Homework 8

Labor Economics

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Due Mon, April 13

1 Setup

Individual knows $\Omega_a = (x_a, y, \epsilon_a)$. His wage is

$$\log w_a^* = \alpha_1 + \alpha_2 x_a + \epsilon_a$$

We observe

$$\log w_a = \log w_a^* + \nu_a$$

Individual gets per-period payoff

$$u(\Omega_a) = \begin{cases} y + \gamma_1 + \gamma_2 y & \text{if } p_a = 0 \text{ (eg, no work)} \\ y + \exp\{f(x_a) + \epsilon_a\} & \text{if } p_a = 1 \text{ (eg, work)} \end{cases}$$

Assume iid shocks:

$$\begin{pmatrix} \epsilon_a \\ \nu_a \end{pmatrix} \sim N\left(0, \begin{bmatrix} \sigma_\epsilon^2 & \\ 0 & \sigma_\nu^2 \end{bmatrix}\right)$$

Objective is

$$\max_{\{p_a\}_{a=1}^A} \beta^{a-1} E\left[u(\Omega) | \Omega_a\right]$$

2 Recursive formulation

Write problem recursively for lazy:

$$V_a^0(x, y, \epsilon_a) = \gamma_1 + (1 + \gamma_2)y + \beta E[V_{a+1}(x+1, y, \epsilon_{a+1})]$$

and working:

$$V_a^1(x, y, \epsilon_a) = \exp\{f(x_a) + \epsilon_a\} + y + \beta E[V_{a+1}(x+1, y, \epsilon_{a+1})]$$

Value is

$$V_a(x, y, \epsilon_a) = \max\left\{V_a^0(x, y, \epsilon_a), V_a^1(x, y, \epsilon_a)\right\}$$

We normalize the value of afterlife to 0 after assuming earthly actions can't affect it

$$V_{A+1}(x, y, \epsilon) = 0$$

Let $\mathcal{W}_a$ be the event that we we work, which is

$$p_a = 1 \quad \Leftrightarrow \quad \epsilon_a \geq \underbrace{\log\left(\gamma_1 + \gamma_2 y + E[V^0_{a+1}(x,y)] - E[V^1_{a+1}(x,y)]\right) - \alpha_1 - \alpha_2 x_a}_{g(x,y,a)}$$

Then

$$\Pr(\mathcal{W}_a) = 1 - \Phi(g(x,y,a)/\sigma_\epsilon)$$

3 Backward induction

3.1 Last period

Last period's value is

$$V_A(x,y,\epsilon_A) = \max\{\gamma_1 + \gamma_2 y, \quad \exp(\alpha_1 + \alpha_2 x + \epsilon_A)\} + y$$

Now

$$g(x,y,A) = \log(\gamma_1 + \gamma_2 y) - (\alpha_1 + \alpha_2 x)$$

so

$$\Pr(\mathcal{W}_A) = 1 - \Phi\left(\frac{g(x,y,A)}{\sigma_\epsilon}\right) = \pi(x,y,A)$$

Expected terminal value is

$$\begin{aligned} E[V_A(x,y)] = y + [1 - \pi(x,y,A)]\,(\gamma_1 + \gamma_2 y) + \\ \pi(x,y,A)\left[\exp\{\alpha_1 + \alpha_2 x\}\underbrace{\frac{1 - \Phi\left(\frac{g(x,y,A) - \sigma^2_\epsilon}{\sigma_\epsilon}\right)}{\pi(x,y,A)}\exp\{\frac{1}{2}\sigma^2_\epsilon\}}_{E[e^\epsilon_A|\mathcal{W}^A]}\right] \end{aligned}$$

This can be written as

$$E[V_A(x,y)] = y + [1 - \pi(x,y,A)]\,[\gamma_1 + \gamma_2 y] + \exp\left\{\alpha_1 + \alpha_2 x + \frac{\sigma^2_\epsilon}{2}\right\}\left[1 - \Phi\left(\frac{g(x,y,A) - \sigma^2_\epsilon}{\sigma_\epsilon}\right)\right]$$

3.2 Other periods

This means

$$g(x,y,a) = \log\left(\gamma_1 + \gamma_2 y + \overbrace{E[V^0_{a+1}(x,y)] - E[V^1_{a+1}(x,y)]}^{\Delta EV(x,y,a)}\right) - \alpha_1 - \alpha_2 x_a$$

and

$$\mathcal{W}_a = \{\epsilon_a \geq g(x,y,a)\}$$

so

$$\Pr(\mathcal{W}_a) = 1 - \Phi\left(\frac{g(x,y,a)}{\sigma_\epsilon}\right) = \pi(x,y,a)$$

and

$$
\begin{aligned}
E[V_a(x,y)] = y + [1-\pi(x,y,a)]\left\{\gamma_1 + \gamma_2 y + E[V_{a+1}(x,y)]\right\} \\
+ \pi(x,y,a)E[V_{a+1}(x+1,y)] + \exp\left\{\alpha_1 + \alpha_2 x + \frac{\sigma_\epsilon^2}{2}\right\}\left[1 - \Phi\left(\frac{g(x,y,a)-\sigma_\epsilon^2}{\sigma_\epsilon}\right)\right]
\end{aligned}
$$

Note that we could simply use the general definition for V_a and $g(x,y,a)$ and specify $V_{A+1} = 0$. This would be a bit neater (ie, for each agent, have $A+1$ periods and just say $V_{A+1} = 0$... then start recursion at $a = A$.

4 Estimation

We have states $\Omega_{ia} = (x,y,a,\epsilon)_{ia}$ and control $p_{ia} \in \{0,1\}$.
Immediately we can get parameters governing distribution of non-labor income from a kernel density estimation of observed y_i values. Or, since we know if we know the underlying distribution we just need μ_y and σ_y which can be estimated by $N^{-1}\sum_i y_i$ and $\widehat{SE(y_i)}$. Number of periods is irrelevant and are consistent as $N \to \infty$. .
Remaining parameters are

$$
\theta = \{\alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma_\epsilon^2, \sigma_\nu^2\}
$$

We'll need functions(?) or matrices of(?)

$$
V_a^0(x,y) \quad V_a^1(x,y) \quad g(x,y,a) \quad \pi(x,y,a) \quad w(x,\epsilon;\alpha_1,\alpha_2)
$$

Note that $g(x,y,a)$ is a function of a because it has $\Delta E[V_{a+1}(x,y)]$

4.1 Plan

1. Given θ, calculate functions I think we are just starting at $t = A$ here and so $\Delta E[V_{a+1}(x,y)] = 0$ and we do not need to fix a θ. Can recover g without.

2. Run probit for working in A

3. Estimate wages as

$$
\log w = \underbrace{\alpha_1 + \alpha_2 x + \sigma_\epsilon \lambda(y,x,a)}_{E[\log w_a^* | \mathcal{W}_a]} + \nu
$$

   where

$$
\lambda(x,y,a) = E\left[\frac{\epsilon}{\sigma}\middle|\frac{\epsilon}{\sigma} \underbrace{\geq}_{\text{notes were } \leq} \frac{g(x,y,a)}{\sigma_\epsilon}\right] = \frac{\phi(g/\sigma)}{1-\Phi(g/\sigma)}
$$

4. Compute $\Delta E[V_A(x,y)]$

5. Run probit for $A-1$...