Homework 7

Labor Economics

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1 Setup

Wages are

$$\begin{aligned} y_0 &= \delta_0 + \beta_0 x + \theta + \epsilon_0 \\ y_1 &= \delta_1 + \beta_1 x + \alpha_1 \theta + \epsilon_1 \end{aligned}$$

Also define the utility shifter function C and an index function I

$$\begin{aligned} C &= \gamma_0 + \gamma_2 z + \gamma_3 x + \alpha_C \theta \\ I &= E[y_1 - y_0 - C |\, \mathcal{F}] \\ &= \underbrace{(\delta_1 - \delta_0 - \gamma_0)}_{\widetilde{\delta}} + \underbrace{(\beta_0 - \beta_1 - \gamma_3)}_{\widetilde{\beta}} x_i - \gamma_2 z + \underbrace{(\alpha_1 - 1 - \alpha_C)}_{\widetilde{\alpha}} \theta - \epsilon_c \end{aligned}$$

The distribution of shocks is

$$\left.\begin{pmatrix} \epsilon_{i,0} \\ \epsilon_{i,1} \\ \epsilon_{i,C} \end{pmatrix}\right|_{x_i, z_i, \theta_i} \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & 0 & 0 \\ 0 & \sigma_1^2 & 0 \\ 0 & 0 & \sigma_C^2 \end{pmatrix}\right]$$

The information set for the agent is $\mathcal{F}$. The preference shock $\epsilon_C \in \mathcal{F}$, but $\{\epsilon_0, \epsilon_1\} \notin \mathcal{F}$. The decision rule is

$$s = 1 \Longleftrightarrow E[I \geq 0 |\, \mathcal{F}]$$

2 Q1

There is no unobserved heterogeneity in this model since we know θ. Thus,

$$E\left[\begin{matrix} y_0 \\ y_1 \end{matrix} \middle| x_i, \theta_i, s = k\right] = \begin{bmatrix} \delta_0 + x\beta_0 + \theta \\ \delta_1 + x\beta_1 + \alpha_1 \theta \end{bmatrix} + \underbrace{E\left[\begin{matrix} \epsilon_0 \\ \epsilon_1 \end{matrix} \middle| x_i, \theta_i, \epsilon_c : big/small\right]}_{0}$$

This is straight-up OLS, which means we recover $\{\delta, \beta, \alpha_1, \sigma_0^2, \sigma_1^2\}$.

$$\begin{aligned} \Pr[S = 1 |\, \mathcal{F}] &= \Pr\left[\epsilon_c \leq \widetilde{\delta} + \widetilde{\beta} x - \gamma_2 z + \widetilde{\alpha}\theta \middle| \mathcal{F}\right] \\ &= \Phi\left[\frac{\overbrace{[(\delta_1 - \delta_0) + (\beta_1 - \beta_0)x + (\alpha_1 - 1)\theta]}^{\text{known number}} - \gamma_0 - \gamma_2 z - \gamma_3 x - \alpha_c \theta}{\sigma_c} \middle| \mathcal{F}\right] \end{aligned}$$

Now we can get $\left\{\gamma_0, \gamma_2, \gamma_3, \alpha_c, \sigma_c^2\right\}$

3 Q2

Now we don't know θ but agents do. However, we do have two measurement equations $m \in \{A, B\}$:

$$\begin{aligned} M_{iA} &= x_i^M \beta_A^M + \theta_i + \epsilon_{iA}^M \\ M_{iB} &= x_i^M \beta_B^M + \alpha_B \theta_i + \epsilon_{iB}^M \end{aligned}$$

where $\epsilon_m^M \sim N(0, \sigma_m^{M2})$ are i.i.d. We can write

$$\begin{aligned} E[y_1|x, z, s=1] &= \delta_1 + \beta_1 x + E[\epsilon_1 + \alpha_1 \theta | x, z, I \geq 0] \\ &= \delta_1 + \beta_1 x + \alpha_1 E[\theta | x, z, I \geq 0] \\ &= \delta_1 + \beta_1 x + \alpha_1 \sigma^* E\left[\frac{\theta}{\sigma^*} \middle| 0 \leq \widetilde{\delta} + \widetilde{\beta} x - \gamma_2 z + \underbrace{(\alpha_1 - \alpha_0 - \alpha_c)\theta - \epsilon_c}_{\eta}\right] \\ &= \delta_1 + \beta_1 x + \alpha_1 \sigma^* E\left[\frac{\theta}{\sigma^*} \middle| \eta \geq -\left(\widetilde{\delta} + \widetilde{\beta} x - \gamma_2 z\right)\right] \end{aligned}$$

Define $\eta \equiv (\alpha_1 - \alpha_0 - \alpha_c)\theta - \epsilon_c$. Then

$$\begin{pmatrix} \eta \\ \theta \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^{*2} & (\alpha_1 - \alpha_0 - \alpha_c)\sigma_\theta^2 \\ (\alpha_1 - \alpha_0 - \alpha_c)\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix}\right]$$

where $\sigma^{*2} = (\alpha_1 - \alpha_0 - \alpha_c)^2 \sigma_\theta^2 + \sigma_c^2$. We can project θ onto η, which means

$$\theta = \frac{\operatorname{Cov}(\eta, \theta)}{\operatorname{Var} \eta} \eta + \nu = \frac{(\alpha_1 - \alpha_0 - \alpha_c)\sigma_\theta^2}{\sigma^{*2}} \eta + \nu$$

where

$$\nu \sim N\left(0, \sigma_\theta^2\left(1 - \rho_{\eta\theta}^2\right)\right) \qquad \text{and} \qquad \rho_{\eta\theta} = \frac{(\alpha_1 - \alpha_0 - \alpha_c)\sigma_\theta^2}{\sigma^* \sigma_\theta}$$

Letting $t \equiv -(\widetilde{\delta} + \widetilde{\beta} x - \gamma_2 z)/\sigma^*$, we can now write

$$\begin{aligned} E[y_0|x, z, s=1] &= \delta_1 + \beta_0 x + \alpha_0 \frac{(\alpha_1 - \alpha_0 - \alpha_c)\sigma_\theta^2}{\sigma^*} \overbrace{\frac{-\phi(t)}{\Phi(t)}}^{\lambda_0} \\ E[y_1|x, z, s=1] &= \delta_1 + \beta_1 x + \alpha_1 \frac{(\alpha_1 - \alpha_0 - \alpha_c)\sigma_\theta^2}{\sigma^*} \underbrace{\frac{\phi(t)}{1 - \Phi(t)}}_{\lambda_1} \end{aligned}$$

4 Histograms

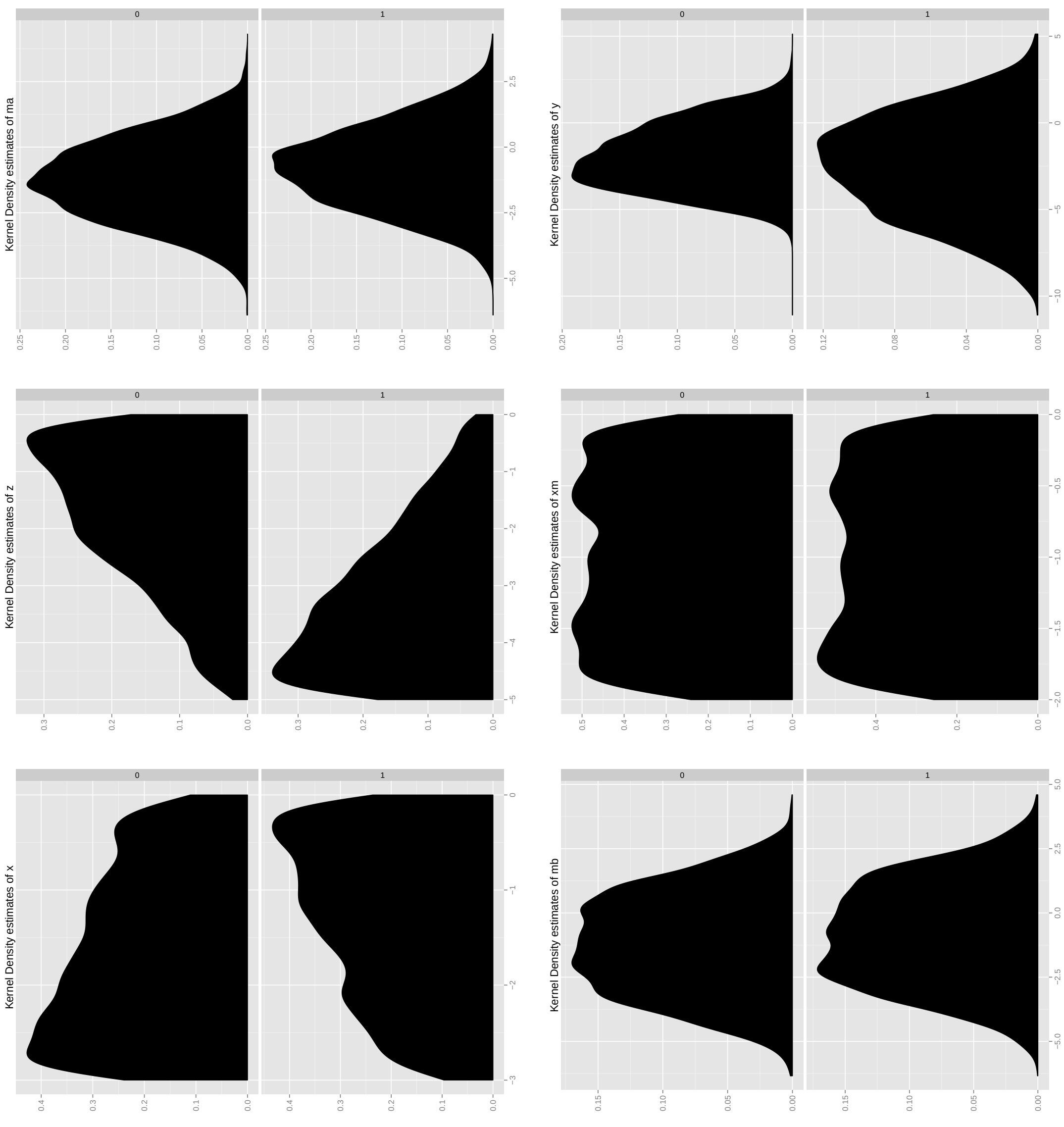

Figure 1: Plots