UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**COMP3004**
ASSESSMENT PATTERN	:	**COMP3004B**
MODULE NAME	:	**Computational Complexity**
DATE	:	**20 May 2016**
TIME	:	**10:00 am**
TIME ALLOWED	:	**2 hours 30 mins**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

2014/15

2015/16-COMP3004B-001-EXAM-3

TURN OVER

Computational Complexity, COMP3004, 2015/2016

Answer BOTH of the TWO questions.

Marks for each part of each question are indicated in square brackets

Calculators are NOT permitted

1. **Preliminaries:** Let $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ denote the natural numbers.

   a. Let $L \subseteq \{a, b\}^*$ be the infinite language

   $$L = \{\epsilon, ab, aabb, aaabbb, aaaabbbb, \ldots\}$$

   Design a TM that decides L (draw a state-transition diagram); and describe in detail how your TM functions.

   [10 marks]

   b. Consider the connected-graph decision problem. An instance of the problem is an undirected graph G. This decision problem asks if the graph is connected or not.

   Give an encoding that encodes instances of this decision problem into a language $L \subseteq \{0, 1\}^*$, and give an example of a string in L and a string not in L for your encoding.

   [10 marks]

   c. What is *Church's thesis*?

   [5 marks]

   d. Define

   $$S := \{X \subseteq \mathbb{N} : \sum_{x \in X} x = \infty\}$$

   thus S is the set of all subsets X of $\mathbb{N}$ such that the sum of the numbers in each set X is infinite. Which of the two possibilities holds?

   let { = 10, “,” = 11, “}”=12

   Then we operate on a base-13 scale

   for {1,2} the Natural number

   representation, this is a injection

   1. S is countable
   2. S is not countable.

   Give a detailed argument which justifies your answer. P(N) is countable

   S = P(N) / Pf(N)

   so S is countable

   [10 marks]

e. Let **HPL** denote the halting problem language and let,

$$\bar{L} := \{x \in \{0,1\}^* : x \notin L\}.$$

Now given the following relationship between A and **HPL**

$$A \leq \textbf{HPL} \leq \bar{A}.$$

Does there exist an $A \subseteq \{0,1\}^*$ such that

1. A is finite. 1,2 not possible
2. A is recursive.
3. A is countably infinite HALT
4. A is recursively enumerable. A = HALT, HATL <= HALT <= HALT-
5. A is not recursive but recursively enumerable. A = HALT, HATL <= HALT <= HALT-
6. A is not recursive. A = HALT, HALT <= HALT <= HALT-
7. A is not recursively enumerable. not possible
8. A and $\bar{A}$ are not recursively enumerable. not possible

For each sub-part when an "A" exists give a particular example and then argue $A \leq \textbf{HPL} \leq \bar{A}$. When an "$A$" does not exist, argue why not. Finally for sub-part 8, full credit will be given for good intuitive arguments.

[15 marks]

[Total=50 marks]

2. a. Define the complexity classes **NP**, **NPC** and **PSPACE**.

[6 marks]

An undirected irreflexive graph (V, E) consists of a set V of vertices, and a set E of two element subsets of V.

An instance of the *graph clique problem* (GCP) is an undirected irreflexive graph $G = (V, E)$ and a positive integer $k \geq 1$. It is a *yes instance* if there is a subset $C \subseteq V$ of size k such that for all pairs of distinct vertices $v \neq w \in C$ we have $\{v, w\} \in E$. It is a *no instance* otherwise.

An instance of the *vertex cover problem* (VCP) is an undirected irreflexive graph $G = (V, E)$ and a positive integer $k \geq 1$. It is a *yes instance* of VCP if there is a subset $S \subseteq V$ of size at most k such that for all $\{v, w\} \in E$ either $v \in S$ or $w \in S$, it is a *no instance* otherwise.

Define undirected graphs $G_1, G_2, G_3(n)$ (for $n \geq 4$) by

$$\begin{aligned} G_1 &= (\{0,1,2,3,4\},\ \{\{0,1\},\{0,2\},\{0,3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}) \\ G_2 &= (\{0,1,\ldots,11\},\ \{\{i,j\} : 2 < |i - j(mod12)|\}) \\ G_3(n) &= (\{0,1,\ldots,n-1\},\ \{\{n-1,0\},\{(i,i+1\} : i < n\}). \end{aligned}$$

b. For each of the graphs G defined above, state the biggest value of k such that (G, k) is a yes instance of GCP.

[8 marks]

c. For each of the graphs G defined above, state the smallest value of k such that (G, k) is a yes instance of VCP (for $G_3(n)$ you may wish to distinguish the cases when n is even from the cases when n is odd).

[8 marks]

d. Prove that GCP belongs to **NP**.

[7 marks]

e. Define a polynomial time reduction: VCP $\leq_p$ GCP.

[8 marks]

f. Define a polynomial time reduction: GCP $\leq_p$ VCP.

[8 marks]

g. Given that VCP is **NPC**, but not assuming anything else, what can you conclude from parts 2.e and 2.f?

[5 marks]

[Total = 50 marks]

END OF PAPER