Problem Sheet 7.

Kripke tree is a graph (directed)

1. (a) $F, V, x. \models \Box q$. false.

(b). $F, V, x \models q \wedge \Box(q \to p)$ true.

(c). $F, V, x. \models \Diamond \Box \bot$. true. $x \to y$.

2.

$\Box \bot$ no outgoing

3. (a) $\Box p \to p$ none of them are reflexive

$\downarrow$

$x \to y$

$y \to x$.

(b) $\Diamond(p \vee \neg p)$

every node has some world to point to

(i). (ii).

Q N

(c)

$\Diamond \Diamond p \to \Diamond p$.

$\Updownarrow$ $\Updownarrow$

$\Diamond \neg \Box \neg p$.

$\Updownarrow$

$\neg \Box \Box \neg p. \to \neg \Box \neg p$.

$\Updownarrow$.

$\Box \neg p \to \Box \Box \neg p$.

all Q, N, N^- are transitive true.

(d). Dense. $\forall x \forall y (E(x,y) \to \exists z \; E(x,z) \wedge E(z,y))$

Q.

(e). $\Box \bot \vee \Diamond \Box \bot$.

no outer $\to$ has at least conn to a point with no outer $\to$.

2. N^-

$n > 0 \to 0$

0

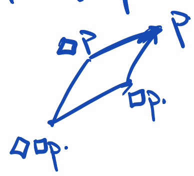
4(a) $F \models \Diamond \top$.

every node in F connects to at least one other node. no end node (world)

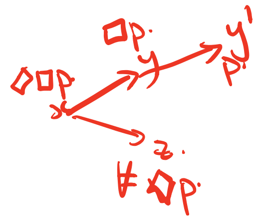
if F is transitive, every other node connect

to each other. !!

(b) $F \models \Diamond\Box p \to \Box\Diamond p$



Convergence if $(x,y)\,(x,z) \in R \to (y,w),(z,w) \in R$.



∴ no $\Diamond\Box p \to \Box\Diamond p$

$G \not\models \Diamond\Box p \to \Box\Diamond p$ not convergent.

Problem Sheet 8.

1. a. 1. false.
2. true
3. false.
4. true.
5. $\Diamond p$ ✓.
$\Box(p \to \Box\neg q)$
$\Leftrightarrow \Box(\neg p \vee \Box\neg q)$. true.

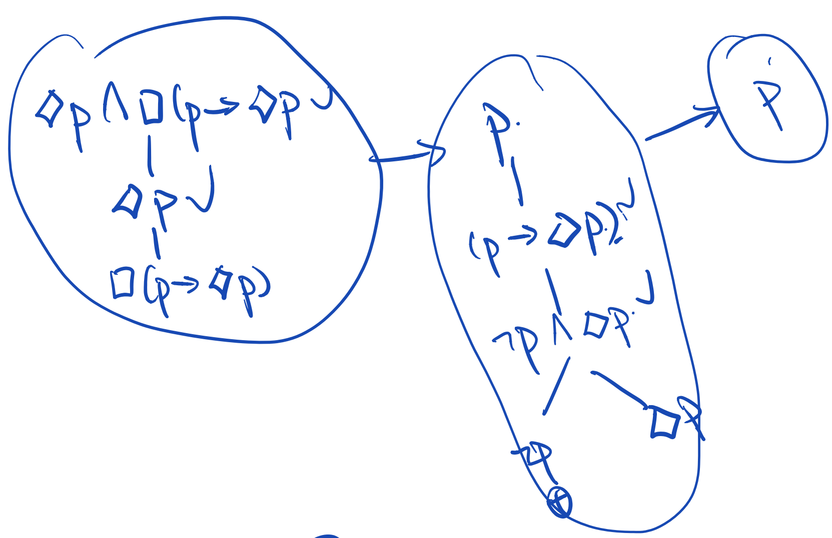
b.

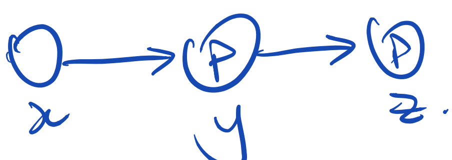
$\Box p \to p$. reflexivity. false.

$\Box p \to \Box\Box p$. transitive true.

$\Box(p \wedge q) \Leftrightarrow (\Box p \wedge \Box q)$. true. axiom

c.





$\Diamond p, \ \Box(p\to\Diamond\neg p), \ \Box(p\vee\Diamond p)$

$\downarrow$

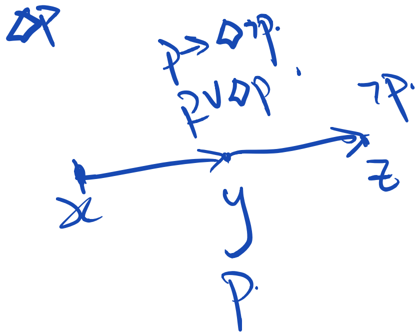
$p, \ p\to\Diamond\neg p, \ p\vee\Diamond p$

$\{p, p, p\to\Diamond\neg p\}$

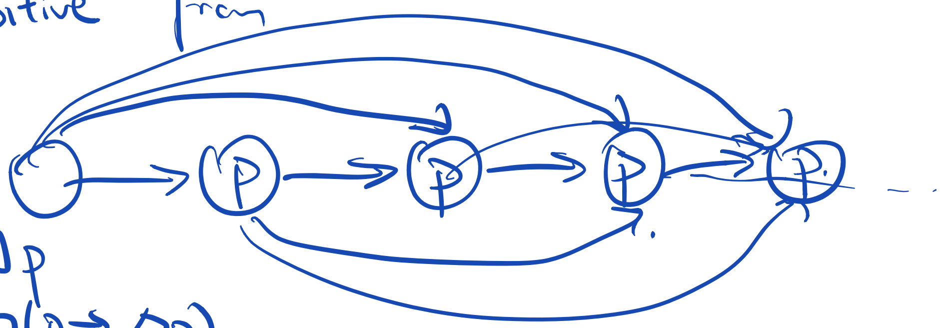
$\{p, \Diamond p, p\to\Diamond p\}$

$\{p, p, \Diamond\neg p\}$

$\{p, \Diamond p, \Diamond\neg p\}$

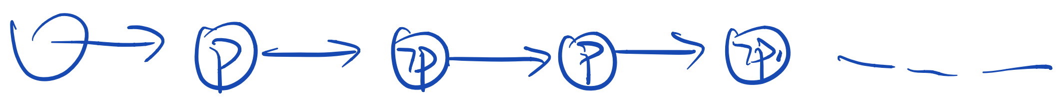


for transitive frame

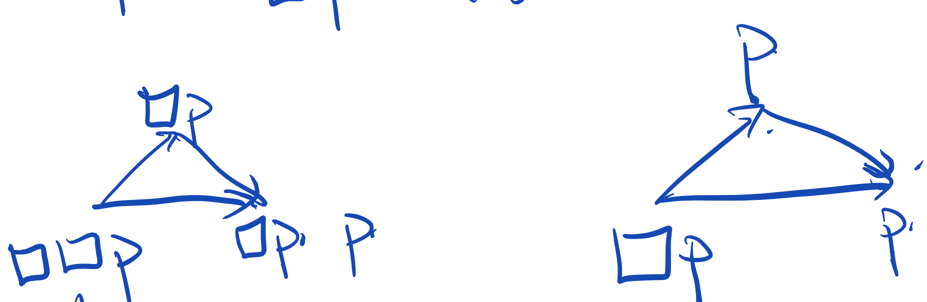


$\Diamond p$

$\wedge\, \Box(p\to\Diamond p)$



d. $\Box\Box p\to\Box p$, dense.



$\downarrow$

$\Box p$