COMP0147_18-19

6301: HW7

~~**Due Monday 13 March**.~~
~~Hand in solutions to questions 1(a)i, 1(b)ii, 3(b), 3(c), 4.~~

1. (a) Find $\phi(245)$ and calculate:
   i. $4^{169} \mod 245$,
   ii. $13^{1696968} \mod 245$.

   (b) Solve the congruences
   i.
   $$x^{101} \equiv 2 \mod 245,$$
   ii.
   $$y^{29} \equiv 1 \mod 245.$$

2. Solve the congruence $x^{11} \equiv 5 \mod 41$.

3. Consider the matrices:
$$A = \begin{pmatrix} 3 & 2 & 9 & 1 \\ 3 & 1 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 2 & 2 & 9 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}.$$
   (a) Calculate AB, BC
   (b) $(AB)C$ and $A(BC)$. Check that they are equal (associativity).
   (c) Calculate C^{-1}.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear map defined by the matrix
$$\begin{pmatrix} 2 & 1 \\ 3 & -1 \\ 5 & 4 \end{pmatrix}.$$
   Is this map injective?