Problem Sheet 1.

Ex. 1.

1. $((p\vee q)\wedge(\neg p\to\neg q)) = (p\vee q)\wedge(p\vee\neg q)$

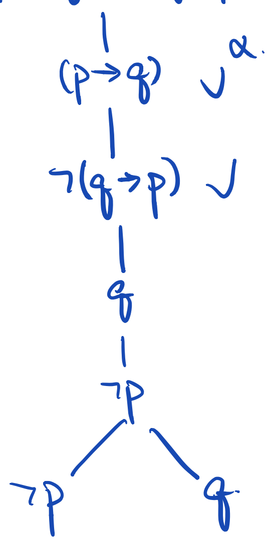
$= [(p\vee q)\wedge p]\vee[(p\vee q)\wedge\neg q]$

$= p\vee(q\wedge p)\vee(p\wedge\neg q)$

$= p\vee(p\wedge\neg q)$

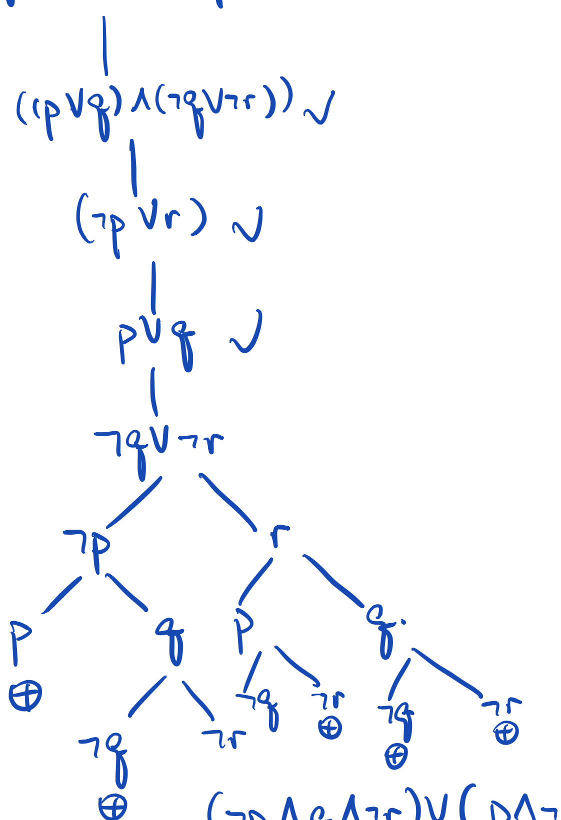
$= p.$

2. $\neg((p\to q)\to(q\to p)).$ ✓



$\neg p\vee q$

3. $((p\vee q)\wedge(\neg q\vee\neg r))\wedge(\neg p\vee r)$ ✓



$(\neg p\wedge q\wedge\neg r)\vee(p\wedge\neg q\wedge r).$

Ex 2.

1. $\exists n\ \neg\exists n_0\ (E(n_0, n) \vee E(n, n_0))$.

2. $\forall n\ (R(n) \vee B(n)) \vee \neg(R(n) \wedge B(n))$

3. $\forall n\ (B(n) \to \exists n_0\ (R(n_0) \wedge (E(n_0, n) \vee E(n, n_0))))$

4. $\forall x \forall y\ (x = y \vee E(x,y) \vee \exists z (E(x,z) \wedge E(z,y)) \vee \exists z \exists w (E(x,z) \wedge E(z,w) \wedge E(w,y)))$

5. reflexive

$\forall x\ E(x,x)$.

Symmetric

$\forall x \forall y\ E(x,y) \to E(y,x)$

transitive

$\forall x \forall y \forall z\ E(x,y) \wedge E(y,z) \to E(x,z)$

Ex. 3.

1. true.
2. true
3. false.
4. true.
5. true

Problem Sheet 2.

1.

$\neg(p \to p)$ α

p

$\neg p$.

$\otimes$

$\neg(p \to p)$ is not satisfiable.

$\therefore p \to p$ is valid.

$\neg(p \vee \neg p)$.

$\neg p$

p

$\otimes$

$p \vee \neg p$ is valid.

$\neg((p \to q) \to (p \to (q \vee r)))$ ✓

$p \to q$ ✓

2. $(p \to q) \to (p \to (q \vee r))$ is valid.

$$\neg(p \to (q \lor r)) \checkmark$$

$$p$$

$$\neg(q \lor r) \checkmark$$

$$\neg q$$

$$\neg r$$

$$\begin{array}{cc} \neg p & q \\ \otimes & \otimes \end{array}$$

$$\neg(((p \to r) \land (q \to r)) \to ((p \lor q) \to r)) \checkmark$$

$$((p \to r) \land (q \to r)) \checkmark$$

$$\neg((p \lor q) \to r) \checkmark$$

$$p \to r \checkmark$$

$$q \to r \checkmark$$

$$p \lor q \checkmark$$

$$\neg r$$

$$\begin{array}{cc} \neg p & r \\ & \otimes \end{array}$$

$$\begin{array}{cc} p & q \\ \otimes & \end{array}$$

$$\begin{array}{cc} \neg q & r \\ \otimes & \otimes \end{array}$$

$\therefore$ $((p \to r) \land (q \to r)) \to ((p \lor q) \to r)$ is valid.

$$\neg(((p \lor q) \to r) \to ((p \to r) \land (q \to r))) \checkmark$$

$$((p \lor q) \to r)$$

$\therefore$ $(((p \lor q) \to r) \to (p \to r) \land (q \to r))$

is valid.

$\neg((p\to r)\wedge(q\to r))$ ✓

$\neg(p\to r)$

$\neg(q\to r)$

p

$\neg r$

q

$\neg r$

$\neg(p\vee q)$ | r

$\neg p$ | ⊗

⊗

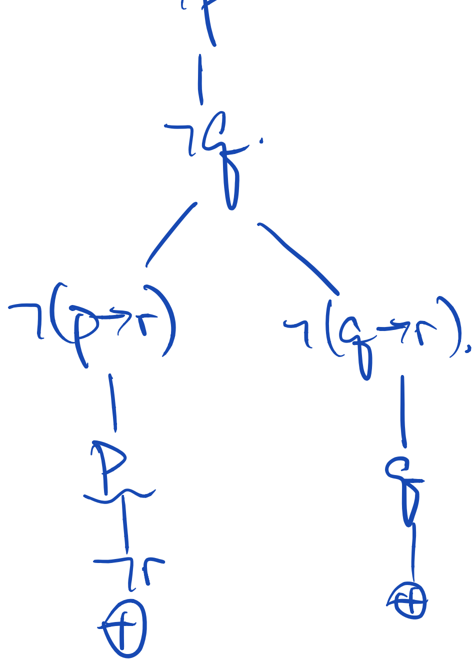
$\neg(((p\vee q)\to r)\to((p\to r)\wedge(q\to r)))$ ✓

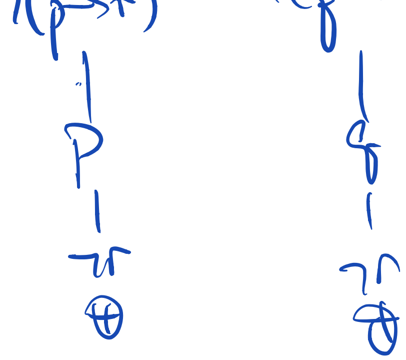
$(p\vee q)\to r$ ✓

$\neg((p\to r)\wedge(q\to r))$ ✓

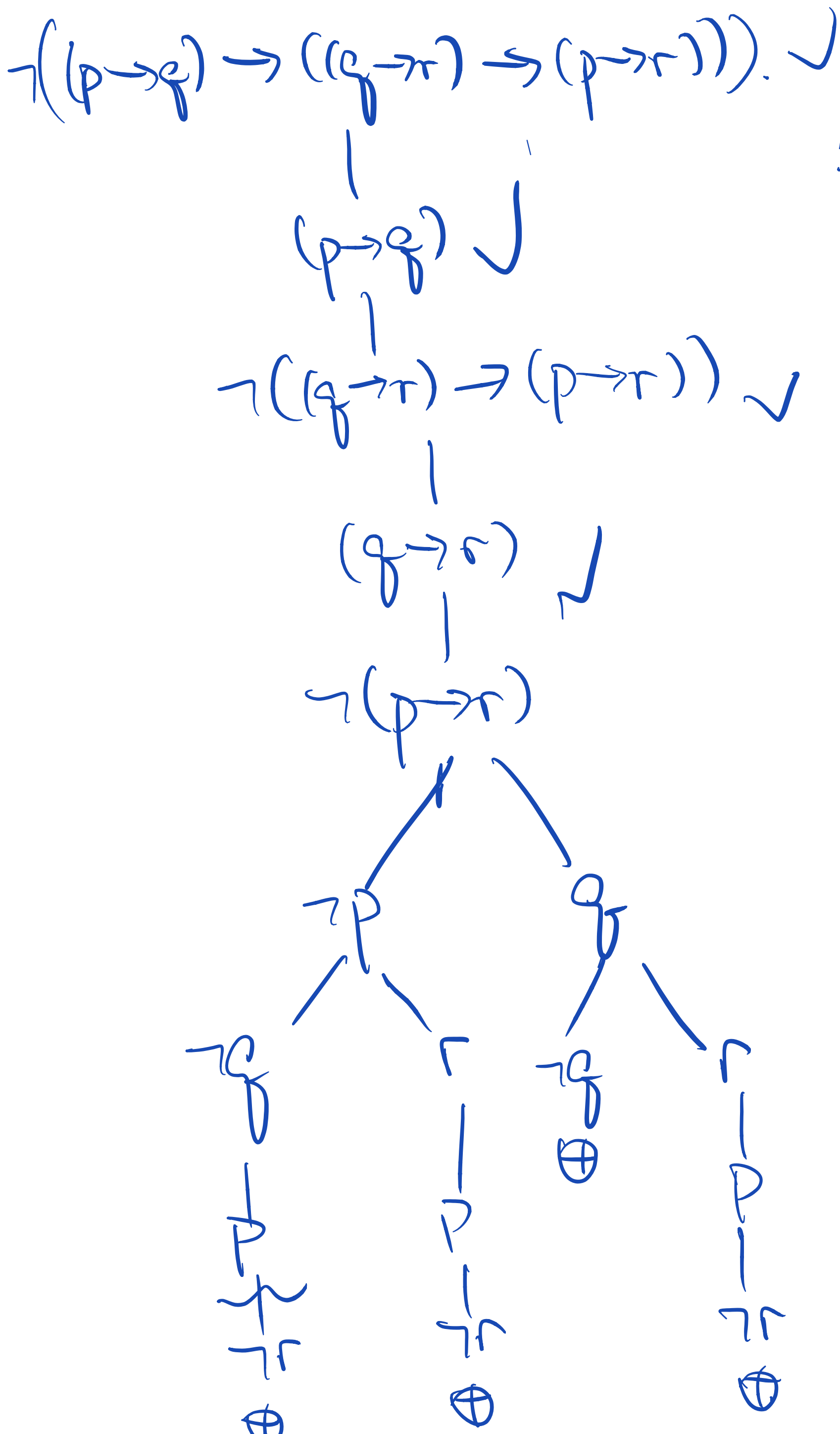
$\neg(p\vee q)$ | r

$\neg p$ | $\neg(p\to r)$ | $\neg(q\to r)$





∴ $((p \vee q) \to r) \to ((p \to r) \wedge (q \to r))$



∴ $((p \to q) \to ((q \to r) \to (p \to r)))$ is valid.

2. two prop are equivalent.

have exact same truth table

$\Downarrow$

i.e. for all valuations v $v(\phi) = v(\phi')$

3. a $\vee$ or $\wedge$ fmlas.

$\Downarrow$

disjunctn of clauses, where each clause is a conjunctn of literals.

$\uparrow$

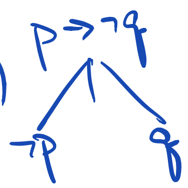
either a prop or a negation of prop

4. take a open branch, conjuct the literals in the branch as a clause

disjunction connects each clause $\rightarrow$ DNF

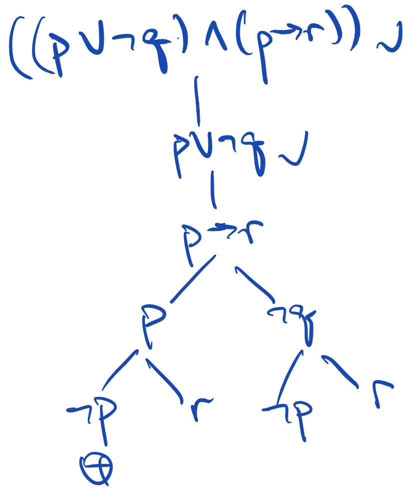
5.

(a)



$\neg p \vee q$.

(b)



$(p \wedge r) \vee (\neg p \wedge \neg q) \vee (\neg q \wedge r)$

$(p \vee \neg q) \wedge (\neg p \vee r)$

$((p \vee \neg q) \wedge \neg p) \vee ((p \vee \neg q) \wedge r)$

$(\neg q \wedge \neg p) \vee (p \wedge r) \vee (\neg q \wedge r)$

(c). $\neg((p \to q) \to (q \to p))$ ✓

$(p \to q)$

$(\neg p \wedge q)$

$\neg(q \to p)$ ✓

q

$\neg p$

$\neg p \qquad q$

(d) $\neg(((p \wedge q) \to r) \leftrightarrow ((p \to r) \vee (q \to r)))$

$\neg(p \wedge q)$ ✓ $\qquad r \qquad (p \to r) \qquad (q \to r)$

$\neg p \quad \neg q \qquad \neg p \quad r \qquad \neg q \quad r$

$\neg p \vee r \vee \neg q \vee r$

$\Leftrightarrow \neg p \vee \neg q \vee r$

$\neg p \vee \neg q \vee r$

f. [redacted]

6 $\qquad \neg((p \to q) \to (q \to p))$ ✓

$(p \to q)$ ✓

$\neg(q \to p)$ ✓

q

$\neg p$

$\neg p$

q

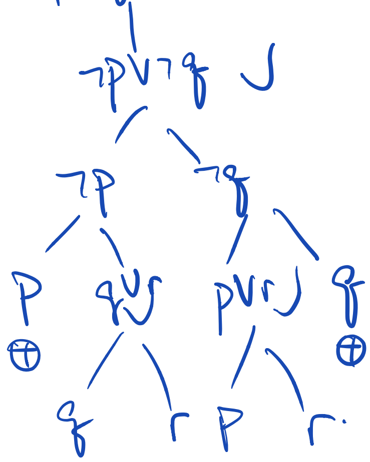
$(\neg p \wedge q)$.

$v(p) = \bot$

$v(q) = T$

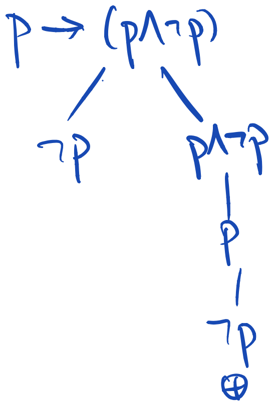
$((p \vee q \vee r) \wedge (\neg p \vee \neg q))$ ✓

$p \vee q \vee r$



$(\neg p\wedge q)\vee(\neg p\wedge r)$
$\vee(p\wedge\neg q)\vee(\neg q\wedge r)$

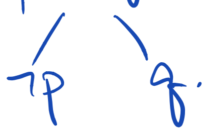
$v(p)=\bot$
$v(q)=\top$
$v(r)=\top$.



$\exists R$: $v(p)=\bot$

7. no

$\phi = p\to q$



ϕ is not valid
if $v(p)=\top$ $v(q)=\bot$.

8. find a DNF for $\neg\phi$.

Assume $DNF(\neg\phi) = A\vee B\vee C$ ——..

$$\phi = \neg(A\vee(B\vee C\text{---})).$$
$$= \neg A\wedge\neg(B\vee C\text{---})$$
$$= \neg A\wedge\neg B\wedge\neg C \text{ ——..}$$

$A = A_0\wedge A_1\wedge A_2$ ——..
$\neg A = \neg(A_0\wedge A_1\text{---})$
$= \neg A_0\vee\neg A_1\vee\neg A_2$ ---.

$\therefore\ \phi = (\neg A_0\vee\neg A_1\text{---})\wedge(\neg B_0$ ——-
it's the CNF of ϕ