1(a) $10x \equiv 14 \pmod{17}$ $\phi(17)=16$ $10^{16} \equiv 1 \pmod{17}$

$10^{-1} \equiv 10^{15} \equiv 10 \pmod{17}$

k	$10^k \pmod{17}$
1	10
2	15
4	4
8	16
15	12

$12 \times 14 = 120+48 = \underline{168} \pmod{17}$

$= 15 \pmod{17}$

$15 + 17k \quad (k \in \mathbb{Z})$

$10^{-1} \times 14$

$\gcd(10, 17)$

i	r_i	q_i	s_i	t_i
0	17		1	0
1	10	1	0	1
2	7	1	1	-1
3	3	2	-1	2
4	1	3	3	-5
5	0			

$\gcd(17, 10) = 1 = 3 \times 17 - 5 \times 10$

$10^{-1} \equiv -5 \pmod{17}$

$= 12 \pmod{17}$

(b) $10x \equiv 14 \pmod{21}$

$\phi(21) = (3-1) \times (7-1) = 2 \times 6 = 12.$

$10^{12} \equiv 1 \pmod{21}$

$10^{11} \equiv ? \pmod{21}$

$x \equiv 19 \times 14 \equiv 14 \pmod{21}$

$x = 14 + 21k \quad (k \in \mathbb{Z})$

$\gcd(10, 21).$

i	r_i	q_i	s_i	t_i
0	21		1	0
1	10	2	0	1
2	1	10	1	-2.
	0			

$1 = 1 \times 21 - 2 \times 10$

$\therefore \quad 10^{11} \equiv (-2) \pmod{21}$

$\equiv 19 \pmod{21}$

2. $\phi(71) = (71-1) = 70 \Rightarrow 5^{70} \equiv 1 \pmod{71}$

$5^{210} \pmod{71} = (5^{70})^3 \pmod{71}$

$= 1 \pmod{71}$

$12^{70} \equiv 1 \pmod{71}$

$12^{143} \equiv (12^{70})^2 \cdot 12^3 \equiv 12^3 \pmod{71}$

$144 \equiv 2 \pmod{71}$

$144 \times 12 = 24 \pmod{71}$

3. $\phi(2m)$

$\phi(n) \cdot \phi(n) \quad n=2 \quad \phi(n) = (2-1) = 1.$

$m = p_0^{\boxed{k_1}} \cdots p_n^{k_n}$

$p=2.$

$k \geq 1$

$m = p_0^{k_0} \cdot p_1^{k_1} \cdots p_n^{k_n}$

$\phi(m) = (p_0-1) \times (p_1-1) \times \cdots \times (p_n-1)$

4. $\phi(300) = (2-1) \times (3-1) \times (5-1) = 2 \times 4 = 8$

$\Downarrow$

$300 = 2^2 \times 3 \times 5^2.$

$7^{2000} = (7^8)^{250} = 1 \pmod{300}$