

1 Prologue

1.1 Basic Properties of Numbers

All variables are assumed to be numbers.

1.1.1 Properties of Addition

Rule 1. $\forall(a, b, c) : a + (b + c) = (a + b) + c$

Rule 2. $\forall(a, b) : a + b = b + a$

Rule 3. $\forall a : a + 0 = 0 + a = a$

Rule 4. $\forall a \exists(-a) : a + (-a) = (-a) + a = 0$

Theorem 1. $\forall a : a + x = a \iff x = 0$

Proof.

$$a + x = a \quad (1.1.1.1)$$

$$\overset{Function}{(-a)} + \overset{Function}{(a+x)} = \overset{Function}{(-a)} + a \quad (1.1.1.2)$$

$$\overset{Rule\ 1}{((-a) + a)} + x = \overset{Rule\ 4}{0} \quad (1.1.1.3)$$

$$\overset{Rule\ 4}{0 + x} = 0 \quad (1.1.1.4)$$

$$\overset{Rule\ 3}{x} = 0 \quad (1.1.1.5)$$

□

1.1.2 Properties of Multiplication

Rule 5. $\forall(a, b, c) : a(bc) = (bc)a$

Rule 6. $\forall(a, b, c) : ab = ba$

Rule 7. $\forall a : a \cdot 1 = 1 \cdot a = a$

Premise 1. $\forall a : 1 \neq 0$

Rule 8. $\forall a \neq 0 \exists a^{-1} : aa^{-1} = a^{-1}a = 1$

Theorem 2. $\forall(a, b, c) : ab = ac \wedge a \neq 0 \iff b = c$

Proof.

$$ab = ac \quad \wedge a \neq 0 \quad (1.1.2.1)$$

$$\overset{\text{Function } a^{-1}.}{a^{-1}(ab)} = \overset{\text{Function } a^{-1}.}{a^{-1}(ac)} \quad \wedge a \neq 0 \quad (1.1.2.2)$$

$$\overset{\text{Rule 6}}{(a^{-1}a)b} = \overset{\text{Rule 6}}{(a^{-1}a)c} \quad \wedge a \neq 0 \quad (1.1.2.3)$$

$$\overset{\text{Rule 8}}{1 \cdot b} = \overset{\text{Rule 8}}{1 \cdot c} \quad (1.1.2.4)$$

$$\overset{\text{Rule 7}}{b} = \overset{\text{Rule 7}}{c} \quad (1.1.2.5)$$

□

1.1.3 Combined Properties

Rule 9. $\forall(a, b, c) : \quad a(b + c) = ab + ac$