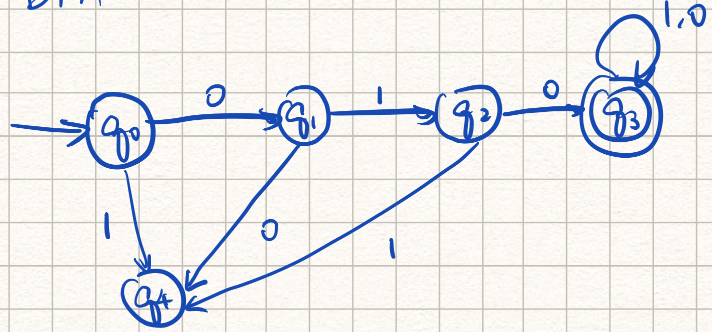
1 DFA.

1.



$Q = \{q_0, q_1, q_2, q_3, q_4\}$.

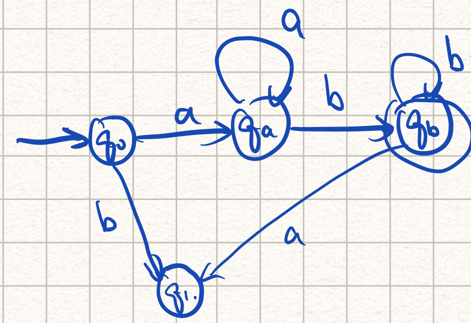
$\Sigma = \{0, 1\}$.

δ $(q_0, 0) = q_1$
$(q_0, 1) = q_4$
$(q_1, 0) = q_4$
$(q_1, 1) = q_3$.

$q_0 = q_0$

$F = \{q_3\}$.

2.



$Q = \{q_0, q_1, q_a, q_b\}$

$\Sigma = \{a, b\}$.

$\delta(q_0, a) = q_a$ ---.

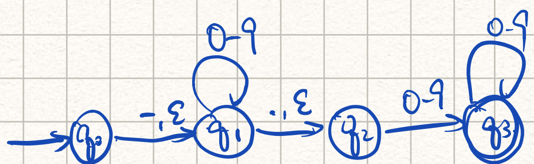
$q_0 = q_0$

$F = \{q_b\}$.

3. $\{w \mid w$ start and end with the same number$\}$.

2. NFA.

4.



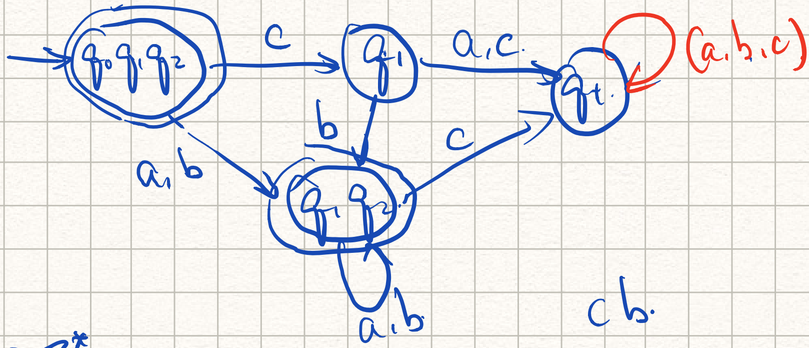
5 No. The technique works as DFA is deterministic,
meaning you could only be in one state when reading a string
For, NFA, as you can be in multiple states,
some strings might not be able to reject

6. Construct NFA N'.

All original N's accept states are connected with a $\xrightarrow{\varepsilon}$ transition
from a new start state. original start state becomes accept

state.

7. 



cb.

8. $010\Sigma^*$

a^+b^+

$(0\Sigma^*0)\cup(1\Sigma^*1)$

$cb\{a,b\}^*\cup\{a,b\}^*$

1. Regex

$R ::= a$
ε
ϕ } base case

$R_1 \cup R_2$.
$R_1 \circ R_2$.
R_1^*.

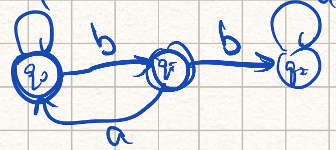
$R_1 \cup R_2$ $w_1 \in R_1$ $w_2 \in R_2$.
$w = w_1 \cup w_2 \in R$.
$w_1^r \in R_1^r$, $w_2^r \in R_2^r$
$w^r = w_1^r \cup w_2^r \in w^r$.
$R^r = R_1^r \cup R_2^r$

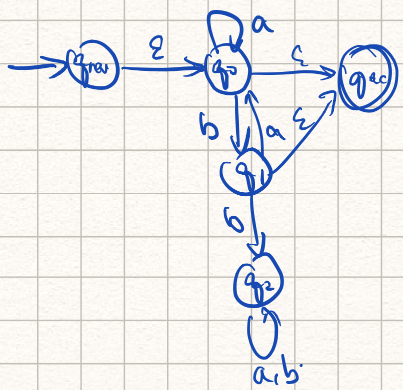
$R_1 \circ R_2$. $w_1 \in R_1$ $w_2 \in R_2$
$w = w_1 \circ w_2 \in R$.
$w^r = w_2^r \circ w_1^r \in R^r$
$R^r = R_2^r \circ R_1^r$

R_1^* $w \in R_1$
$w = w_1 \cdots w_n$
$w^r = w_n^r \cdots w_1^r \in R_1^r$
$R^r = R_1^{r*}$

2.

a   a, b.





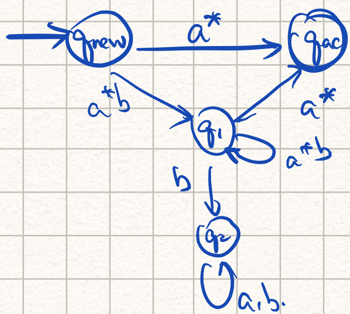
Remove q_0

$q_n \to q_{ac}$: a^*

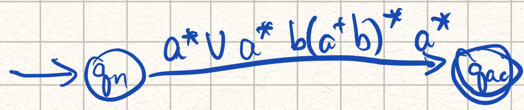
$q_n \to q_1$: a^*b

$q_1 \to q_{ac}$: a^*

$q_1 \to q_1$: $aa^*b = a^+b$



Remove q_1: $q_n \to q_{ac}$: $a^* | a^*b(a^+b)^*a^*$



2. Pumping lemma.

3. $\{ww^R \mid$ for some $w \in \Sigma^*\}$ Assume pumping length p

$a^p b b a^p$ $\quad |xy| \le p \quad xy^iz \in L \quad |y| > 0$

$|xy| \le p$

y contain all a's

pump y $\quad \underbrace{a^{p+k}}\, b\, b\, \underbrace{a^p}$

not match.

4. Given $L = \{a^i b^j \mid i > j\}$ is not regular.

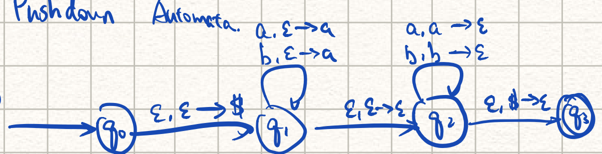
Construct $w = a^p b^{p-1}$

$|xy| \le p$ $\quad$ y must contain all a (at least 1 a)

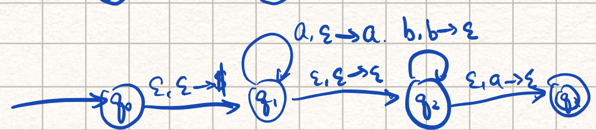
If y's pumped down $\quad a^{p-k} b^{p-1}$ contains equally or less as than bs

3. Pushdown Automata.

5



6



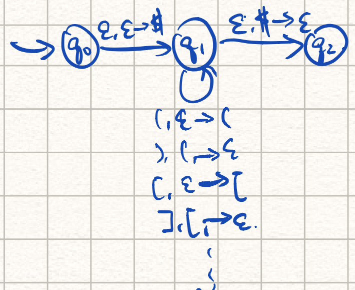
1. CFG

1. $S \to (S) \mid [S] \mid \{S\} \mid \varepsilon$.

2. odd number of 0's/1's

$w = \{0,1\}^*$ with $len(w) \equiv 1 \pmod 2$

2. PDA.



3. CFL Pumping lemma.

$L = \{a^i b^j c^i d^j \mid i, j \geq 0\}$ is not a CFL.

$uvxyz$.

$|vxy| \leq p \quad |vy| > 0$

$w = a^p b^p c^p d^p$.

$|vxy| \leq p$. $v\ y$ can only contain two / one adjacent letter(s).

if $v\ y$ contain one letter each / same / different
pumping up breaks the language

if $v\ y$ contain two adjacent letters

pumping up also breaks

1. Pumping lemma for CFLs

$L = \{a^i b^j c^k \mid k = i * j\}$ is not a CFL.

Construct $w = a^p b^p c^{p \times p}$ for a pumping length p

$\Downarrow$

$uvxyz$. $\underline{|vxy| \leq p}$ $|vy| > 0$.

$|vxy|$ either Case 1: only have one alphabet.

if i pump up $(p+k) \times p \neq p \times p$ $k > 0$

$p \times p \neq p \times p + k$

Case 2: have two alphabets.

a, b, X.

$(p+k_1) \cdot (p+k_2) > \underline{p \times p}$

b, c.

uv^2xy^2z. best case

p $\quad$ $\frac{1-b}{p-1-c} \Rightarrow p-c$ not enough.

Pumping lemma false.

2. L contains equal number of 0's and 1's

Base case: If no 0 exsist, scan 1. if no 1 exsist ACCEPT
else reject.

Inductive case: Scan for first ~~scan~~ 0, → mark $\cancel{0}$
then reset head, scan for first $\cancel{1}$
if not found Base case

$L = \{0^{n^2} \mid n \geq 0\}$.

Base case: if there is no 0 left ACCEPT.

Inductive case: delete $2n+1$ 0s each turn.
n start from 0.
use another tape to keep track of n.

3

4. Let $L = \{\langle M\rangle, M$ is a DFA and $L(M) = \Sigma^*\}$.

L is decidable. (b) (c).

5. Let $L = \{\langle M\rangle \mid M$ is a DFA that doesn't accept any string ... odd 1's$\}$

DFA M' accept string with odd number of 1's

feed $(M \cap M')$ $\longrightarrow$ accept if $L(M \cap M') = \phi$.