COMP0009 Revision Notes

Formal Logic: Syntax (how to write), Semantics (what it means), proof system (Inference) deduction proofs. (how to reason)

Propositional Logic

prop ::= p | q | r | ...

fm ::= prop | ¬fm | fm ∘ fm

↑ an operand ∧, ∨ or →

Literal: prop or its negation p / $\neg p$

Main Connective: $(p \wedge q) \vee \neg(q \to r)$

the connective with the largest scope (most outside, highest lvl)

Semantics

v → evaluation. $\top$ truth, $\bot$ false.

$v(\neg\phi) = \top \iff v(\phi) = \bot$

$v(\phi \wedge \psi) = \top \iff v(\phi) = v(\psi) = \top$.

$v(\phi \vee \psi) = \top \iff v(\phi) = \top$ or $v(\psi) = \top$.

$v(\phi \to \psi) = \top \iff v(\neg\phi) = \top$ or $v(\psi) = \top$

Validity, Satisfiability, Equivalence.

ϕ is valid if $v(\phi) = \top$ for all types of valuations v. (always true)

ϕ is satisfiable if $\exists v$ $v(\phi) = \top$. (true at least once)

ϕ and ψ is logically eq. iff $\forall v$ $v(\phi) = v(\psi) \Rightarrow \phi \equiv \psi$

All valid fmla are satisfiable

Predicate Logic

Language $L(C, F, P)$

C constant symbols.

F function symbols f^n (n-ary)

P a nonempty predicate (relation) symbol set P^n (n-ary)

$tm ::= v : v \in Var \mid c : c \in C \mid f^n(tm \ldots tm) : f^n \in F$.

$3 + (x \times 2) \iff +(3, \times(x, 2))$.

$2, 3 \in C \quad x \in Var.$

$+, \times \in f^2.$

$atom := p^n(tm, \dots tm) : p^n \in P.$

$x + y < 2 \times y - 1 \quad < \text{ is a } P^2$

$fm := atom \mid \neg fm \mid (fm \vee fm) \mid \exists v\, fm : v \in Var.$

$(fm \wedge fm)$

$(fm \to fm)$

$\forall x \phi$ as abbr. $\neg \exists x \neg \phi \qquad \forall x \phi \leftrightarrow \neg \exists x \neg \phi$