Chapter 1 Homework

28 August 2023, due 6 September

You are tasked with modeling an outbreak of the common cold (*Rhinovirus*) in a city. First, you can assume that population size N is approximately fixed for the duration of the outbreak. It is known that for *Rhinovirus*, individuals are either susceptible or infected, and that someone may be infected multiple times during a season. In other words, the progression of the disease is

$$S \rightarrow I \rightarrow S \tag{1}$$

where S, I are the number of individuals in the susceptible and infected classes at time t **(thus $\mathbf{S + I = N}$)**. The rate of transition from S to I is given by the probability of contacting a sick individual (I/N) multiplied by the transmission rate for a contact ($\beta : \beta > 0$). Finally, individuals recover from a common cold at a rate γ ($\gamma > 0$).

- Write down a difference equation that describes the dynamics of this system. (NB: Only one equation is needed.)
- What are the two fixed points for the system? What do the fixed points tell us about the conditions on β and γ?
- Using recursion, vary the three parameters (N, β, γ) of the system and plot the results for each combination; choose 3 different values for each parameter. Include at least one combination where the condition in the previous item is broken. Assume that $I_0 = 1$. Do the plots confirm your analytical results?

Next, consider a slight modification on the biology of the common cold such that individuals spend sometime after infection where they a not susceptible to infection (i.e., temporary immunity). Now the progression of the disease is

$$S \rightarrow I \rightarrow R \rightarrow S \tag{2}$$

where R is the "recovered" class with temporary immunity. This time, let γ be the transition rate from $I \rightarrow R$ and μ be the transition rate from $R \rightarrow S$.

- Repeat the exercises for the SIS system for the SIRS system. This time, do not vary the parameter N; rather, vary the parameter $\mu : \mu > 0$. (NB: you will now need a system of equations.)