Homework 8

Labor Economics

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1 Setup

Individual knows $\Omega_a = (x_a, y, \epsilon_a)$. His wage is

$$\log w_a^* = \alpha_1 + \alpha_2 x_a + \epsilon_a$$

We observe

$$\log w_a = \log w_a^* + \nu_a$$

Individual gets per-period payoff

$$u(\Omega_a) = \begin{cases} y + \gamma_1 + \gamma_2 y & \text{if } p_a = 0 \text{ (eg, no work)} \\ y + \exp\{f(x_a) + \epsilon_a\} & \text{if } p_a = 1 \text{ (eg, work)} \end{cases}$$

Assume iid shocks:

$$\begin{pmatrix} \epsilon_a \\ \nu_a \end{pmatrix} \sim N\left(0, \begin{bmatrix} \sigma_\epsilon^2 & \\ 0 & \sigma_\nu^2 \end{bmatrix}\right)$$

Objective is

$$\max_{\{p_a\}_{a=1}^A} \beta^{a-1} E\left[u(\Omega)|\Omega_a\right]$$

2 Recursive formulation

Write problem recursively for lazy:

$$V_a^0(x, y, \epsilon_a) = \gamma_1 + (1 + \gamma_2)y + \beta E[V_{a+1}(cyanx + 1, y, \epsilon_{a+1})]$$

and working:

$$V_a^1(x, y, \epsilon_a) = \exp\{f(x_a) + \epsilon_a\} + y + \beta E[V_{a+1}(x + 1, y, \epsilon_{a+1})]$$

Value is

$$V_a(x, y, \epsilon_a) = \max\left\{V_a^0(x, y, \epsilon_a), V_a^1(x, y, \epsilon_a)\right\}$$

We normalize the value of afterlife to 0 after assuming earthly actions can't affect it

$$V_{A+1}(x, y, \epsilon) = 0$$

Let $\mathcal{W}_a$ be the event that we we work, which is

$$p_a = 1 \quad \Leftrightarrow \quad \epsilon_a \geq \underbrace{\log\left(\gamma_1 + \gamma_2 y + E[V^0_{a+1}(x,y)] - E[V^1_{a+1}(x,y)]\right) - \alpha_1 - \alpha_2 x_a}_{g(x,y,a)}$$

Then

$$\Pr(\mathcal{W}_a) = 1 - \Phi(g(x,y,a)/\sigma_\epsilon)$$

3 Backward induction

3.1 Last period

Last period's value is

$$V_A(x,y,\epsilon_A) = \max\{\gamma_1 + \gamma_2 y, \quad \exp(\alpha_1 + \alpha_2 x + \epsilon_A)\} + y$$

Now

$$g(x,y,A) = \log(\gamma_1 + \gamma_2 y) - (\alpha_1 + \alpha_2 x)$$

so

$$\Pr(\mathcal{W}_A) = 1 - \Phi\left(\frac{g(x,y,A)}{\sigma_\epsilon}\right) = \pi(x,y,A)$$

Expected terminal value is

$$E[V_A(x,y)] = y + [1 - \pi(x,y,A)]\,(\gamma_1 + \gamma_2 y) + \pi(x,y,A)\left[\exp\{\alpha_1 + \alpha_2 x\}\underbrace{\frac{1 - \Phi\left(\frac{g(x,y,A) - \sigma_\epsilon^2}{\sigma_\epsilon}\right)}{\pi(x,y,A)}\exp\{\frac{1}{2}\sigma_\epsilon^2\}}_{E[e^{\epsilon}_A|\mathcal{W}^A]}\right]$$

This can be written as

$$E[V_A(x,y)] = y + [1 - \pi(x,y,A)]\,[\gamma_1 + \gamma_2 y] + \exp\left\{\alpha_1 + \alpha_2 x + \frac{\sigma_\epsilon^2}{2}\right\}\left[1 - \Phi\left(\frac{g(x,y,A) - \sigma_\epsilon^2}{\sigma_\epsilon}\right)\right]$$

3.2 Other periods

This means

$$g(x,y,a) = \log\left(\gamma_1 + \gamma_2 y + \beta\,\overbrace{E[V^0_{a+1}(x,y)] - E[V^1_{a+1}(x,y)]}^{\Delta EV(x,y,a)}\right) - \alpha_1 - \alpha_2 x_a$$

and

$$\mathcal{W}_a = \{\epsilon_a \geq g(x,y,a)\}$$

so

$$\Pr(\mathcal{W}_a) = 1 - \Phi\left(\frac{g(x,y,a)}{\sigma_\epsilon}\right) = \pi(x,y,a)$$

and

$$E[V_a(x,y)] = y + [1-\pi(x,y,a)]\{\gamma_1 + \gamma_2 y + E[V_{a+1}(x,y)]\}$$
$$+ \pi(x,y,a)E[V_{a+1}(x+1,y)] + \exp\left\{\alpha_1 + \alpha_2 x + \frac{\sigma_\epsilon^2}{2}\right\}\left[1 - \Phi\left(\frac{g(x,y,a)-\sigma_\epsilon^2}{\sigma_\epsilon}\right)\right]$$

Note that we could simply use the general definition for V_a and $g(x,y,a)$ and specify $V_{A+1} = 0$. This would be a bit neater (ie, for each agent, have $A+1$ periods and just say $V_{A+1} = 0$... then start recursion at $a = A$.

4 Estimation

We have states $\Omega_{ia} = (x,y,a,\epsilon)_{ia}$ and control $p_{ia} \in \{0,1\}$.
Immediately we can get parameters governing distribution of non-labor income from a kernel density estimation of observed y_i values. Or, since we know if we know the underlying distribution we just need μ_y and σ_y which can be estimated by $N^{-1}\sum_i y_i$ and $\widehat{SE(y_i)}$. Number of periods is irrelevant and are consistent as $N \to \infty$. .
Remaining parameters are

$$\theta = \{\alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma_\epsilon^2, \sigma_\nu^2\}$$

We'll need functions(?) or matrices of(?)

$$V_a^0(x,y) \quad V_a^1(x,y) \quad g(x,y,a) \quad \pi(x,y,a) \quad w(x,\epsilon;\alpha_1,\alpha_2)$$

Note that $g(x,y,a)$ is a function of a because it has $\Delta E[V_{a+1}(x,y)]$

4.1 Identification

All parameters are identified through non-linearity in Probit except σ_v which is never identified.

4.2 Plan

1. We use the fact that in period A we have $\Delta E[V_{A+1}(x,y)] = 0.0\ \forall x$
2. Run probit for working in A. Recover all structural parameters.
3. Estimate wages as

$$\log w = \underbrace{\alpha_1 + \alpha_2 x + \sigma_\epsilon \lambda(y,x,a)}_{E[\log w_a^* | \mathcal{W}_a]} + \nu$$

   where

$$\sigma\lambda(x,y,a) = \sigma E\left[\frac{\epsilon}{\sigma}\middle|\frac{\epsilon}{\sigma} \geq \frac{g(x,y,a)}{\sigma_\epsilon}\right] = \frac{\phi(g/\sigma)}{1-\Phi(g/\sigma)}$$

4. Compute $E[V_A(x,y)]$ and $E[V_A(x+1,y)]$ for all relevant x given observed work history for person i at $A-1$.
5. Use $\Delta E[V_A(x,y)]$ to repeat procedure from Step 2.

Each period you get a set of estimates of the structural parameters from the probit and a second set of estimates for wage function parameters and σ_ε. So you have A sets of

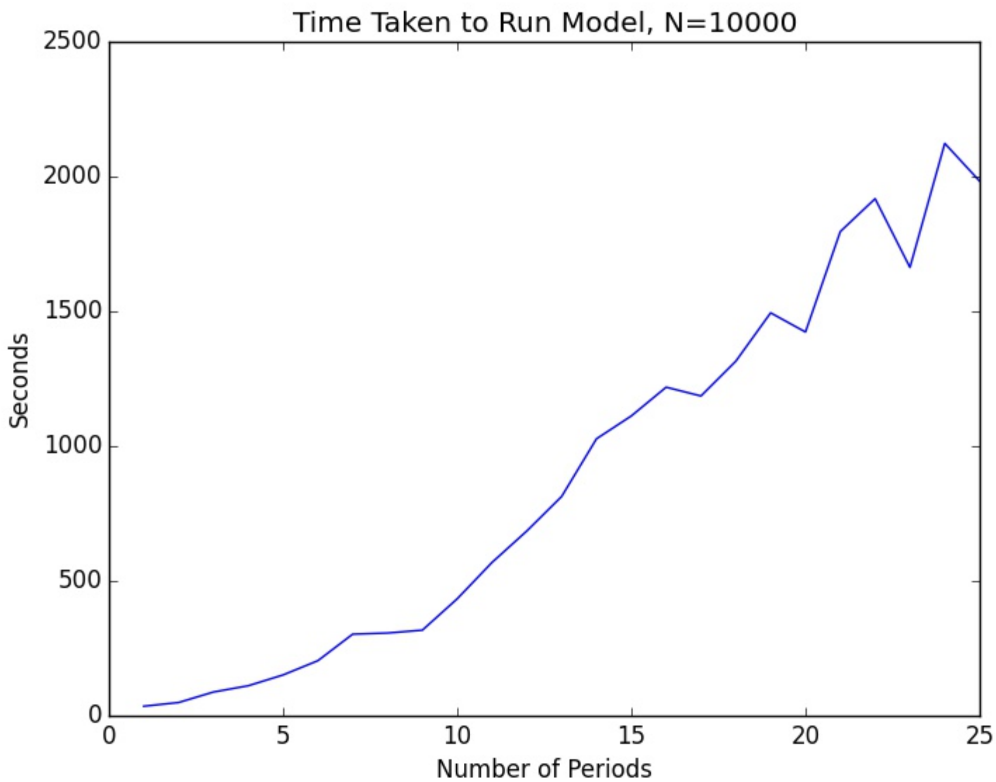
$$\widehat{\theta}_{MLE} = \{\widehat{\gamma}_1,\ \widehat{\gamma}_2,\ \widehat{\alpha}_1,\ \widehat{\alpha}_2,\ \widehat{\alpha}_3,\ \widehat{\sigma}_e\}$$

and A sets of

$$\widehat{\beta}_{OLS} = \{\widehat{\alpha}_1,\ \widehat{\alpha}_2,\ \widehat{\alpha}_3,\ \widehat{\sigma}_e\}$$

5 Results

First ran the model with $N = 10,000$ for 25 periods to get a sense of how long things took.



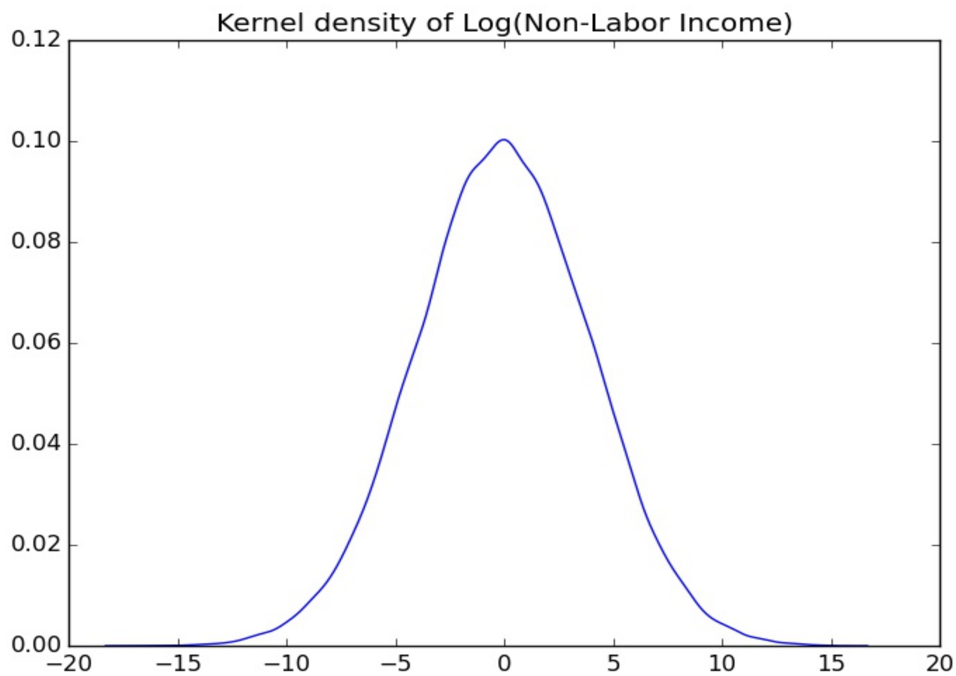
Then we upped $N = 35,000$ and ran for 11 periods to get more percision on our estimates. It took 62 minutes to run through all periods starting at $A = 11$, then $A = 10$, to $A = 1$. Estiamtes provided below. First a bit of background about our economy.

Listing 1: Functions used

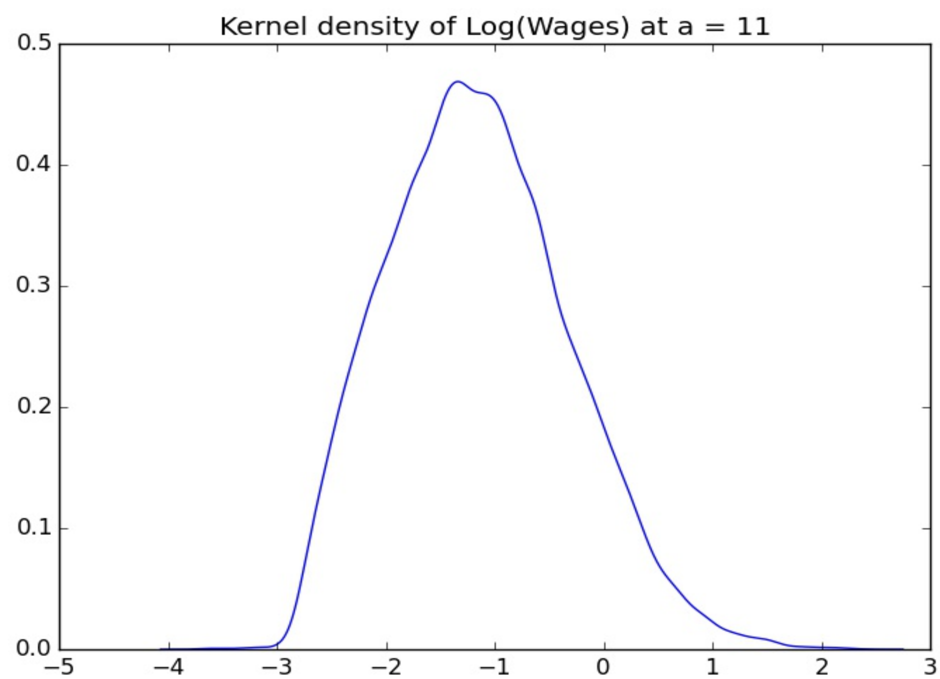
```
###########################
# Specify Model Parameters
###########################
# A     = 11          # Number of Periods
γ_1   = 0.300         # Leisure Coefficient
γ_2   = 0.500         # Consumption-Leisure Interaction Coefficient
δ     = 0.1           # Discount Rate
N     = 35000         # Number of Individuals
σ_e   = 1.000         # Standard Error of Wage Shock
σ_v   = 0.100         # Standard Error of Measurement Error
α_1   = 5.000         # Wage Function Parameter
α_2   = 0.500         # Wage Function Parameter
α_3   = -0.100        # wage function parameter
μ_y   = 0.0           # Mean of non-labor income
σ_y   = 2.0           # std dev of non-labor income
# yL = 0            # Minimum Non-Labor Income
# yH = 1000           # Top Non-Labor Income

θ_real = [γ_1; γ_2; α_1; α_2; α_3; σ_e]
β_real = [α_1; α_2; α_3; σ_e]
```

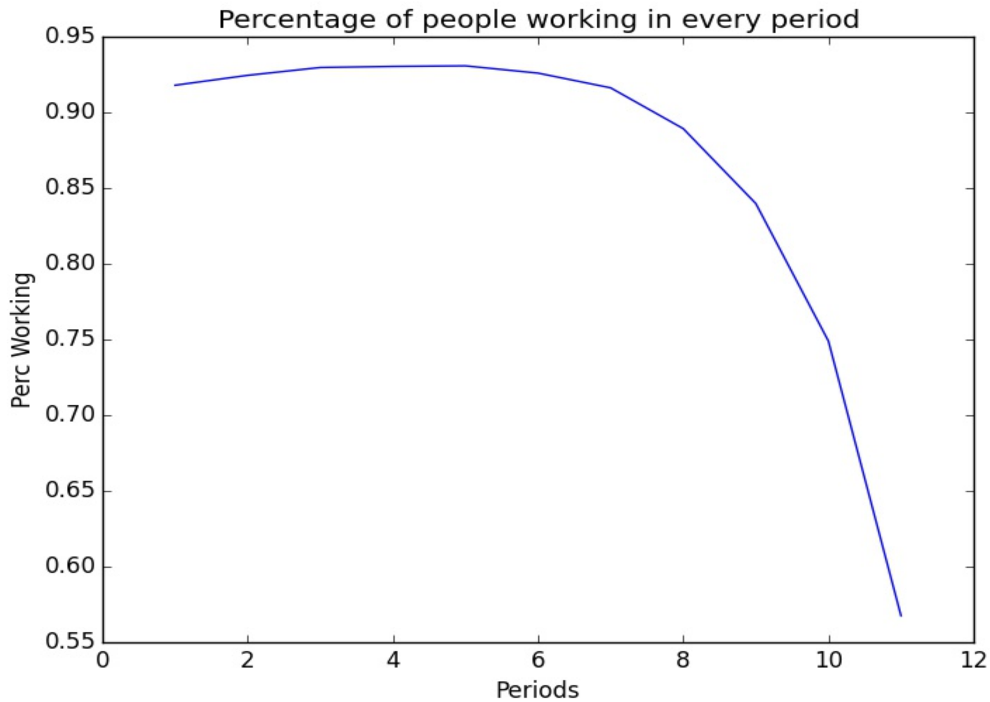
5.1 Non-labor income



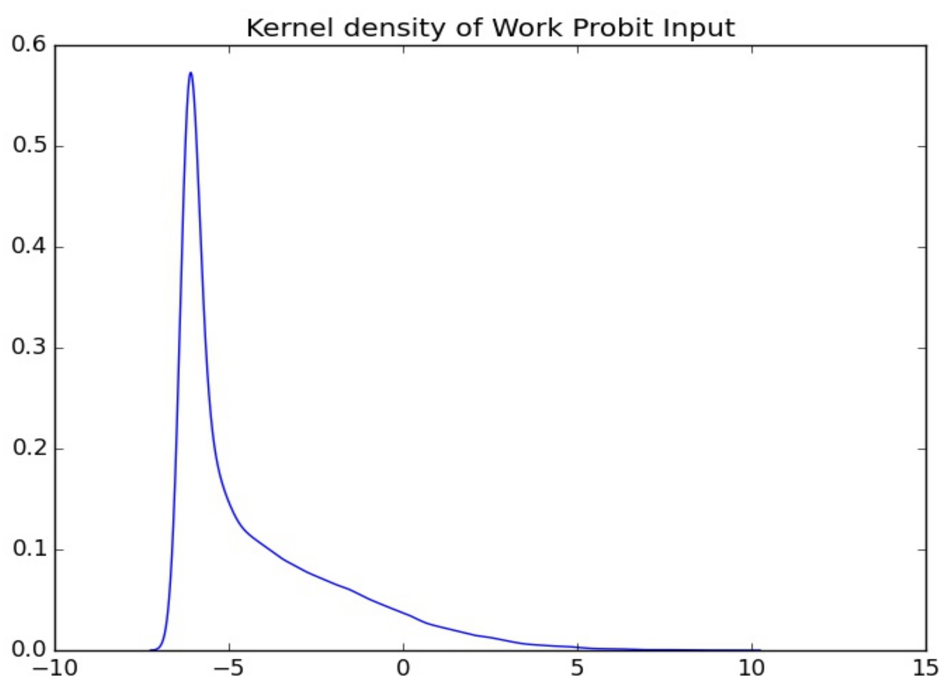
5.2 Wage Income at Period 11



5.3 Percentage working every period



5.4 Input in the Probit for decision to work



5.5 Sample Output

Listing 2: Functions used

```
Eval 3: value = 3938.49124
Eval 4: value = 3393.73034
Eval 5: value = 2335.27842
Eval 10: value = 2823.92179
Eval 20: value = 2381.45825
Eval 30: value = 2366.84353
Eval 40: value = 1.7165000000000002e54
Eval 50: value = 2335.01217
Eval 75: value = 2334.88533
Eval 100: value = 2334.88218
Eval 125: value = 2334.88033
Eval 150: value = 2334.88032
Eval 175: value = 2334.88031
Eval 200: value = 2334.88031
Eval 250: value = 2334.88031
Eval 300: value = 3018.2841
Eval 350: value = 2334.89949
Eval 400: value = 2334.88031
Eval 450: value = 2334.88031
Eval 500: value = 2343.89512
Eval 600: value = 2334.88031
Eval 700: value = 2335.3637
Eval 800: value = 2334.88031
Eval 900: value = 2334.88488
Eval 1000: value = 2334.88031
Results of Optimization Algorithm
 * Algorithm: Nelder-Mead
 * Starting Point:
     [0.616155778922203,0.508501689628141,5.0377041439308625,1.2182100632738138,1.198399291904949,1.0062741405941

 * Minimum:
     [0.6161618008355569,0.5085065629459316,5.037713726302649,1.5515365722879093,1.5317258009190446,1.00627413594

 * Value of Function at Minimum: 2334.880305
 * Iterations: 91
 * Convergence: true
   * |x - x'| < NaN: false
   * |f(x) - f(x')| / |f(x)| < 1.0e-12: true
   * |g(x)| < NaN: false
   * Exceeded Maximum Number of Iterations: false
 * Objective Function Calls: 182
 * Gradient Call: 0elapsed time: 3731.772961383 seconds
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 1:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 2334.88



 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     0.616

γ_2      0.5     0.509

α_1      5.0     5.038

α_2      0.5     1.552

```

```
α_3      -0.1    1.532

σ_e      1.0     1.006

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     5.399

α_2      0.5     NaN

α_3      -0.1    NaN

σ_e      1.0     1.027
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 2:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 1902.961



 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     1.129

γ_2      0.5     0.514

α_1      5.0     5.009

α_2      0.5     0.517

α_3      -0.1    -0.092

σ_e      1.0     0.984

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     5.604

α_2      0.5     NaN

α_3      -0.1    NaN

σ_e      1.0     0.95
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 3:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 1687.306



 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     -0.0

γ_2      0.5     0.503
```

```
α_1      5.0      4.983

α_2      0.5      0.608

α_3      -0.1     -0.141

σ_e      1.0      0.996

 OLS Parameters:
         TRUE     ESTIMATED

α_1      5.0      5.589

α_2      0.5      -0.004

α_3      -0.1     0.004

σ_e      1.0      1.02
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 4:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 1732.686



 MLE Parameters:
         TRUE     ESTIMATED

γ_1      0.3      -0.0

γ_2      0.5      0.512

α_1      5.0      5.014

α_2      0.5      0.49

α_3      -0.1     -0.102

σ_e      1.0      0.993

 OLS Parameters:
         TRUE     ESTIMATED

α_1      5.0      5.464

α_2      0.5      -0.09

α_3      -0.1     0.022

σ_e      1.0      0.971
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 5:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 1853.577



 MLE Parameters:
         TRUE     ESTIMATED
```

```
γ_1      0.3     2.266

γ_2      0.5     0.531

α_1      5.0     5.11

α_2      0.5     0.457

α_3      -0.1    -0.095

σ_e      1.0     0.963

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     4.993

α_2      0.5     -0.03

α_3      -0.1    0.009

σ_e      1.0     0.98
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 6:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 2222.365



 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     0.146

γ_2      0.5     0.157

α_1      5.0     3.811

α_2      0.5     0.46

α_3      -0.1    -0.09

σ_e      1.0     1.002

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     4.312

α_2      0.5     0.043

α_3      -0.1    -0.005

σ_e      1.0     1.014
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 7:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]

 LLN value at θ true: 2950.578
```

```
 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     0.677

γ_2      0.5     0.499

α_1      5.0     5.004

α_2      0.5     0.521

α_3      -0.1    -0.103

σ_e      1.0     0.977

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     3.698

α_2      0.5     -0.026

α_3      -0.1    0.002

σ_e      1.0     0.963
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 8:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]


 LLN value at θ true: 4320.6



 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     -0.0

γ_2      0.5     0.435

α_1      5.0     4.817

α_2      0.5     0.532

α_3      -0.1    -0.104

σ_e      1.0     1.004

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     2.576

α_2      0.5     0.046

α_3      -0.1    -0.006

σ_e      1.0     0.966
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 9:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]

 LLN value at θ true: 6327.785
```

```
 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     0.243

γ_2      0.5     0.507

α_1      5.0     5.038

α_2      0.5     0.496

α_3      -0.1    -0.1

σ_e      1.0     1.009

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     1.452

α_2      0.5     -0.03

α_3      -0.1    0.003

σ_e      1.0     1.0
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 10:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]

 LLN value at θ true: 9500.981

 MLE Parameters:
         TRUE    ESTIMATED

γ_1      0.3     0.094

γ_2      0.5     0.148

α_1      5.0     3.97

α_2      0.5     0.444

α_3      -0.1    -0.096

σ_e      1.0     0.979

 OLS Parameters:
         TRUE    ESTIMATED

α_1      5.0     0.168

α_2      0.5     -0.03

α_3      -0.1    0.001

σ_e      1.0     0.981
There where 35000 workings and 11 periods
It took 3731.773 seconds to run

 Percentage that worked in period 11:
[0.92,0.92,0.93,0.93,0.93,0.93,0.92,0.89,0.84,0.75,0.57]

 LLN value at θ true: 16365.872

 MLE Parameters:
```

```
          TRUE    ESTIMATED

γ_1       0.3     0.373

γ_2       0.5     0.587

α_1       5.0     5.142

α_2       0.5     0.491

α_3       -0.1    -0.099

σ_e       1.0     0.985

 OLS Parameters:
          TRUE    ESTIMATED

α_1       5.0     -1.598

α_2       0.5     -0.015

α_3       -0.1    0.001

σ_e       1.0     1.023

NumPeriods: Time Taken:
11                3731.772961383
```

5.6 Main Code

Listing 3: Functions used

```julia
# Set Working Directory
cd("C:/Users/nick/skydrive/projects/laborecon/ps8")
# Call Packages
using DataFrames
using Optim
using Distributions
using PyPlot
using KernelDensity
pwd()

include("code/functions.jl")
include("code/functions_probit.jl")

A_min = 11
A_max = 11
time_taken = zeros(A_max)

for A in A_min:A_max

    tic()

    ###########################
    # Specify Model Parameters
    ###########################
    # A     = 11          # Number of Periods
    γ_1   = 0.300         # Leisure Coefficient
    γ_2   = 0.500         # Consumption-Leisure Interaction Coefficient
    δ     = 0.1           # Discount Rate
    N     = 35000         # Number of Individuals
    σ_e   = 1.000         # Standard Error of Wage Shock
    σ_v   = 0.100         # Standard Error of Measurement Error
    α_1   = 5.000         # Wage Function Parameter
    α_2   = 0.500         # Wage Function Parameter
```

```
α_3   = -0.100       # wage function parameter
μ_y   = 0.0          # Mean of non-labor income
σ_y   = 2.0          # std dev of non-labor income
# yL = 0           # Minimum Non-Labor Income
# yH = 1000          # Top Non-Labor Income

θ_real = [γ_1; γ_2; α_1; α_2; α_3; σ_e]
β_real = [α_1; α_2; α_3; σ_e]
# srand(12345)

β = 1/(1+δ)

################
# Start Dataset
################

Y_nl = exp(rand(Normal(μ_y,σ_y^2),N))
# Y_nl = rand(Uniform(yL,yH),N)
df = DataFrame(
    ID = squeeze(kron([1:N],int(ones(A,1))),2), # ID
    A  = repmat([1:A],N),                       # Indicate Period
    Y  = squeeze(kron(Y_nl,ones(A,1)),2),       # Non-Labor Income
    e  = rand(Normal(0,σ_e^2),N*A),               # Wage Shock
    v  = rand(Normal(0,σ_v^2),N*A)                # Measurement Error
    )
head(df)

# generate empty value function
# pre-allocation makes code cleaner later
for tt in A+1:-1:1
    for x in 0:tt
        df[symbol("EV_$(tt)_x$(x)")] = 0.0   # what are the $$'s? can i get some? is that a
            different pkg?
        df[symbol("V_$(tt)_x$(x)")]  = 0.0
        df[symbol("w_$(tt)_x$(x)")]  = 0.0
        df[symbol("V_$(tt)_x$(x)_no")] = 0.0
        df[symbol("V_$(tt)_x$(x)_yes")] = 0.0
        df[symbol("p_$(tt)_x$(x)")]      = 0.0
    end
end
head(df)

# fill in values
for tt in A:-1:1
    for x in 0:tt
        println("$tt and $x")

            # Convenience
            X_a    = int(x.*ones(N)) # give everyone X = x at a
            E_a    = (df[:e])[df[:A].==tt] # errors this period
            y      = df[:Y][df[:A].== tt] # N-vec of non-labor income at a

            ##### WAGES
            w_a_x = symbol("w_$(tt)_x$(x)") # wage[ A= a, X = x]
            w_a   = symbol("w_$(tt)") # observed wage at a

            #####  VALUES
            # generated before
            EV_a1_x    = symbol("EV_$(tt+1)_x$(x)") # of being at a+1 with x (didn't work)
            EV_a1_x1   = symbol("EV_$(tt+1)_x$(x+1)") # being at a+1 with x+1 (did work)

            # generated here
            V_a_x      = symbol("V_$(tt)_x$(x)") # being at a with x
            EV_a_x     = symbol("EV_$(tt)_x$(x)") # exp of being at a with x
            V_a_x_no   = symbol("V_$(tt)_x$(x)_no") # of working this period
            V_a_x_yes  = symbol("V_$(tt)_x$(x)_yes") # not working this period

            ##### POLICY FUNCITON
```

```
            p_a_x     = symbol("p_$(tt)_x$(x)") # actual policy function



            # Add Wages to Dataset
            # True Wage
            df[w_a_x][df[:A]      .==tt] = wage_eqn(θ_real,X_a,E_a)

            # value of not working
            df[V_a_x_no][df[:A]   .==tt] = leisure_value_t(θ_real, tt) + (df[EV_a1_x])[df[:A
                ].==tt]
            # value of working
            df[V_a_x_yes][df[:A]  .==tt] = y + wage_eqn(θ_real, X_a,E_a) + df[EV_a1_x1][df[:A
                ].==tt]

            # Choose between and record
            df[p_a_x][df[:A]      .==tt] = df[V_a_x_no][df[:A].==tt] .< df[V_a_x_yes][df[:A
                ].==tt]
            (df[V_a_x])[df[p_a_x] .==false] = (df[V_a_x_no])[df[p_a_x].==false]
            (df[V_a_x])[df[p_a_x] .==true] = (df[V_a_x_yes])[df[p_a_x].==true]

        # calculate expected value of being at current period
        # Correct expectations for selection
        # Weight values by probability of occurence (cond'l on e_{ia})
        Π = Π_work(θ_real, X_a,tt)
        (df[EV_a_x])[df[:A].==tt] =
            (1-Π).*( leisure_value_t(θ_real, tt)
            + β*df[EV_a1_x][df[:A].==tt] )
            + Π.*( y + β*df[EV_a1_x1][df[:A].==tt] )
            + exp(.5*σ_e^2)*wage_eqn(θ_real,X_a, zeros(N)).*
            ( 1 - normcdf( (g(θ_real,X_a,tt) - σ_e^2)./σ_e ) )
    end
end
head(df)


DATA = DataFrame(
    ID = squeeze(kron([1:N],int(ones(A,1))),2), # ID
    A  = repmat([1:A],N),                       # Indicate Period
    Y  = squeeze(kron(Y_nl,ones(A,1)),2),       # Non-Labor Income
    )
# generate empty observed data set
for tt in 1:A
    DATA[symbol("P_$(tt)")] = 0.0   # do they work in period a
    DATA[symbol("W_$(tt)")] = NaN # wage in period a
    DATA[symbol("X_$(tt)")] = 0.0  # experience in period a
end
head(DATA)

for jj in 1:A

    P_a = symbol("P_$(jj)") # observed decision
    W_a = symbol("W_$(jj)")
    X_a = symbol("X_$(jj)")
    X_a1 = symbol("X_$(jj+1)")

    X_vec = convert(Array{Float64},(DATA[X_a])[df[:A].== jj])
    for x in 0:jj
        # println("$jj,$x")
        tP_a = (df[symbol("p_$(jj)_x$(x)")][ df[:A].==jj ])
        tP = DATA[P_a][df[:A].== jj]
        tP[x.==X_vec] = tP_a[x.==X_vec]
        DATA[P_a][df[:A].== jj] = tP

        WORKED = DATA[P_a][df[:A].== jj]
        tW_a = (df[symbol("w_$(jj)_x$(x)")][ df[:A].==jj ])
        # don't forget measurement error, + v
        tW = DATA[W_a][df[:A].== jj] + df[:v][df[:A].== jj]
```

```
        tW[WORKED.==true] = tW_a[WORKED.==true]
        DATA[W_a][df[:A].== jj] = tW
    end

    if jj < A
        (DATA[X_a1])[df[:A].== jj+1] = X_vec + (DATA[P_a][df[:A].== jj])
    end
end
# DATA


# percentage that work in period A
perc_a = zeros(A)
for a in 1:A
    perc_a[a] = sum( DATA[symbol("P_$(a)")][DATA[:A].== a ]  )/N
end

##################################
######## Estimation
##################################


θ = θ_real

ntheta = length(θ_real)
nbeta  = length(β_real)

θ_MLE = zeros(A,ntheta)
β_ols = zeros(A,nbeta)
Σ_w       = zeros(A)
Σ_ols     = zeros(nbeta,nbeta,A)


for tt in A+1:-1:1
    DATA[symbol("EV_$(tt)_x")] = 0.0
    DATA[symbol("EV_$(tt)_x1")] = 0.0
end

for tt in A:-1:1  # for every period, starting at A and working backward

    initials = ones(ntheta)
    initials = θ

    probit_opt = []
    global count = 1
    for i =1:5
      probit_opt = optimize(probit_wrapper,vec(initials),autodiff = true,
          ftol=1e-12,
          iterations = 3000)
      initials = probit_opt.minimum
    end
    show(probit_opt)

    θ_MLE[tt,:] = probit_opt.minimum



    # Second stage

    θ_hat = probit_opt.minimum

    W_a = DATA[symbol("W_$(tt)")][DATA[:A].==tt]
    X_a = DATA[symbol("X_$(tt)")][DATA[:A].==tt]
    P_a = DATA[symbol("P_$(tt)")][DATA[:A].==tt]

    W_a = W_a[P_a .== true]
    X_a = X_a[P_a.== true]
```

```
        Y_mat = log(W_a)

        X_mat = [ones(int(sum(P_a))) X_a X_a.^2 λ(θ_hat)[P_a.==true]]

        if det(X_mat'*X_mat) > 0.0
            (β_ols[tt,:], Σ_w[tt], Σ_ols[:,:,tt]) = least_sq(X_mat,Y_mat)
        else
            X_mat = [ones(int(sum(P_a))) λ(θ_hat)[P_a.==true]]
            (test, ~, ~) = least_sq(X_mat,Y_mat)
            β_ols[tt,:] = [test[1] NaN NaN test[2]]
        end


        # Calculate EV_a_x and EV_a_x1 (current period)
        ## assumes value for next period already exists
        EV_hat(θ_hat,tt)

    end

    time_taken[A] = toc()

    θ_names = ["γ_1" "γ_2" "α_1" "α_2" "α_3" "σ_e"]
    β_names = ["α_1" "α_2" "α_3" "σ_e"]

    for tt in 1:A
        println("There where $N workings and $A periods")
        println("It took $(round(time_taken[A],3)) seconds to run" )

        println("\n Percentage that worked in period $tt:")
        println("$(round(perc_a,2)) \n")
        println("\n LLN value at θ true: $(round(probit_LL(θ_MLE[tt,:][:]),3)) \n \n")

        println("\n MLE Parameters: \n \t TRUE \t ESTIMATED")
        for ii in 1:ntheta
            println("")
            println("$(θ_names[ii]) \t $(round(θ_real[ii],3)) \t $(round(θ_MLE[tt,ii],3))")
        end

        println("\n OLS Parameters: \n \t TRUE \t ESTIMATED")
        for ii in 1:nbeta
            println("")
            println("$(β_names[ii]) \t $(round(β_real[ii],3)) \t $(round(β_ols[tt,ii],3))")
        end

    end

end # end of loop over A


for ii in 1:N
    temp = 1:A
    println("NumPeriods: Time Taken:")
    println("$(temp[ii]) \t\t $(time_taken[ii])")
end



fig5 = figure
fig5 = plot([1:11],time_taken)
fig5 = title("Time Taken to Run Model, N=$(N)")
fig5 = xlabel("Number of Periods")
fig5 = ylabel("Seconds")
savefig("./plots/time_taken.jpg")




    # # map g function
```

```
    # k = kde(probit_input(θ_real))

    fig2 = figure()
    fig2 = plot(k)
    fig2 = title("Kernel density of Work Probit Input")
    savefig("./plots/KdenX.jpg")


    k_y = KernelDensity.kde(log(df[:Y][df[:A].== 1]))
    fig3 = figure()
    fig3 = plot(k_y)
    fig3 = title("Kernel density of Log(Non-Labor Income)")
    savefig("./plots/Yden_normal.jpg")

    fig4 = figure()
    fig4 = plot([1:A],perc_a)
    fig4 = xlabel("Periods")
    fig4 = ylabel("Perc Working")
    fig4 = title("Percentage of people working in every period")
    savefig("./plots/perc_working.jpg")


    W_11 = (DATA[:W_11][df[:A].== 11])
    Wages = log(W_11[isnan(W_11) .== 0])
    show(Wages)
    k_w = KernelDensity.kde(Wages)
    fig11 = figure()
    fig11 = plot(k_w)
    fig11 = title("Kernel density of Log(Wages) at a = 11")
    savefig("./plots/logWages_normal.jpg")
```

5.7 Functions

Listing 4: Functions used

```
function leisure_value_t(
        θ::Array{Float64},
        a::Int64)

    p_vec = unpackparams(θ)
    γ_1 = p_vec["γ_1"]
    γ_2 = p_vec["γ_2"]

    # calls df but for :Y and :A same as DATA
        γ_1 + (1 + γ_2).*(df[:Y])[df[:A] .== a]
end

function wage_eqn(
        θ::Array{Float64},
        X_a::Union(Array{Int64}, Int64, DataArray),
        e  ::Union(Array{Float64}, Float64, DataArray)
        )

  p_vec = unpackparams(θ)
  α_1 = p_vec["α_1"]
  α_2 = p_vec["α_2"]
  α_3 = p_vec["α_3"]

        exp( α_1 + α_2.*X_a + α_3.*(X_a.^2) + e )
end

function obs_wage_eqn(
        θ::Array{Float64},
        X_a::Union(Array{Int64}, Int64, DataArray),
```

```
        e  ::Union(Array{Float64}, Float64, DataArray),
        v  ::Union(Array{Float64}, Float64, DataArray)
        )

        exp( log(wage_eqn(θ,X_a,e)) + v )
end

function g(
        θ::Array{Float64},
        X_a::Union(Array{Int64}, Int64, DataArray),
        a  ::Int64)

        x         = unique(X_a)
        EV_a1_x   = symbol("EV_$(a+1)_x$(x[1])")
        EV_a1_x1  = symbol("EV_$(a+1)_x$(x[1]+1)")
        EV_1      = (df[EV_a1_x1])[df[:A] .== a]
        EV_0      = (df[EV_a1_x])[df[:A] .== a]
        y         = (df[:Y])[df[:A] .== a]

        log(
                 leisure_value_t(θ,a)
                 - y + β*(EV_0 - EV_1)  )
    - wage_eqn(θ,X_a,zeros(N))
end

function Π_work(
        θ::Array{Float64},
        X_a::Union(Array{Int64}, Int64, DataArray),
        a  ::Int64
        )

  p_vec = unpackparams(θ)
  σ_e = p_vec["σ_e"]

        1 - normcdf( g(θ,X_a,a)./σ_e )
end

function E_g_fun(
    θ::Array{Float64},
    X_a::Union(Array{Int64}, DataArray),
    tt::Int64)

    p_vec = unpackparams(θ)
    γ_1   = p_vec["γ_1"]
    γ_2   = p_vec["γ_2"]
    α_1   = p_vec["α_1"]
    α_2   = p_vec["α_2"]
    α_3   = p_vec["α_3"]
    σ_e   = p_vec["σ_e"]

    X_a       = DATA[symbol("X_$(tt)")][DATA[:A].==tt]
    X_a_p1    = X_a + 1
    y         = DATA[:Y][DATA[:A].== tt] # N-vec of non-labor income at a
    # will update entry for current period
    EV_a_x    = symbol("EV_$(tt)_x") # exp of being at a with x
    EV_a_x1   = symbol("EV_$(tt)_x1") # exp of being at a with x+1
    # use next period's in calculation
    EV_a1_x   = symbol("EV_$(tt)_x") # exp of being at a with x
    EV_a1_x1  = symbol("EV_$(tt)_x1") # exp of being at a with x+1
    EV_a1_1   = (DATA[EV_a_x1])[DATA[:A] .== tt]
    EV_a1_0 = (DATA[EV_a_x])[DATA[:A] .== tt]

    log_term = leisure_value_t(θ,tt) - y + β*(EV_a1_0 - EV_a1_1)
    log_term[log_term .<= 0] = eps()
    g_fun = log( log_term ) - wage_eqn(θ,X_a,zeros(N))

end
```

```julia
function EV_hat(
    θ::Array{Float64},
    tt::Int64
    )

    p_vec = unpackparams(θ)
    γ_1   = p_vec["γ_1"]
    γ_2   = p_vec["γ_2"]
    α_1   = p_vec["α_1"]
    α_2   = p_vec["α_2"]
    α_3   = p_vec["α_3"]
    σ_e   = p_vec["σ_e"]

    X_a      = DATA[symbol("X_$(tt)")][DATA[:A].==tt]
    X_a_p1   = X_a + 1
    y        = DATA[:Y][DATA[:A].== tt] # N-vec of non-labor income at a
    # will update entry for current period
    EV_a_x   = symbol("EV_$(tt)_x") # exp of being at a with x
    EV_a_x1  = symbol("EV_$(tt)_x1") # exp of being at a with x+1
    # use next period's in calculation
    EV_a1_x  = symbol("EV_$(tt)_x") # exp of being at a with x
    EV_a1_x1 = symbol("EV_$(tt)_x1") # exp of being at a with x+1
    EV_a1_1  = (DATA[EV_a_x1])[DATA[:A] .== tt]
    EV_a1_0 = (DATA[EV_a_x])[DATA[:A] .== tt]


    # EV if not working in a
    Π = 1 - normcdf( E_g_fun(θ,X_a,tt)./σ_e )
    (DATA[EV_a_x])[DATA[:A].==tt] =
        (1-Π).*( leisure_value_t(θ,tt)
        + β*DATA[EV_a1_x][DATA[:A].==tt] )
        + Π.*( y + β*DATA[EV_a1_x1][df[:A].==tt] )
        + exp(.5*σ_e^2)*wage_eqn(θ,X_a, zeros(N)).*
        ( 1 - normcdf( (E_g_fun(θ,X_a,tt) - σ_e^2)/σ_e ) )

    # EV if working in a
    Π = 1 - normcdf( E_g_fun(θ,X_a_p1,tt)./σ_e )
    (DATA[EV_a_x1])[DATA[:A].==tt] =
        (1-Π).*( leisure_value_t(θ,tt)
        + β*DATA[EV_a1_x][DATA[:A].==tt] )
        + Π.*( y + β*DATA[EV_a1_x1][df[:A].==tt] )
        + exp(.5*σ_e^2)*wage_eqn(θ,X_a_p1, zeros(N)).*
        ( 1 - normcdf( (E_g_fun(θ,X_a_p1,tt) - σ_e^2)/σ_e ) )
end


function unpackparams(θ::Array{Float64})
  d = minimum(size(θ))
  θ = squeeze(θ,d)
  γ_1 = θ[1]
  γ_2 = θ[2]
  α_1 = θ[3]
  α_2 = θ[4]
  α_3 = θ[5]
  σ_e = θ[6]

  return [
  "γ_1" => γ_1,
  "γ_2" => γ_2,
  "α_1" => α_1,
  "α_2" => α_2,
  "α_3" => α_3,
  "σ_e" => σ_e
  ]
end
```

```
function least_sq(
        X::Union(Array,DataArray,Float64),
        Y::Union(Array,DataArray,Float64);
        N=int(size(X,1)), W=1
        )

  l = minimum(size(X))
  A = X'*W*X
  if sum(size(A))== 1
    inv_term = 1./A
  else
    inv_term = A\eye(int(size(X,2)))
  end
  β = inv_term * X'*W*Y
  if l == 1
    sigma_hat = sqrt(sum((1/N).* (Y - (β*X')')'*(Y - (β*X')'  ) ) ) #sum converts to Float64
  else
    sigma_hat = sqrt(sum((1/N).* (Y - (X*β))'*(Y - (X*β)  ) ) ) #sum converts to Float64
  end
  VCV = (sigma_hat).^2 * inv_term * eye(l)
  return β, sigma_hat, VCV
end



function λ(t)
    normpdf( probit_input(t) )./(1-normcdf(probit_input(t)))
end
```

5.8 Probit Functions

Listing 5: Functions used

```
## Normal PDF
function normpdf(x::Union(Vector{Float64}, Float64, DataArray) ;mean=0,var=1) # a type-union
     should work here and keep code cleaner
    out = Distributions.pdf(Distributions.Normal(mean,var), x)
    out + (out .== 0.0)*eps(1.0) - (out .== 1.0)*eps(1.0)
end

## Normal CDF
function normcdf(x::Union(Vector{Float64}, Float64, DataArray);mean=0,var=1)
    out = Distributions.cdf(Distributions.Normal(mean,var), x)
    out + (out .== 0.0)*eps(1.0) - (out .== 1.0)*eps(1.0)
end


function probit_wrapper(θ::Array{Float64})

        probit_LL(θ)
end

function probit_LL(θ::Vector{Float64})

        P = DATA[symbol("P_$(tt)")][DATA[:A].== tt]

        g_over_sig = normcdf( probit_input(θ))
        out = P.*log( 1 - g_over_sig ) + (1-P).*log(g_over_sig)

        # clean output
        out[isnan(out).==1] = - 1e50
```

```
        out = - sum( out )

        countPlus!(out)
        return out
end

function probit_input(θ::Array{Float64})
        p_vec = unpackparams(θ)
        γ_1 = p_vec["γ_1"]
        γ_2 = p_vec["γ_2"]
        α_1 = p_vec["α_1"]
        α_2 = p_vec["α_2"]
        α_3 = p_vec["α_3"]
        σ_e = p_vec["σ_e"]

        Y_a   = DATA[:Y][DATA[:A].== tt]
        X_a   = DATA[symbol("X_$(tt)")][DATA[:A].==tt]
        EV_x  = DATA[symbol("EV_$(tt+1)_x")][DATA[:A].==tt]
        EV_x1 = DATA[symbol("EV_$(tt+1)_x1")][DATA[:A].==tt]

        term  = [ones(N) Y_a]*[γ_1; γ_2 ] + β*(EV_x - EV_x1)

        term[term .<= 0] = NaN

        g_over_sig  = (log(term) - [ones(N) X_a X_a.^2]*[α_1; α_2;α_3])./σ_e

        return g_over_sig
end
```

```
function printCounter(count)
        if count <= 5
                denom = 1
        elseif count <= 50
                denom = 10
        elseif count <= 200
                denom = 25
        elseif count <= 500
                denom = 50
        elseif count <= 2000
```

```
                denom = 100
        else
                denom = 500
        end
        mod(count, denom) == 0
end


function countPlus!()
  global count += 1
  if printCounter(count)
    println("Eval $(count)")
  end
end


function countPlus!(out::Float64)
  global count += 1
  if printCounter(count)
    println("Eval $(count): value = $(round(out,5))")
  end
    return count
end
```