For $ax = b$. $x = b \cdot a^{-1}$

Similarly $Ax = b \Rightarrow x = A^{-1} \cdot b$ for good A.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$$x_1 = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

Example: $\begin{cases} 3x + 4y = 7 \\ 6x + 5y = 11 \end{cases}$ $R_2 - 2R_1 \Rightarrow (6x+5y) - (6x+8y) = -3y = -3$.

$y = 1 \quad x = 1$.

$$\begin{cases} ax + by = b_1 \\ cx + dy = b_2 \end{cases} \qquad R_2 - \frac{c}{a}R_1 \Rightarrow cx + dy - (cx + \frac{bc}{a}y) = (b_1 - b_2)$$

$$\left(\frac{ad-bc}{a}\right)y = (b_1 - b_2)$$

$$y = \frac{(b_1 - b_2)\,a}{ad-bc}$$

$$\Downarrow$$

$$(ad-bc) \neq 0$$

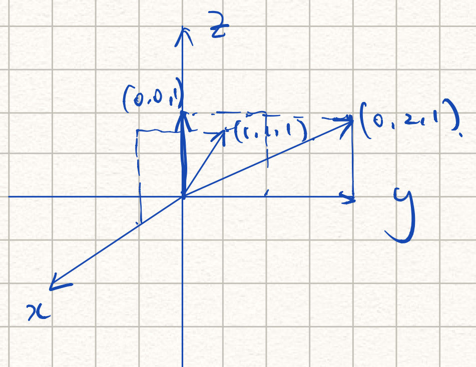
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

if $ad-bc$ is not zero, there is one unique solution y

If $ad - bc = 0$. infinite solution / no solution.

$$\begin{aligned} x + y + z &= \cdots \\ 2y + z &= \cdots \\ z &= \cdots \end{aligned}$$



To understand $A \cdot B = C$ (matrix products)

$x = A^{-1} \cdot b$.

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \quad \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{pmatrix}$$

$(2,3)\times\begin{pmatrix}6\\8\end{pmatrix}$ $\quad$ $(2,3)\times\begin{pmatrix}7\\9\end{pmatrix}$

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix} = \begin{pmatrix} 2\cdot 6 + 3\cdot 8 = 36 & 2\cdot 7 + 3\cdot 9 = 41 \\ \vdots & \vdots \end{pmatrix}$$

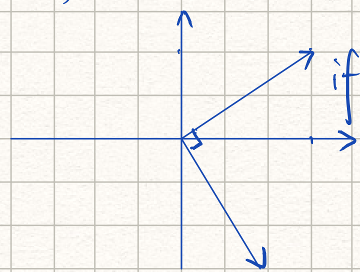
$$c_{ik} = (a_{i1} \cdots a_{in}) \cdot \begin{pmatrix} b_{1k} \\ \vdots \\ b_{nk} \end{pmatrix} = a_{i1}\cdot b_{1k} + a_{i2}\cdot b_{2k} + \cdots + a_{in}\cdot b_{nk}$$

ith row $\quad$ kth column

$$= \sum_{j=1}^{n} a_{ij}\cdot b_{jk}$$

$$E = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$$(2 \;\; -3) \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 6 - 6 = 0$$



if two perpendicular

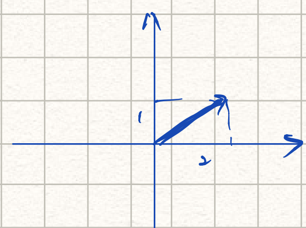
their scalar product is 0.

$C_{11} = a_{11}\cdot 1 + \cdots 0 \cdots = a_{11}$.

$C_{12} = a_{11}\cdot 0 + a_{12}\cdot 1 + \cdots 0 = a_{12}$.

$C_{13} = \quad a_{13} \cdots$

$$(2, 1)\begin{pmatrix} 2 \\ 1 \end{pmatrix} = d^2 = (\sqrt{5})^2$$



$$A \cdot A^{-1} = E.$$

$$A \cdot \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nn} \end{pmatrix} = E.$$

$$\det(A\cdot A^{-1}) = \det(E) = 1$$

$$A = \begin{pmatrix} \lambda_1 & & * \\ 0 & \ddots & \\ 0 & \cdots 0 & \lambda_n \end{pmatrix}.$$

$$\det(A) = \lambda_1 \cdot \lambda_2 \cdots \lambda_n \neq 0$$

Row operations

$$\left.\begin{matrix} 3x + 5y = 7 \\ 4x + y = 1 \end{matrix}\right\} \quad A = \begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$\det[A] = 3 - 20 = -17$$

$R_2 - \frac{4}{3} R_1$

$$A = \begin{pmatrix} 3 & 5 \\ 0 & -\frac{17}{3} \end{pmatrix} \quad b = \begin{pmatrix} 7 \\ -\frac{25}{3} \end{pmatrix}$$

y

$$\det[A] = 3 \cdot \left(-\frac{17}{3}\right) - 0 = -17.$$