Problem Sheet 5.

1. a. 1. $\neg P^2(x,y)$ literal.

2. $\exists x(P^2(x,y) \vee P^2(y,x))$ δ

3. $\neg\exists x \forall y\, P'^2(x,y)$ γ

4. $\neg(\cdots \vee \cdots)$ α

b. let a be a new const. not in the Tableau,
substitute the bound var? x by a in the fmla, and remove the quantifier.

for $\exists x\, \phi(x)$ → add $\phi(a)$ below every leaf node

$\neg\forall x \phi(x)$ $\Leftrightarrow \exists x \neg\phi(x)$ add $\neg\phi(a)$ — — →

tick the current node.

c. expand α nodes first, as they keep in the same branch.
δ nodes next, as it remains in one branch, and have no prerequisites, just make a new const. C.
β nodes then, as it introduces new branches, would potentially complicate tableau if expanded first.

for γ nodes, expand it wherever it can close a branch.

(expand γ nodes in queue to ensure fairness make sure all closed terms t in Tableau gets a γ expansion).

d. $\forall x \forall y (P^2(x,y) \to \neg P^2(y,x)) \wedge \exists x P^2(x,x)$ ✓

$\forall x \forall y (P(x,y) \to \neg P(y,x)$

$\exists x\, P(x,x)$. ✓

$P(a,a)$

$P(a,a) \to \neg P(a,a)$

$\neg P(a,a)$ ⊕ $\quad$ $\neg P(a,a)$ ⊕.

not satisfiable.

2. $\exists x\, Q(x) \wedge \forall x \exists y\, P(x,y)$ ✓

$\exists x\, Q(x)$ ✓

Satisfiable.

$\forall x \exists y P(x,y)$

|

$Q(a)$

|

$\exists y P(a,y)$ ✓

|

$P(a,b)$

|

$\exists y P(b,y)$

|

$P(b,c)$

|

|

3. $\exists x \forall y P(x,y) \wedge \neg \forall x \exists y P(y,x)$ ✓

|

$\exists x \forall y P(x,y)$ ✓

|

$\neg \forall x \exists y P(y,x) \Leftrightarrow \exists x \neg \exists y P(y,x)$ ✓

|

$\forall y P(a,y)$

|

$\neg \exists y P(y,b) \Leftrightarrow \forall y \neg P(y,b)$

|

$P(a,b)$

$\neg P(a,b)$

$\oplus$.

not satisfiable.

Problem Sheet 6.