Categorical Features



Decision Trees - Construction

Which attributes do we choose?

• In which order?

In the case of continuous attributes, how do we

choose the threshold value?

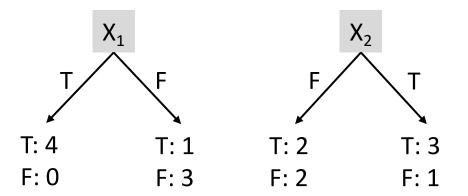
Construction Algorithm

- Learning the simplest (smallest) decision tree is an NP complete problem (translation: it is hard !!!)
- Greedy heuristic:
 - 1. Pick an attribute to split at a non-terminal node
 - 2. Split example into groups based on attribute value
 - 3. For each group:
 - No examples -> return majority of parent node
 - All examples from same class -> return class
 - Else: loop to step 1

Construction Algorithm

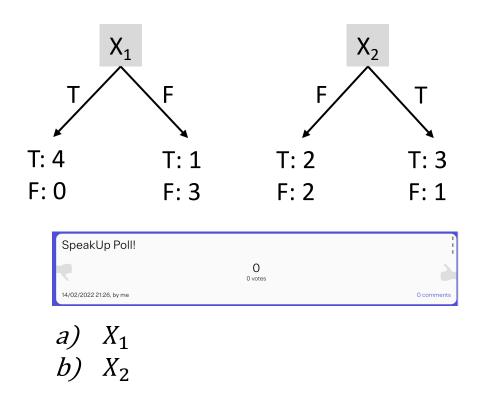
- Learning the simplest (smalle complete problem (translation best attribute ?
- Greedy heuristic:
 - 1. Pick an attribute to split at a non-terminal node
 - 2. Split example into groups based on attribute value
 - 3. For each group:
 - No examples -> return majority of parent node
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Picking the best attribute



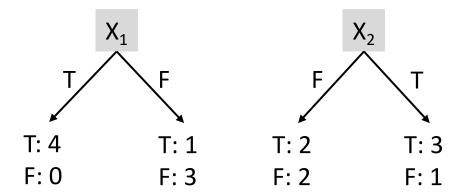
X ₁	X ₂	Υ
Т	Т	Т
Т	F	Т
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F
F	Т	F
F	F	F

On which attribute would you split?



X ₁	X ₂	Υ
Т	Т	Т
Т	F	Т
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F
F	Т	F
F	F	F

Picking the best attribute



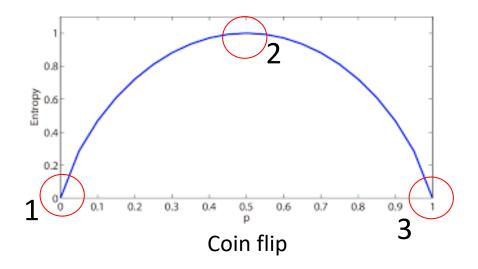
How t	o do	it forma	lly?
-------	------	----------	------

-> Information theory !!!

X ₁	X ₂	Υ
Т	Т	Т
Т	F	Т
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F
F	Т	F
F	F	F

Entropy

 Describes the level of "uncertainty" or "surprise" about a random variable's possible outcome



- 1. Will always land on heads
- 2. 50/50
- 3. Will always land on tail

Entropy

$$H(X) = -\sum_{x \in X} p(x) \cdot \log_2 p(x)$$

Information content or surprise of event x.

Intuition: low probability = high surprise/information content.

Entropy: Expected surprise/information content.

Conditional Entropy

P(X,Y)

X	Cloudy	Not Cloudy
Rain	0.24	0.01
No Rain	0.25	0.50

Specific Conditional Entropy: what is the entropy of cloudiness Y, given that it is raining?

$$H(Y|X=x) = -\sum_{y \in Y} p(y|x) \cdot \log_2 p(y|x)$$

Conditional Entropy

P(X,Y)

X	Cloudy	Not Cloudy
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• Specific Conditional Entropy: what is the entropy of cloudiness Y, given that it is raining?

$$H(Y|X=x) = -\sum_{v \in Y} p(y|x) \cdot \log_2 p(y|x)$$

• Expected Conditional Entropy: what is the expected entropy of cloudiness Y, given "raininess" X?

$$H(Y|X) = \sum_{x \in Y} p(x) \cdot H(Y|X = x)$$

Information Gain

P(X,Y)

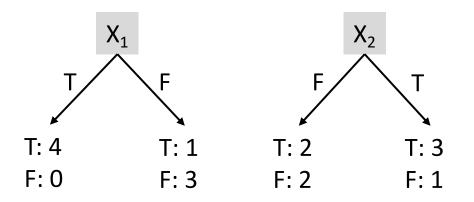
X	Cloudy	Not Cloudy
Rain	0.24	0.01
No Rain	0.25	0.50

• Information Gain: how much information about cloudiness (Y) do we get by discovering whether it is raining (X)?

$$I(Y;X) = H(Y) - H(Y|X)$$

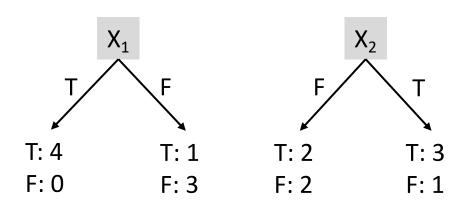
Picking the best attribute

Should I pick X_1 or X_2 to gain the most information about Y? Is $I(Y; X_1) > I(Y; X_2)$?



X ₁	X ₂	Υ
Т	Т	Т
Т	F	Т
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F
F	Т	F
F	F	F

Picking the best attribute

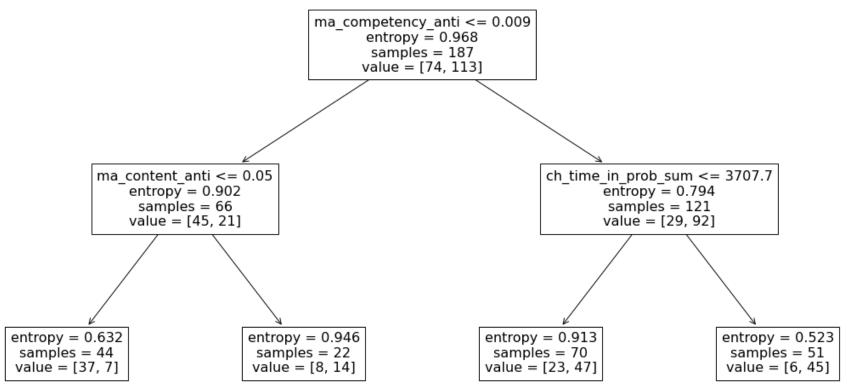


X ₁	X ₂	Υ
Т	Т	Т
Т	F	Т
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F
F	Т	F
F	F	F

What makes a good tree?

- Not too small: needs to handle important, but possibly subtle distinctions in data
- Not too big:
 - Avoid overfitting to training examples
 - Computational efficiency (avoid redundant, spurious attributes)
- Pruning strategies:
 - Stop splitting a node when the number of samples falls below a certain threshold
 - Grow a full tree, do bottom-up cross-validation: two leaves can be merged and labeled with the majority class if classification accuracy (on a validation set!) does not get worse

Decision Tree - Example



Agenda

- Traditional Classification Methods:
 - Decision Trees
 - Random Forest
 - K-Nearest Neighbor
 - Logistic Regression
- Performance Metrics
- Classification of Time Series

Random Forest - Idea

- Ensemble Method:
 - Take a collection of weak (simple) learners
 - Combine their predictions to obtain a better result
- Bagging: train learners on different samples of the data and then combine their output (e.g., majority vote or average)

Grow K trees on datasets sampled from the original data set (size N) with replacement (bootstrap samples), d = number of features.

- Draw K bootstrap samples of size N (bootstrap means that different samples can have elements in common)
- Grow each decision tree by selecting a random set of m out of d features at each node, and choosing the best feature to split on.
- Aggregate the predictions of the trees (majority vote or average) to produce the final prediction

Grow K trees on datasets sample N) with replacement (bootstrap features.

Each tree is trained on different data

et (size

- Draw *K bootstrap samples* of size N (bootstrap means that different samples can have elements in common)
- Grow each decision tree by selecting a random set of m out of d features at each node, and choosing the best feature to split on.
- Aggregate the predictions of the trees (majority vote or average) to produce the final prediction

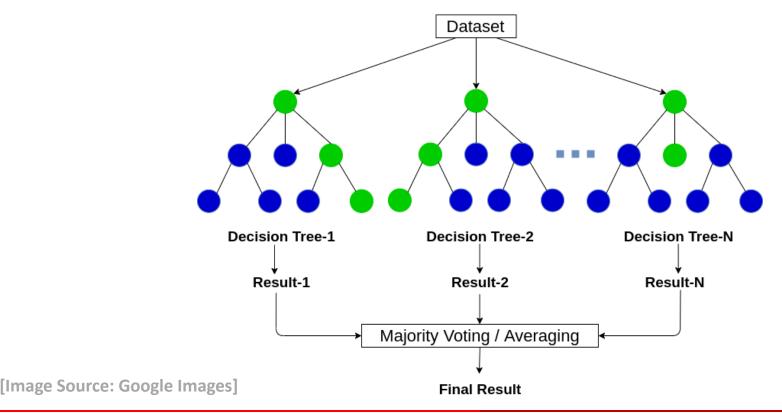
Grow K trees on datasets sampled from the original data set (size N) w Corresponding nodes in different trees 1, d = number of

feat

Corresponding nodes in different trees will (usually) not use the same feature for splitting

• [

- Grow each decision tree by selecting a random set of m out of d features at each node, and choosing the best feature to split on.
- Aggregate the predictions of the trees (majority vote or average) to produce the final prediction



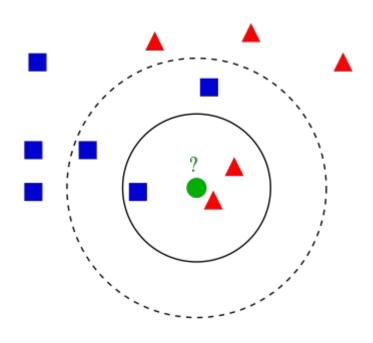
Summary

- Decision trees are simple, but
 - sensitive to small perturbations in the data
 - tend to overfit
- Random forests
 - Reduce overfitting in decision trees and can improve accuracy
 - Are versatile (classification, regression, continuous/categorical variables)
 - Easy to implement and parallelize
 - Still a popular algorithm in practice (for dense data)
 - Not so easy to interpret...

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- Traditional Classification Methods:
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kNN-Illustration



- Alternative to parametric models are non-parametric models
- Learning amounts to simply storing training data:
- Test instances are classified using similar training instances
- kNN: k-nearest neighbors

kNN- Algorithm

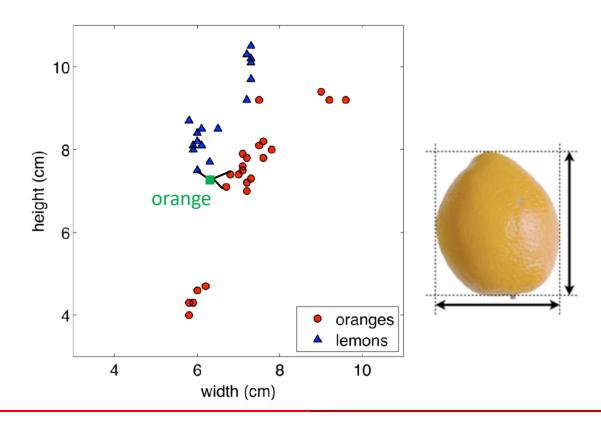
- Training data: samples x_n (n = 1, ..., N) with class labels c (c = 1, ..., C)
- Classification of test sample x^*
 - Find the k nearest neighbors of x^*
 - Predicted class \hat{c}^* = majority vote of k nearest neighbors
 - Option: give nearest neighbors different weights

kNN- Algorithm

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Often based on distance, i.e. the smaller the distance, the higher the weight

Example: 3-nearest neighbor



How do we choose k?

- Larger k may lead to better performance
- But if we set k too large we may end up looking at samples that are not neighbors (are far away from the query)
- Find k using leave-one-out cross-validation:
 - For each point x_n in the <u>training data set</u>, find the k nearest neighbors from the set of all *other* training samples
 - Predict \hat{c}_n as the majority vote of the k nearest neighbors
 - Measure the classification accuracy for different values of k
 - Choose k with the highest classification accuracy on the <u>training data set</u>

kNN: what distance could we use?

• Comparing the number of sessions students had in a MOOC:

```
u_1: [5,3,7,8,10,2] u_2: [1,9,10,2,3,2] u_3: [4,5,2,8,7,6]
```

Comparing users by the movies they have watched:

```
u_1: {"Frozen", "The Horse Whisperer", "Follow Me", "Notting Hill" } u_2: {"Die Hard 1", "The Father", "Frozen", "Black Panther", "Casablanca" } u_3: {The Dark Knight", "Die Hard 1", "Wonder Woman", "Black Panther", "Logan", "Up" }
```

Comparing weeks days of users (W: Work, O: Off, S: Sick)

```
u_1: [O, W, W, O, W] u_2: [W, W, W, W, W] u_3: [W, W, S, W, O]
```

Comparing sequences of actions of users

```
u_1: [O, A, P, E, T, F, G, F, G, H, I, O, N, K, U, P, E, L] u_2: [O, S, I, E, P, L] u_3: [O, R, C, C, T, A, A, S, S, P, L]
```

• Comparing the relative amount of time users spent on watching videos, solving quizzes, etc.

```
u_1: [0.2, 0.3, 0.1, 0.1, 0.3] u_2: [0.8, 0.1, 0.0, 0.0, 0.1] u_3: [0.1, 0.5, 0.3, 0.0, 0.1]
```

kNN: similarity metrics

Euclidean Distance: simple & fast for numbers

$$d(x,y) = \|x - y\|$$

Jaccard Distance: for set data

$$d(X,Y) = 1 - \frac{|X \cap Y|}{|X \cup Y|}$$

Hamming distance: for strings (with the same length)

$$d(x,y) = \sum_{i=1}^{n} (x_i \neq y_i)$$

kNN: similarity metrics

- Levensthein distance: minimal number of single character edits (insertion, deletion, substitution) to change one string into the other
- Longest common subsequence (LCS): string similarity measure, find the longest common subsequence between two sequences
- Kullback-Leibler Divergence: measures difference between two probability distributions (relative entropy)

$$D_{KL}(P||Q) = \sum_{x \in X} P(x) \cdot \log(\frac{P(x)}{Q(x)})$$

kNN: similarity metrics

- There are many more similarity metrics (e.g., Cosine distance,
 Manhattan distance, Mahalanobis distance)
- These are all standard metrics we will look detailed into metrics for comparing sequences in later lectures

Summary

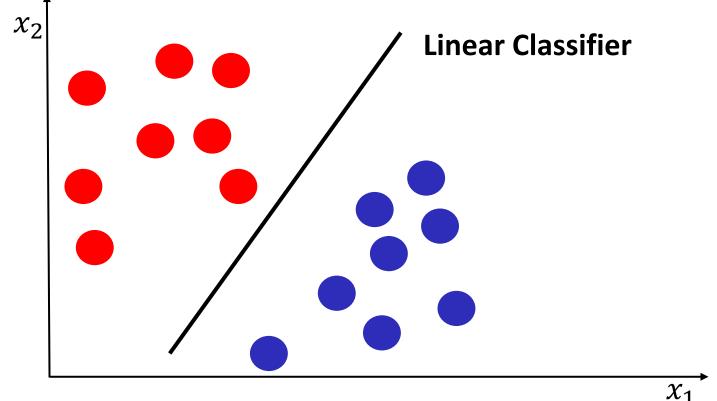
- Naturally forms complex decision boundaries
- Works well if we have a lot of samples
- Issues:
 - Complexity Scales linearly with the number of samples
 - High-dimensional data: to work well, we need a number of samples that grows exponentially with dimension

Agenda

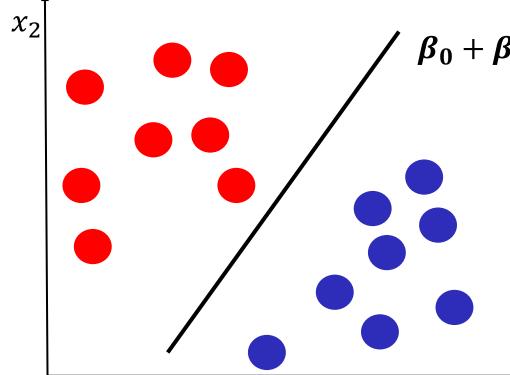
Traditional Classification Methods:

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Logistic Regression as a binary classifier



Logistic Regression as a binary classifier



$$\boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \boldsymbol{x}_1 + \boldsymbol{\beta}_2 \boldsymbol{x}_2 = \boldsymbol{\beta} \widetilde{\boldsymbol{x}} = \boldsymbol{0}$$

How could we assign a class probability to every point from $\beta \tilde{x}$?

 x_1

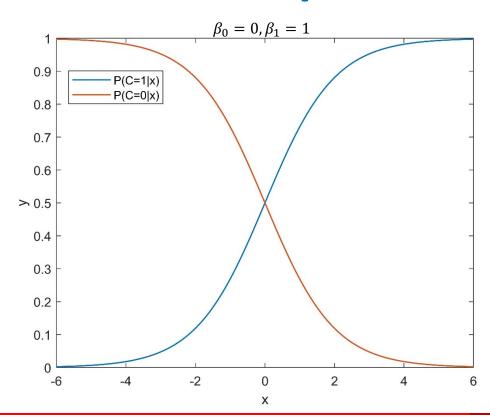
Logistic regression as a binary classifier

We use a logistic function !!!

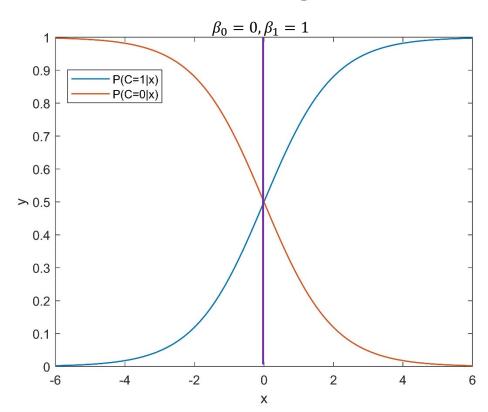
$$P(C = 1|x) = \frac{1}{1 + e^{-\beta \tilde{x}}}$$
 \longrightarrow $P(C = 0|x) = 1 - P(C = 1|x)$

- Binary classification: We predict C=1 if P(C=1|x)>0.5 and C=0 otherwise.
- Why logistic function ?
 - Finding the parameters: convex optimization problem (easy to solve)
 - Smooth function

Example with 1 dimension



Example with 1 dimension



Set threshold to 0.5, i.e. predict C = 1 if P(C = 1|x) > 0.5

Logistic Regression as a binary classifier

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 = 0$$

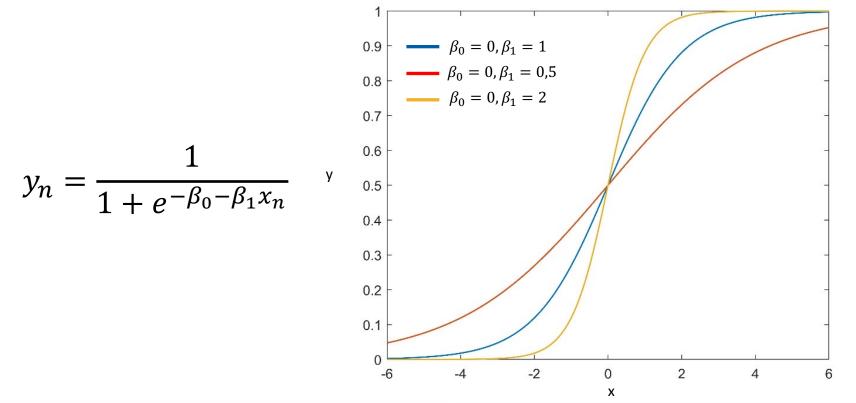
$$2\beta_0 + 2\beta_1 x_1 + 2\beta_2 x_2 = 0$$

$$5\beta_0 + 5\beta_1 x_1 + 5\beta_2 x_2 = 0$$

These 3 equations define the same boundary (same solutions). If we use them for logistic regression, will the resulting classifier be different?

Answer: yes. The probabilities will grow faster towards 0 or 1 the further we get away from the decision boundary.

Example with 1 dimension



Logistic regression with multiple classes

Multi-class classification: we do one regression per class:

•
$$\beta_1 \widetilde{x} = \beta_{1,0} + \beta_{1,1} x_1 + \beta_{1,2} x_2 + \cdots + \beta_{1,D} x_D$$

•
$$\beta_2 \widetilde{x} = \beta_{2,0} + \beta_{2,1} x_1 + \beta_{2,2} x_2 + \cdots + \beta_{2,D} x_D$$

•
$$\beta_K \widetilde{x} = \beta_{K,0} + \beta_{K,1} x_1 + \beta_{K,2} x_2 + \cdots + \beta_{K,D} x_D$$

D: dimension

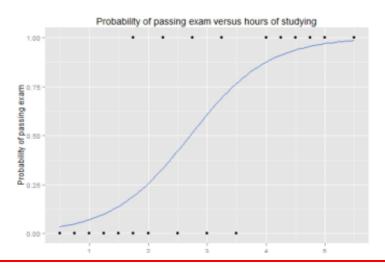
K: number of classes

- There exists multiple methods to compute the probability of each classes.
- One of them is the SoftMax function: $P(C = k|x) = \frac{e^{\beta}k^{\widetilde{x}}}{\sum_{i=1}^{K}e^{\beta}j^{\widetilde{x}}}$

Example: passing the exam

• **Problem**: given the number of hours the student spent learning, will (s)he pass the exam?

Hours	0.50	0.75	1.00	1.25	1.50	1.75	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.25	4.50	4.75	5.00	5.50
Pass	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	1	1	1



Hours of study	Probability of passing exam
1	0.07
2	0.26
3	0.61
4	0.87
5	0.97

Summary

- Advantages:
 - Natural probabilistic view of class predictions
 - Quick to train
 - Good accuracy for many simple data sets
 - Interpretability of model coefficients
- Disadvantages:
 - Linear decision boundary.

Traditional Methods

- Decision Trees and Random Forest
- K-Nearest Neighbor
- Logistic Regression

These are just examples of classification algorithms, there are others out there (e.g., Naïve Bayes, Support Vector Machines)

Agenda

- Traditional Methods:
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 - K-Nearest Neighbor
 - Logistic Regression
- Performance Metrics
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Classification: Accuracy

$$Acc = \frac{|agreements|}{N}$$

where *agreements* means that the predicted outcome is equal to the observed outcome

- Easy to interpret
- Works for binary and multi-class
- General agreement across fields: accuracy is not a good standalone performance metric

Classification: Accuracy

$$Acc = \frac{|agreements|}{N}$$

where *agreements* means that the predicted outcome is equal to the observed outcome

- Easy to interpret
- Works for binary and multi-class



 General agreement across fields: accuracy is not a good standalone performance metric

Classification: Balanced accuracy

$$Acc_{bal} = \frac{1}{|C|} \cdot \sum_{c \in C} Acc_c$$

where |C| denotes the number of classes

- Easy to interpret: average accuracy over all classes
- Takes into account class imbalance
- Works also for the multi-class case

Classification: Confusion Matrix

		True Outcome			
		Positive	Negative		
icted	Positive	True Positive (tp)	False Positive (fp)		
Predicted Outcome	Negative	False Negative (fn)	True Negative (tn)		

Confusion matrix for a binary classification task (two classes denoted as positive and negative class)

Classification: Confusion Matrix

			True Outcome				
		А	В	С			
tcome	А	15	7	5			
Predicted Outcome	В	5	82	1			
Predic	С	1	6	13			

Confusion matrix for a classification task with 3 classes: A, B, C

Classification: Confusion Matrix

		True Outcome				
		А	В	С		
tcome	А	21	0	0		
Predicted Outcome	В	0	95	0		
Predic	С	0	0	19		

Confusion matrix for a **Perfect** classifier

Classification: Specificity, sensitivity,...

		True Outcome		
		Positive	Negative	
Predicted Outcome	Positive	True Positive (tp)	False Positive (fp)	
Predi Outc	Negative	False Negative (fn)	True Negative (tn)	

$$specificity = \frac{tn}{tn + fp} \quad sensitivity = \frac{tp}{tp + fn}$$

Classification: Specificity, sensitivity,...

		True Outcome			
		Positive	Negative		
Predicted Outcome	Positive	True Positive (tp)	False Positive (fp)		
Predi	Negative	False Negative (fn)	True Negative (tn)		

$$specificity = \frac{tn}{tn + fp} \quad sensitivity = \frac{tp}{tp + fn}$$

- Sensitivity: what is the proportion of positive I identified correctly?
 - High Sensitivity: Most of the positive were identified
- Specificity: what is the proportion of negative I identified correctly?
 - High Specificity: Most of the negative were identified

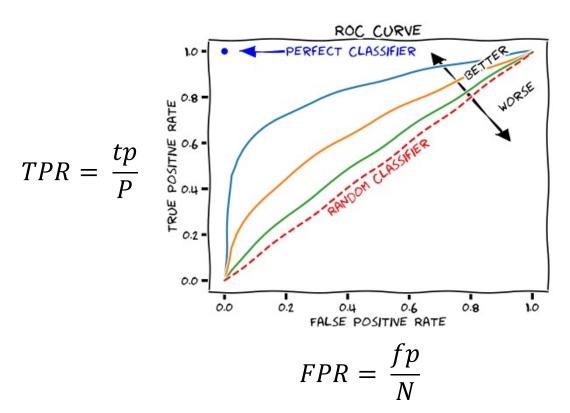
Classification: Specificity, sensitivity,...

		True Outcome			
		Positive	Negative		
Predicted Outcome	Positive	True Positive (tp)	False Positive (fp)		
Pred Outc	Negative	False Negative (fn)	True Negative (tn)		

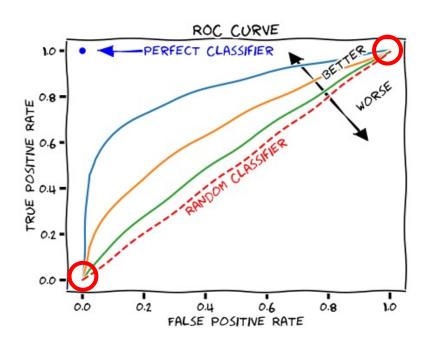
$$specificity = \frac{tn}{tn + fp} \quad sensitivity = \frac{tp}{tp + fn}$$

- These metrics are used in addition to accuracy
- Sensitivity and specificity are often used in the fields of Psychology, Learning Sciences, etc.

ROC curve



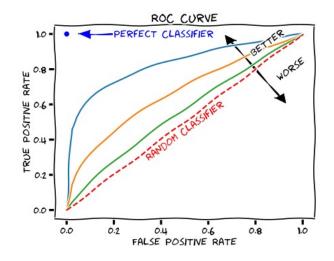
Computing the ROC curve



- Given a classifier f(x) predicting some type of score (e.g., a probability)
- Choose a threshold t:
 - $f(x_i) \ge t$: predict positive class
 - $f(x_i) < t$: predict negative class
- Compute TPR and FPR given t
- Repeat for different thresholds (e.g., in case of probabilities choose t = 0,0.1, ... 1)

Area under the ROC curve (AUC)

- The AUC denotes the area under the ROC curve
- A perfect classifier has an AUC of 1
- A random classifier has an AUC of 0.5
- Often used as a performance metric in more technical fields (e.g., educational data mining)
- The AUC can be extended to the multi-class case by considering all possible pairs of classes



Classification: Summary

• Do:

- Carefully think about the choice of metric (or combination)
- Some ideas:
 - Use accuracy plus sensitivity and specificity [binary]
 - Use (balanced) accuracy plus AUC [binary + multi-class]
 - Use just AUC [binary + multi-class]

• Don't:

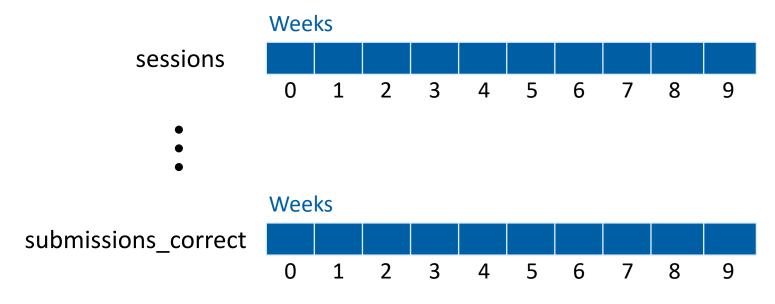
- Use accuracy as a standalone metric
- Compute "all" possible metrics that come to your mind

Agenda

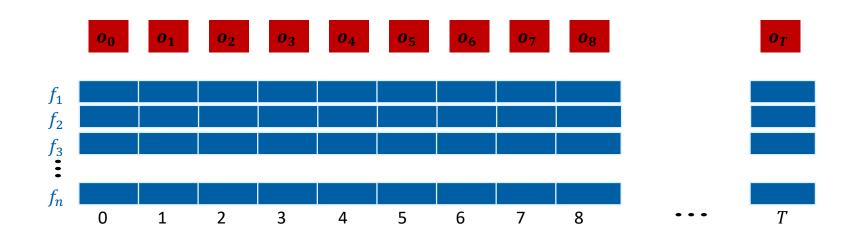
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- Classification of Time Series

Time Series – Our flipped classroom case

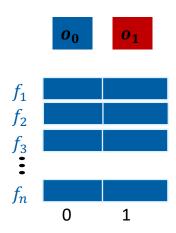




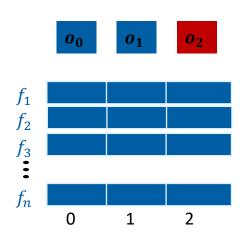
- Prediction of a categorical target variable after t < T time steps, where T is the total number of time steps
- Prediction of a variable in time step t+1, based on time steps 0, ..., t.



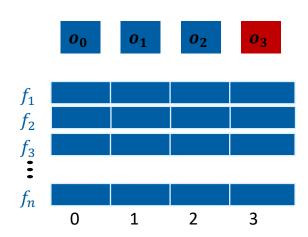
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Last Week:

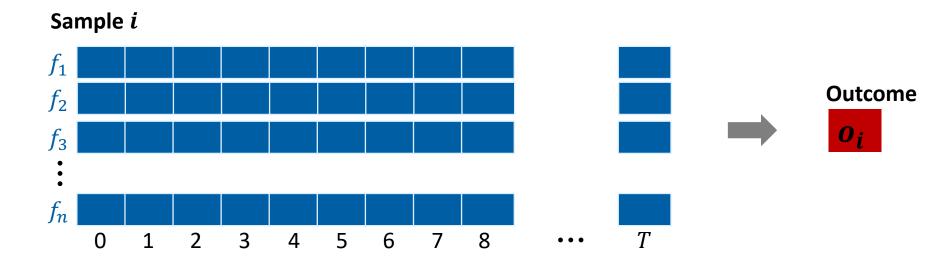
 we have solved this task for a numerical target variable (students' performance in weekly quizzes) using a linear mixed effect model

This Week:

- for a binary target variable, this task can be solved using a logistic (mixed effect) model
- The other presented algorithms are not suitable for this task

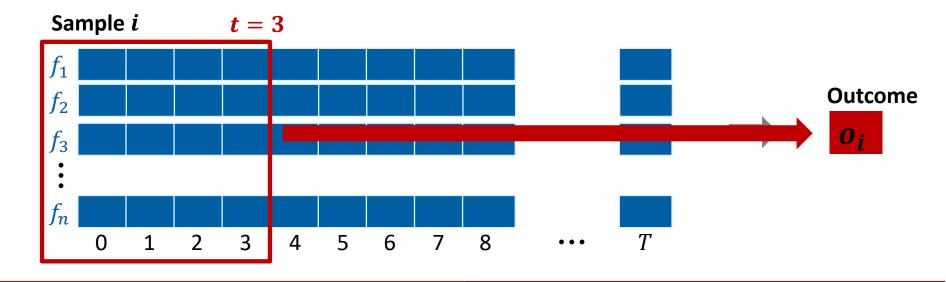
Time Series – Prediction Task

• Prediction of a **binary** target variable after t < T time steps, where T is the total number of time steps



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Your Turn

- Predict whether students will pass the course after t = 5 weeks (i.e. after half of the course)
- We provide you the train-test split to use in the Jupyter Notebook
- You can choose the classifier: Decision Tree, Random Forest, or k-Nearest Neighbor

Your Turn – Some Hints

- For Decision Tree and Random Forest you will need to use one of the following:
 - Flattening
 - Aggregation (hint: we have aggregated features last week)
- For k-Nearest Neighbor, you can compute pairwise distances between vectors
 - If you have several features, compute a pairwise distance matrix
 separately for each feature and then sum the distance matrices up
 - Distance matrices can have different scales (hint: MinMaxScaler from sklearn)

Your Turn – Feedback

Do you want feedback or have questions? Upload your Jupyter Notebook here:

https://go.epfl.ch/mlbd-activities