Problem Sheet 3.

1. a. if ϕ is valid. let $M=(D,I)$

$M,A \models \phi \Leftrightarrow \forall A_0 \quad M,A_0 \models \phi.$

$\Leftrightarrow \neg\exists A_0 - M,A_0 \models \neg\phi.$

$\Leftrightarrow M,A \not\models \neg\phi.$

b. $\forall x \exists y\, (E^2(x,y) \wedge \forall z (E^2(y,z) \vee y=z))$

Graph 1 satisfies. b, c connects to every other node

$a \to b. \to \{N\}$

$d \to c \to \{N\}$

$b \to c \to \{N\}$

$c \to b. \to \{N\}$

Graph 2 doesn't. d no $\to$.

3 ——— $d \to b.$ $d \to c$

but b doesn't have $E(b,d)$ So. falsify.

but c doesn't have $E(c,a)$ So falsify

c.

1. $\forall x\; N(x) \vee Col(x)$

2. $(\forall x\; \neg E(x,x)) \wedge (\forall x \forall y. (E(x,y) \to E(y,x)))$

3. $\forall x\; N(x) \to \exists i\; (Col(i) \wedge p(x,i) \wedge \forall y\; (N(y) \to \exists j (Col(j) \wedge p(y,j) \wedge (x=y \vee \neg(i=j))$
$)$

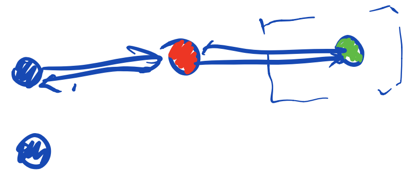
every graph node has a unique color

$\forall x\; N(x) \to \exists i\; (C(i) \wedge p(x,i) \wedge \forall j\, (C(j) \to (i=j) \vee \neg p(x,j)))$

4. $\forall x \; N(x) \rightarrow (\forall y \; E(x,y) \vee E(y,x) \rightarrow \exists i \; \exists j \; p(x,i) \wedge p(y,j) \wedge \neg(i=j))$

5. $\exists i \; \exists j \; \exists k \; \forall x \; N(x) \rightarrow p(x,i) \vee p(x,j) \vee p(x,k)$

6.



Problem Sheet 4.

1. $\neg(\exists x \forall y Cxy \rightarrow \forall x \exists y Cyx)$ ✓

$\exists x \forall y Cxy$ ✓

$\neg \forall x \exists y Cyx \Leftrightarrow \exists x \neg \exists y Cyx$ ✓

$\forall y Cay$

$\neg \exists y Cyb \Leftrightarrow \forall y \neg Cyb$.

Cab.

$\neg Cab$

$\oplus$

2. $\forall x (\exists y Fxy \rightarrow Gbx), \forall x \forall y Fyx. \vdash \exists x Gxx.$

$\forall x (\exists y Fxy \rightarrow Gbx) \wedge \forall x \forall y Fyx \wedge \neg \exists x Gxx$ ✓

$\forall x (\exists y Fxy \rightarrow Gbx)$

$\forall x \forall y Fyx$

$\neg \exists x Gxx \Leftrightarrow \forall x \neg Gxx$

$\neg\exists x\, Gxx \Leftrightarrow \forall x\, \neg Gxx.$

$\exists y\, Fby \to Gbb.$ ✓

$Fbc \to Gbb$ ✓

$\neg Fbc$ $\qquad$ $Gbb.$

Fbc $\qquad$ $\neg Gbb$

$\otimes$ $\qquad$ $\otimes$

3. $\forall x \forall y (\exists z Fyz \to Fxy),\ Fab \vdash \forall y \exists x\, Fyx.$

$\forall x \forall y (\exists z Fyz \to Fxy) \wedge Fab \wedge \neg \forall y \exists x\, Fyx.$

$\forall x \forall y (\exists z Fyz \to Fxy)$

$Fab.$

$\neg \forall y \exists x\, Fyx \Leftrightarrow \exists y \neg \exists x\, Fyx.$ ✓

$\neg \exists x\, Fcx \Leftrightarrow \forall x \neg Fcx.$

$\neg Fca.$

$\exists z Faz \to Fca.$ ✓

$\forall z \neg Faz \Leftrightarrow \neg \exists z Faz$ $\qquad$ Fca

$\neg Fab$ $\qquad$ $\otimes$

$\otimes$

4. $\forall x \forall y (Pxy \to \neg Pyx) \vdash \neg \forall x \neg Pxx.$

$\forall x \forall y (Pxy \to \neg Pyx) \wedge \neg \forall x \neg Pxx$ $\checkmark$

|

$\forall x \forall y (Pxy \to \neg Pyx)$

|

$\neg \forall x \neg Pxx \Leftrightarrow \exists x\, Pxx.$ $\checkmark$

|

Paa

|

$Paa \to \neg Paa.$ $\checkmark$

$\neg Paa$ $\otimes$ | $\neg Paa$ $\otimes$

5. $\forall x(Gx \to Hx), \forall x(Hx \to Fx), Ga \vdash \exists x(Gx \wedge Fx)$

$\forall x(Gx \to Hx) \wedge \forall x(Hx \to Fx) \wedge Ga \wedge \neg \exists x(Gx \wedge Fx)$ $\checkmark$

|

$\forall x(Gx \to Hx)$

|

$\forall x(Hx \to Fx)$

|

Ga

|

$\neg \exists x(Gx \wedge Fx) \Leftrightarrow \forall x \neg (Gx \wedge Fx)$

|

$Ga \to Ha.$

$Ha \to Fa.$

$\neg(Ga \wedge Fa)$

$\neg Ga$ $\quad\quad$ $Ha.$

$\otimes$

$\neg Ha$ $\quad\quad$ Fa

$\otimes$

$\neg Ga$ $\quad\quad$ $\neg Fa$

$\otimes.$ $\quad\quad$ $\otimes$

6. $\neg Cb \wedge \neg Cc,\ Ca \to \forall x Cx \vdash \neg Ca.$

$(\neg Cb \wedge \neg Cc) \wedge (Ca \to \forall x Cx) \wedge Ca.$

$\neg Cb$

$\neg Cc$

$Ca \to \forall x Cx$

Ca

$\neg Ca$ $\quad\quad$ $\forall x Cx.$

$\otimes$

$Cb.$

$\otimes$

7. $\exists x Fxb \to \forall x Gx,\ \forall x Fax \vdash \forall x (Hxc \to Gx)$

$(\exists x Fxb \to \forall x Gx) \wedge (\forall x Fax) \wedge (\neg \forall x (Hxc \to Gx))$

$\exists x Fxb \to \forall x Gx$

$\forall x Fax$

$\neg\forall x(Hxc \to Gx) \Leftrightarrow \exists x \neg(Hxc \to Gx)$

$\neg(Hdc \to Gd).$

Hdc

$\neg Gd.$

$\forall x \neg Fxb \Leftrightarrow \neg\exists x Fxb.$ $\qquad$ $\forall x Gx.$

$\neg Fab.$ $\qquad$ $Gd.$ $\oplus$

Fab $\oplus$

8. $\forall x\forall y(\exists z Ayz \to Axy), \neg Att \vdash \neg Ats$

$\forall x\forall y(\exists z Ayz \to Axy) \wedge \neg Att \wedge Ats$ ✓

$\forall x\forall y(\exists z Ayz \to Axy)$

$\neg Att$

Ats

$\exists z Atz \to Att$

$\forall z \neg Atz \Leftrightarrow \neg\exists z Atz$ $\qquad$ $Att.$ $\oplus$

$\neg Afs$

$\oplus$.

9. $\forall x \exists y\, (Ayx \wedge Cxy),\ Awh \vdash \exists x\, Chx.$

$\forall x \exists y\, (Ayx \wedge Cxy) \wedge Awh \wedge \neg \exists x\, Chx$

$\forall x \exists y (Ayx \wedge Cxy)$

Awh

$\neg \exists x\, Chx \Leftrightarrow \forall x \neg Chx.$

$\exists y\, Ayh \wedge Chy.$ ✓

$Aih \wedge Chi$ ✓

Aih

Chi

$\neg Chi$

$\oplus$

10. $\forall x \exists y\, (Cx \Rightarrow (Py \wedge Axy)) \vdash \forall x\, (Gx \Rightarrow \exists y (Py \wedge Axy))$

$\forall x \exists y (Cx \rightarrow (Py \wedge Axy)) \wedge \neg \forall x\, (Cx \Rightarrow \exists y (Py \wedge Axy))$

$\forall x \exists y ((x \Rightarrow (Py \wedge Axy))$

$\neg\ (Cx \Rightarrow \exists y (Py \wedge Axy))$

$\exists x \neg (Cx \to \exists y (Py \wedge \ldots \Leftrightarrow \neg \forall x (Cx \to \exists y (Py \wedge Axy)))$.

$\neg (Ca \to \exists y (Py \wedge Aay))$.

Ca

$\forall y \neg (Py \wedge Aay) \Leftrightarrow \neg \exists y (Py \wedge Aay)$

$\exists y (Ca \to (Py \wedge Aay)) \checkmark$

$Ca \to Pb \wedge Aab$

$\neg Ca$ $Pb \wedge Aab \checkmark$

$\oplus$

Pb

Aab

$\neg (Pb \wedge Aab)$

$\neg Pb$ $\neg Aab$

$\oplus$ $\oplus$