

UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : COMP3004

ASSESSMENT : COMP3004C
PATTERN

MODULE NAME : Computational Complexity

DATE : 20 May 2016

TIME : 10:00 am

TIME ALLOWED : 2 hours 30 mins

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

2015/16

Answer BOTH of the TWO questions.

Marks for each part of each question are indicated in square brackets

Calculators are NOT permitted

1. **Preliminaries:** Let $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ denote the natural numbers.

a. Let $L \subseteq \{a, b\}^*$ be the infinite language

$$L = \{\epsilon, ab, aabb, aaabbb, aaaabbbb, \dots\}$$

Design a TM that decides L (draw a state-transition diagram); and describe in detail how your TM functions.

[10 marks]

b. Consider the connected-graph decision problem. An instance of the problem is an undirected graph G . This decision problem asks if the graph is connected or not.

Give an encoding that encodes instances of this decision problem into a language $L \subseteq \{0, 1\}^*$, and give an example of a string in L and a string not in L for your encoding.

[10 marks]

c. What is *Church's thesis*?

[5 marks]

d. Define

$$S := \{X \subseteq \mathbb{N} : \sum_{x \in X} x = \infty\}$$

thus S is the set of all subsets X of \mathbb{N} such that the sum of the numbers in each set X is infinite. Which of the two possibilities holds?

1. S is countable
2. S is not countable.

Give a detailed argument which justifies your answer.

[10 marks]

e. Let **HPL** denote the halting problem language and let,

$$\bar{L} := \{x \in \{0, 1\}^* : x \notin L\}.$$

Now given the following relationship between A and **HPL**

$$A \leq \mathbf{HPL} \leq \bar{A}.$$

Does there exist an $A \subseteq \{0, 1\}^*$ such that

1. A is finite.
2. A is recursive.
3. A is countably infinite
4. A is recursively enumerable.
5. A is not recursive but recursively enumerable.
6. A is not recursive.
7. A is not recursively enumerable.
8. A and \bar{A} are not recursively enumerable.

For each sub-part when an “ A ” exists give a particular example and then argue $A \leq \mathbf{HPL} \leq \bar{A}$. When an “ A ” does not exist, argue why not. Finally for sub-part 8, full credit will be given for good intuitive arguments.

[15 marks]

[Total=50 marks]

2. a. Define the complexity classes **NP**, **NPC** and **PSPACE**.

[6 marks]

An undirected irreflexive graph (V, E) consists of a set V of vertices, and a set E of two element subsets of V .

An instance of the *graph clique problem* (GCP) is an undirected irreflexive graph $G = (V, E)$ and a positive integer $k \geq 1$. It is a *yes instance* if there is a subset $C \subseteq V$ of size k such that for all pairs of distinct vertices $v \neq w \in C$ we have $\{v, w\} \in E$. It is a *no instance* otherwise.

An instance of the *vertex cover problem* (VCP) is an undirected irreflexive graph $G = (V, E)$ and a positive integer $k \geq 1$. It is a *yes instance* of VCP if there is a subset $S \subseteq V$ of size at most k such that for all $\{v, w\} \in E$ either $v \in S$ or $w \in S$, it is a *no instance* otherwise.

Define undirected graphs $G_1, G_2, G_3(n)$ (for $n \geq 4$) by

$$G_1 = (\{0, 1, 2, 3, 4\}, \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\})$$

$$G_2 = (\{0, 1, \dots, 11\}, \{\{i, j\} : 2 < |i - j(\text{mod } 12)|\})$$

$$G_3(n) = (\{0, 1, \dots, n-1\}, \{\{n-1, 0\}, \{(i, i+1) : i < n\}\}).$$

- b. For each of the graphs G defined above, state the biggest value of k such that (G, k) is a *yes instance* of GCP.

[8 marks]

- c. For each of the graphs G defined above, state the smallest value of k such that (G, k) is a *yes instance* of VCP (for $G_3(n)$ you may wish to distinguish the cases when n is even from the cases when n is odd).

[8 marks]

- d. Prove that GCP belongs to **NP**.

[7 marks]

- e. Define a polynomial time reduction: $\text{VCP} \leq_p \text{GCP}$.

[8 marks]

f. Define a polynomial time reduction: $GCP \leq_p VCP$.

[8 marks]

g. Given that **VCP** is **NPC**, but not assuming anything else, what can you conclude from parts 2.e and 2.f?

[5 marks]

[Total = 50 marks]

END OF PAPER