

# Chapter 1 Homework

31 August 2021

You are tasked with modeling an outbreak of the common cold (*Rhinovirus*) in a city. First, you can assume that population size  $N$  is approximately fixed for the duration of the outbreak. It is known that for *Rhinovirus*, individuals are either susceptible or infected, and that someone may be infected multiple times during a season. In other words, the progression of the disease is

$$S \rightarrow I \rightarrow S \quad (1)$$

where  $S, I$  are the number of individuals in the susceptible and infected classes at time  $t$  (**thus**  $\mathbf{S} + \mathbf{I} = \mathbf{N}$ ). The rate of transition from  $S$  to  $I$  is given by the probability of contacting a sick individual ( $I/N$ ) multiplied by the transmission rate for a contact ( $\beta : \beta > 0$ ). Finally, individuals recover from a common cold at a rate  $\gamma$  ( $\gamma > 0$ ).

- Write down a difference equation that describes the dynamics of this system. (NB: Only one equation is needed.)
- What are the two fixed points for the system? What do the fixed points tell us about the conditions on  $\beta$  and  $\gamma$ ?
- Using recursion, vary the three parameters  $(N, \beta, \gamma)$  of the system and plot the results for each combination; choose 3 different values for each parameter. Include at least one combination where the condition in the previous item is broken. Assume that  $I_0 = 1$ . Do the plots confirm your analytical results?

Next, consider a slight modification on the biology of the common cold such that individuals spend sometime after infection where they are not susceptible to infection (i.e., temporary immunity). Now the progression of the disease is

$$S \rightarrow I \rightarrow R \rightarrow S \quad (2)$$

where  $R$  is the “recovered” class with temporary immunity. This time, let  $\gamma$  be the transition rate from  $I \rightarrow R$  and  $\mu$  be the transition rate from  $R \rightarrow S$ .

- Repeat the exercises for the SIS system for the SIRS system. This time, do not vary the parameter  $N$ ; rather, vary the parameter  $\mu : \mu > 0$ . (NB: you will now need a system of equations.)