Candidate No. ....

Seat No. ....

UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **COMP3004**

ASSESSMENT PATTERN : **COMP3004C**

MODULE NAME : **Computational Complexity**

DATE : **04 May 2017**

TIME : **10:00 am**

TIME ALLOWED : **2 hours 30 mins**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

2015/16, 2016/17

<u>Under no circumstances</u> are the attached papers to be removed from the examination by the candidate.

2016/17-COMP3004C-001-EXAM-109

TURN OVER

Computational Complexity, COMP3004, 2016-17

Answer BOTH of the TWO questions.

Marks for each part of each question are indicated in square brackets.

Calculators are NOT permitted.

Q1. **Preliminaries:** Let $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ denote the natural numbers. Let $\mathrm{LA}(M)$ denote the language a TM M semidecides.

a. Let $L \subseteq \{a, b\}^*$ be the infinite language

$$L = \{ab, aaab, aaaaab, aaaaaaab, aaaaaaaaab, \ldots\}$$

Design a TM that decides L (draw a state-transition diagram); and describe in detail how your TM functions.

[10 marks]

b. Define $f : \mathbb{N} \to \{0, 1\}$ as follows

$$f(x) := \begin{cases} 0 & x \text{ is even} \\ 1 & x \text{ is odd} \end{cases}$$

Design a URM that computes f (give the program); and describe in detail how your URM functions.

[10 marks]

c. Let $\Sigma = \{0, 1\}$ and define $S = \{L : L \text{ is a finite language over } \Sigma\}$. Which of the two possibilities holds?

1. S is countable
2. S is not countable.

Give a detailed argument which justifies your answer.

[10 marks]

[Question 1 cont. over page]

d. Recall that the set of all languages S_Σ with the alphabet $\Sigma := \{0, 1\}$ is an uncountable set. Let $C_1, C_2, U \subseteq S_\Sigma$, where the sets of languages C_1 and C_2 are countable and U is uncountable. The complement of a set C is denoted $\bar{C} := \{L \in S_\Sigma : L \notin C\}$.

1. Is the cardinality of $C_1 \cup C_2$,

   (a) Countable.

   (b) Uncountable.

   (c) Dependent on choice of C_1 and C_2.

   Give a proof (or detailed explanation) which justifies your answer. [2 marks]

2. Is the cardinality of $\bar{C}_1$,

   (a) Countable.

   (b) Uncountable.

   (c) Dependent on choice of C_1.

   Give a proof (or detailed explanation) which justifies your answer. [2 marks]

3. Is the cardinality of $\bar{U}$,

   (a) Countable.

   (b) Uncountable.

   (c) Dependent on choice of U.

   Give a proof (or detailed explanation) which justifies your answer. [3 marks]

4. Is the cardinality of $\bar{C}_1 \cap U$,

   (a) Countable.

   (b) Uncountable.

   (c) Dependent on choice of C_1 and U.

   Give a proof (or detailed explanation) which justifies your answer.

   [3 marks]

[Question 1 cont. on next page]

[Question 1 cont.]

e. Let L_{*1*} denote the language that only contains strings with a "1" in them. Hence $0010110 \in L_{*1*}$ but $00 \notin L_{*1*}$.

Given the decision problem,

"Given a TM M does the language that it semidecides contain a string with a '1' in it?"

Consider the corresponding language L, which is

$$L = \{\text{code}(M) : \text{LA}(M) \cap L_{*1*} \neq \emptyset\}$$

Indicate one of three possibilities

1. L is recursive.
2. L is recursively enumerable but not recursive.
3. L is not recursively enumerable.

Give a proof which justifies your answer.

[10 marks]

Q2. a. Define the following complexity classes

i. **NP-complete**

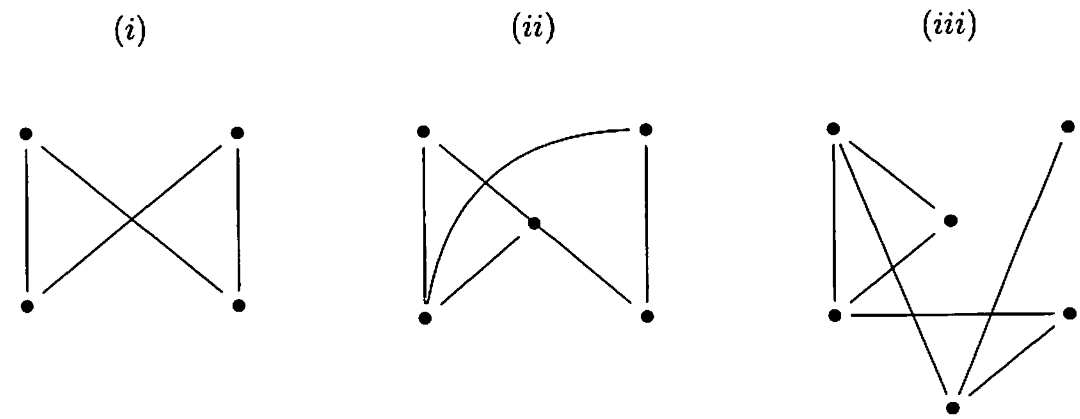
ii. **co-NP.**

iii. **NEXPTIME**

[12 marks]

Consider the following decision problem, the *All-but-one Circuit Problem* (ABCP). An instance is an undirected graph $G = (V, E)$. (V, E) is a *yes* instance of ABCP if V can be enumerated as $v_0, v_1, v_2, \ldots, v_{k-1}$ in such a way that $(v_1, v_2, \ldots, v_{k-1})$ is a circuit, i.e. for each i with $1 \leq i < k-1$ we have $(v_i, v_{i+1}) \in E$ and $(v_{k-1}, v_1) \in E$, otherwise (V, E) is a *no* instance.

b. For each of the following graphs, state whether it is a yes or a no instance of ABCP.



[11 marks]

c. Prove that ABCP is in **NP**.

[12 marks]

d. Prove that ABCP is **NP-complete**. You may assume that the *Hamiltonian Circuit Problem* (HCP) is **NP-complete**.

[15 marks]