Consider $Ax = B \Rightarrow$ matrix.

$A \Downarrow$ matrix, x tuple

$Ax = b$.

Example
$$\begin{cases} x + y + z = -1 \\ 2x + y + z = 0 \\ 3x + 4y + 5z = 0 \end{cases} \qquad A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 4 & 5 \end{pmatrix} \quad b = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Goal: simplify this system

$$\begin{array}{l|ccc|c} R_1 & 1 & 1 & 1 & 1 \\ R_2 - 2R_1 \Rightarrow & (0 & -1 & -1) & (-2) \\ R_3 - 3R_1 \Rightarrow & 0 & 1 & 2 & (-3) \\ & & & & + \\ & 0 & 0 & 1 & (-5) \end{array}$$

$\Downarrow$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 1 & -5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -5 \end{array}\right) \Rightarrow \; R_2 - R_3 \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -5 \end{array}\right)$$

$\Downarrow$

$$R_1 - R_3 \left(\begin{array}{ccc|c} 1 & 1 & 0 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -5 \end{array}\right)$$

$\Downarrow$

$$R_1 - R_2 \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -5 \end{array}\right)$$

$\det(A) \neq 0$

$\Downarrow$

$Ax = b \rightsquigarrow A'x = b'$

$A' = E \qquad Ex = b'$

$x = E^{-1} \cdot b' = E \cdot b' = b'$

By Gaussian Transformation

we can reach A: Entity Matrix

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & & 1 & 0 \\ 0 & \cdots & & \cdots & 1 \end{pmatrix}$$

$A'X = B'$ Happens $A' = E$

$E \cdot X = B'$

$\Downarrow$

$X = B'$

$AX = E$

$EX = B'$

$(A|E) \rightsquigarrow (E|B') \quad B' = A^{-1}$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

swap rolws.

$$\begin{vmatrix} c & d \\ a & b \end{vmatrix} = bc - ad = -\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$Av = \lambda v$. The two vectors does not match on the same line.

Vector $\quad \underset{\sim}{v \neq 0}$

$(A - \lambda)v = 0$

$(A - \lambda E)v = 0 \quad$ vector.

$\because v \neq 0$

$\therefore \underbrace{|A - \lambda E|}_{\text{matrix}} = 0$

$$\begin{cases} x' = 2x + 2y \\ y' = x + 5y \end{cases} \quad A = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix} \quad A - \lambda E = \begin{pmatrix} 2-\lambda & 2 \\ 2 & 5-\lambda \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} 2-\lambda & 2 \\ 2 & 5-\lambda \end{vmatrix} = \lambda^2 - 7\lambda + 10 - 4 = \lambda^2 - 7\lambda + 6 = (\lambda - 1)(\lambda - 6)$$

$\lambda_1 = 1 \quad \lambda_2 = 6$

① $A - \lambda E = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$v_1 + 2v_2 = 0 \qquad v_1 = -2v_2.$

$2v_1 + 4v_2 = 0$

② $A - \lambda E = \begin{vmatrix} -4 & 2 \\ 2 & -1 \end{vmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Take $(-2, 1)$ for example

$$\begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -4+2 \\ -4+5 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$\begin{cases} x' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \end{cases}$

$A = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \quad |A - \lambda E| = \begin{vmatrix} \frac{\sqrt{2}}{2} - \lambda, & \frac{\sqrt{2}}{2} \end{vmatrix} \quad \frac{1}{2} + \lambda^2 - \sqrt{2}\lambda + \frac{1}{2}$

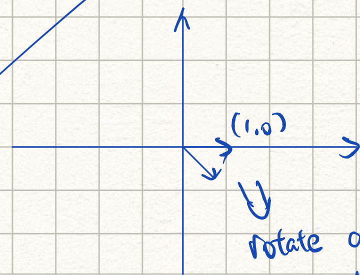
$$y' = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y$$

$$\begin{pmatrix} \ldots \\ -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \end{pmatrix} \qquad \begin{vmatrix} \ldots \\ -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}-\lambda \end{vmatrix} = \ldots$$

$$= \lambda^2 - \sqrt{2}\lambda + 1$$

no real lambda $A v = \lambda v$.

$$A \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2}, & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}, & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}$$



$\Downarrow$ rotate a vector by angle α.

$\det(A) = 1$.
only rotation

$\det(A) \neq 1$.
not only rotation
also scaling

Boolean Tuples.

$\underset{2}{\cdot} \quad \underset{2}{\cdot} \quad \underset{2.}{\cdot} \quad 2^3 = 8$ possibilities

$\nearrow$
Basic Counting Principle.

Cartesian Product. $A \times B = \{(x,y) \mid (x \in A) \wedge (y \in B)\}$.

	A_1	A_2	A_3
B_1			
B_2			

6 choices $= 3 \times 2$

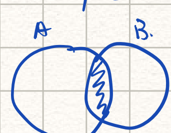
$(1 \; \ldots \; n)$
$\Downarrow$ rearrange
$(a_1 \; \ldots \; a_n)$

$n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1 = n!$ possibilities.

The multiplication principle in counting.

$|A \times B| = |A| \cdot |B|$.

$|A \cup B| = |A| + |B| - |A \cap B|$. Inclusion / Exclusion principle



$|A| = 90.$ $|B| = 90$ $|A \cap B| = 85$

$|A \cup B| = 95$