Problem Sheet 5. (PS 6 worth looking again)

1. a. 1. $\neg P^2(x,y)$ literal.

2. $\exists x (P^2(x,y) \vee P^2(y,z))$ $\delta$

3. $\neg\exists x \forall y\, P'^2(x,y)$ $\gamma$

4. $\neg(\cdots \vee \cdots)$ $\alpha$

b. let a be a new const. not in the Tableau,

substitute the bound var ? x by a in the fmla, and remove the quantifier

for $\exists x\, \phi(x) \rightarrow$ add $\phi(a)$ below every leaf node

$\neg\forall x \phi(x) \Leftrightarrow \exists x \neg\phi(x)$ add $\neg\phi(a)$ — —

tick the current node.

c. expand $\alpha$ nodes first, as they keep in the same branch.

$\delta$ nodes next, as it remains in one branch, and have no prerequisites, just make a new const. c.

$\beta$ nodes then, as it introduces new branches, would potentially complicate tableau if expanded first.

for $\gamma$ nodes, expand it whenever it can close a branch.

( ~~expand~~ $\gamma$ nodes in queue to ensure fairness make sure all closed terms $t$ in Tableau gets a $\gamma$ expansion ).

d. $\forall x \forall y (P^2(x,y) \to \neg P^2(y,x)) \wedge \exists x P^2(x,x)$ ✓

$\forall x \forall y (P^2(x,y) \to \neg P(y,x))$

$\exists x\, P(x,x)$. ✓

$P(a,a)$

$P(a,a) \to \neg P(a,a)$

$\neg P(a,a)$ ⊕ $\qquad\qquad$ $\neg P(a,a)$ ⊕.

not satisfiable.

2. $\exists x\, Q(x) \wedge \forall x \exists y\, P(x,y)$ ✓

$\exists x\, Q(x)$ ✓

Satisfiable.

$\forall x \exists y P(x,y)$

$|$

$Q(a)$

$|$

$\exists y P(a,y) \checkmark$

$|$

$P(a,b)$

$|$

$\exists y P(b,y)$

$|$

$P(b,c)$

$|$

$\vdots$

3. $\exists x \forall y P(x,y) \land \neg \forall x \exists y P(y,x) \checkmark$

$|$

$\exists x \forall y P(x,y) \checkmark$

$|$

$\neg \forall x \exists y P(y,x) \Leftrightarrow \exists x \neg \exists y P(y,x) \checkmark$

$|$

$\forall y P(a,y)$

$|$

$\neg \exists y P(y,b) \Leftrightarrow \forall y \neg P(y,b)$

$|$

$P(a,b)$

$P(a, b)$.

|

$\neg P(a, b)$

$\oplus$.

not satisfiable.

Problem Sheet 6.

1. $\Gamma \vdash \phi$ $\Gamma$ proves $\phi$

$\exists\ \phi_1, \phi_2 \cdots\cdots \phi$,

$\phi_k$ is either axiom

from inference rule.

or fmla in $\Gamma$

$\Gamma$ is hypotheses.

2. $\Gamma \models \phi$. $\forall S\ \ S \models \Gamma \rightarrow S \models \phi$.

3.

Inconsistent

$\Sigma \vdash p \wedge \neg p$ ($\Sigma \vdash \bot$.).

4. $\vdash$ sound.

prove system is sound when all provable fmlas are valid.

$\Gamma \vdash \phi \rightarrow \Gamma \models \phi$.

$\vdash$ strongly complete

all valid fmlas are provable by the proof system.

$\Gamma \models \phi \rightarrow \Gamma \vdash \phi$.

A set $\Sigma$ of sentences is consistent

iff $\Sigma$ has a model

an interpretation under which all formulas are true.
($\Sigma$ is satisfiable)

Soundness implies consistency
if inconsistent $\Gamma \vdash \bot$ ← not valid.

Strong completeness formalizes the concept of logical consequence.

$\Gamma \models \phi \to \Gamma \vdash \phi$.

a inconsistent set can prove anything, so it's still complete but not sound.

5. if every subset of $\Sigma$ has a model, $\Sigma$ has a model. / every finite subset of $\Sigma$ is consistent, $\Sigma$ is consistent.

6. $+(+(\underbrace{+(+(1,1),+(1,1))}_{4}, \underbrace{+(1,+(1,1))}_{3}))$

7. every element in $N$ is named by a closed term.

$0, \; 1, \; +(1,1), \; +(+(1,1),1) \; \dots$

8. $1=1$. $\forall x \forall y\,(x+y=y+x)$.

$1=0$

9. given $F$ is finite

$\exists t \; t<w$ and $t$ is the largest $N$ smaller than $w$.

$I(t) = n \in N$

$I(w) = n+1 \in N$.

$\mathcal{E}$ is true in $I'$ as there is no change in interpretation

$t < w$ is true in $I'$ as $n < n+1$ and anything smaller than $n$ in finite set $F$.

$\therefore$ every statement in $F$ is true
it has a model $\Leftrightarrow$ consistent

10. by compactness theory,

every finite subset $\mathcal{E}^+$ has a model.

$\therefore$ $\mathcal{E}^+$ has a model

11. (read the q).
let $\infty$ be a non-std. number

$\forall x \; x < \infty$

$\forall St(x) \;\; St(x) < Inf(x)$

$\omega, \; \omega+1 \;\; \omega \times \omega$

inf.

$N \models \mathcal{E}$
$N^+ \models \mathcal{E}$.

12. Yes it is true in $N$ and $N \subset N^+$

13. $\forall x(\neg x = 0 \leftrightarrow \exists y(y+1 = x))$
is a fmla of $\mathcal{E}$ which has model and also $N^+$

$m$ cannot need by closed term.
thus not standard.

14. every element of $N^+$ has a

predecessor

15. $\forall P((P(0) \wedge \forall x\,(P(x) \rightarrow P(x+1))) \rightarrow \forall x P(x))$

No. example let $P(x)$ decides if $x$ is a "std" number, expressable thru $L$-term $t$. (not $L^+$) no $w$.

$P(0)$ ✓

$P(x) \rightarrow P(x+1)$ ✓

$P(w)$ ✗ so. doesn't stand.