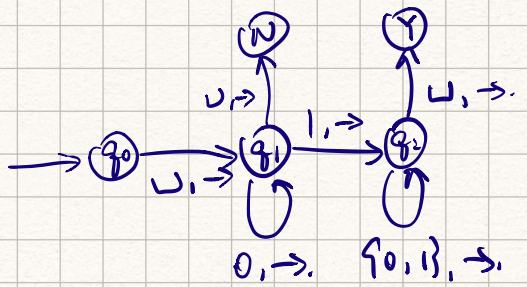
$s \in \Sigma^*$ L.

↑ alphabet

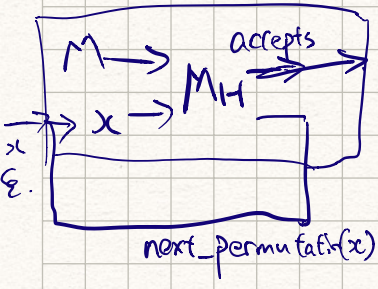
decidable if $s \in L$, then TM reaches halting state accept

recognisable ($s \notin L$ ——— reject.

$s \notin L$, either loop, or reach reject state



b. L_1 X recognisable X apply.



L_2. non-recognisable apply. $ETH \leq_m HALT$

L_3. rec--- ETH apply. $\overline{ETH}$

L_4. decidable recognisable. X apply

L_5. non-recognisable. apply.

$AH = \{\langle N \rangle \mid N \text{ halts on all } x\}$.

$HALT^- \leq_m AH$

c. $L = \{y \in \{0,1\}^* \mid y = \text{code}(M),\ M \text{ halts.}\}$

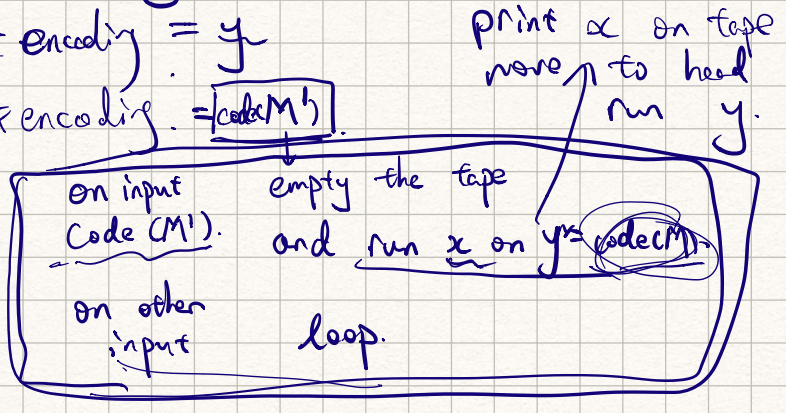
$HALT \leq L$.

$\langle y, x \rangle$ y.

$f(\langle y, x \rangle) = y$. $\quad HALT \leq_m L$

$y \neq \text{encoding} = y$

$y \neq \text{encoding} = \text{code}(M')$.



$HALT \leq_m L$

$\downarrow$

$\overline{HALT} \leq_m \overline{L}$

$\uparrow$

unrecognisable

L

undecidable.

$A \leq_m B$ $\quad HALT \leq_m L$.

$\overline{A} \leq_m \overline{B}$ $\quad \overline{HALT} \leq_m \overline{L}$

unrecognisable.

1. a. $G = (V, E)$

exist path travel every vertex and return to starting point

$\exists\ p = \{v_0 \cdots v_n\}$. $\quad |p| = |V|$.

such that $(v_0, v_1) \in E$

$(v_1, v_2) \in E$

$\vdots$

$(v_n, v_0) \in E$.

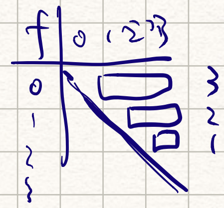
b. f(1→3) 0

0 1 3 2. $k=0$.

0, 1, 2, 3. $2\times 1\times 4\times 2=16$

$k=16$

c. ~~$\log_2(|X|)$~~



$1+2+\cdots(|X|-1)$.

$\frac{|X|\times(|X|-1)}{2}\times\lceil\log_2(m)\rceil + \lceil\log_2(k)\rceil$

f k.

d. $\leq_p$.

$A\leq_p B$ $B\leq_p C$.

∃ dTM. p-time. T_0 $f_{T_0}(a)=b$.

↑ instance of A.

T_1 $f_{T_1}(b)=c$.

$f_{T_0}(f_{T_1}(a))=c$. $T_0 : T_1$ DTM.

p-time p-time × p-time. = p-time

$A\leq_p C$.

e. NDTM.

ND ← pick. $x\in|X|$.

path = {x}.

while |path| < |X|.

pick $x_0\in X$/path.

path = {path + x.}. append.

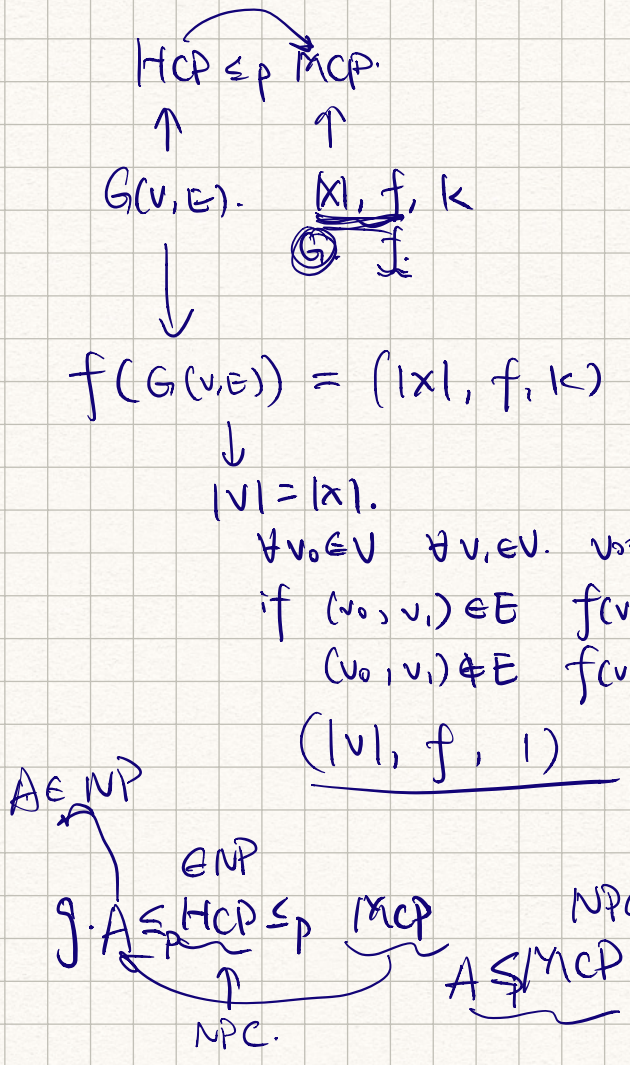
done. path = {x_0, x_1, x_2 ---

Compute $S=f(x_0,x_1)\times\cdots\times f(x_n,x_0)$

check if $S\leq k$ if yes accept

reject.

MCP ∈ NP


HCP ≤p TCP.
G(V,E).
|X|, f, k
f(G(V,E)) = (|X|, f, k)
|V| = |X|.
∀v0∈V ∀v1∈V. v0≠v1
if (v0, v1) ∈E f(v0, v1) = 1.
(v0, v1) ∉E f(v0, v1) = 2.
(|V|, f, 1)
A∈NP
∈NP
∈NP
HCP
TCP
NPC.
NPC.
A ≤p TCP