4-bit to 4-bit Less-Than Comparator Circuit

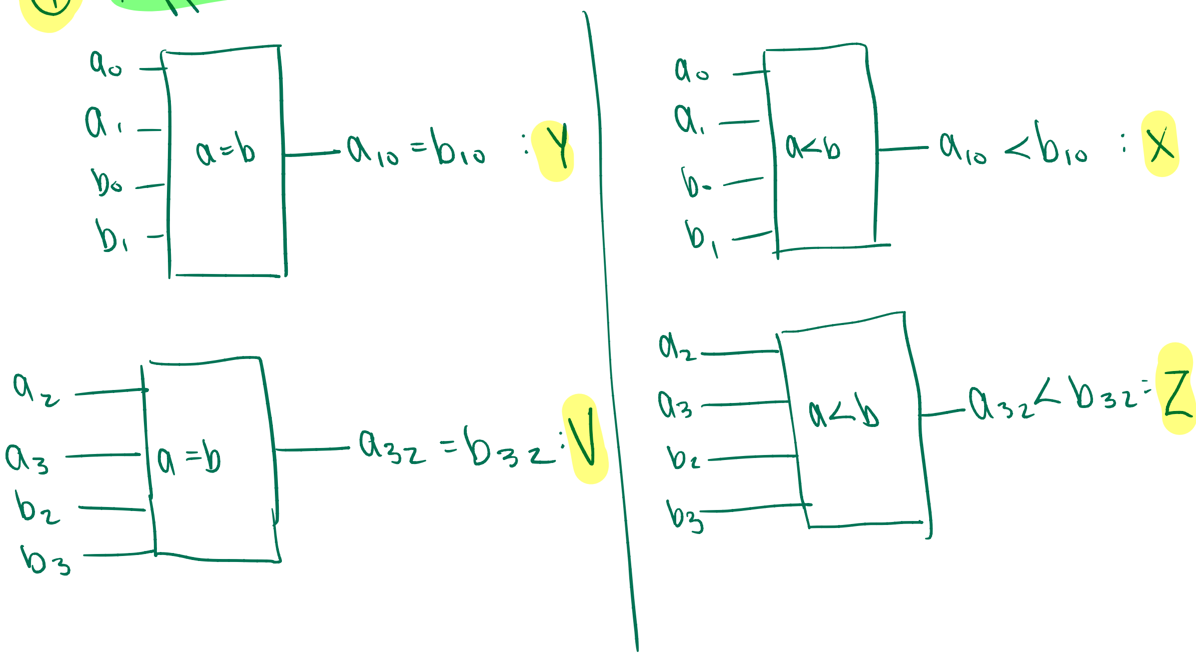
Instructions:

Problem: How do we create a quantum circuit where $a_3a_2a_1a_0 < b_3b_2b_1b_0$ will return 1 if true, 0 if false?

Ex: $1000 < 0011 \rightarrow 0$, $0101 < 0101 \rightarrow 0$, $0001 < 1100 \rightarrow 1$

a_3 a_2 a_1 a_0 b_3 b_2 b_1 b_0

① Approach:



For $a_3a_2a_1a_0$ to be less than $b_3b_2b_1b_0$,

$a_3a_2a_1a_0 < b_3b_2b_1b_0$,

Z is true or V is true and X is true.

$F = Z + V \cdot X$ ← SOP

(2) What is Z, V, X?

V: $a_{32} = b_{32}$

$a_3 a_2 \backslash b_3 b_2$	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	0	0	1	0
10	0	0	0	1

$\overline{a}_3 \overline{a}_2 \overline{b}_3$

$\overline{a}_3 \overline{b}_3 b_2$

$a_2 b_3 b_2$

$a_3 \overline{a}_2 b_3$

$$\boxed{\begin{aligned} V &= \overline{a}_3 \overline{a}_2 \overline{b}_3 \oplus \overline{a}_3 \overline{b}_3 b_2 \oplus a_3 b_3 b_2 \oplus a_0 \overline{a} \; b \\ &= \overline{a}_3 \overline{b}_3 (\overline{a}_2 \oplus b_2) \oplus a_3 b_3 (b_2 \oplus \overline{a}_2) \end{aligned}}$$

$X : a_{10} < b_{10}$

$a_1a_0 \backslash b_1b_0$	00	01	11	10
00	0			
01	0	0		
11	0	0	0	0
10	0	0		0

$\bar{a}_1 \bar{a}_0 b_0$

$\bar{a}_1 b_1$

$\bar{a}_0 b_1 b_0$

$$\boxed{\begin{aligned} X &: \bar{a}_1 \bar{a}_0 b_0 \oplus \bar{a}_1 b_1 \oplus \bar{a}_0 b_1 b_0 \\ &= \bar{a}_0 b_0 (\bar{a}_1 \oplus b_1) \oplus \bar{a}_1 b_1 \end{aligned}}$$

$Z : a_{32} < b_{32}$

$$\boxed{Z = \bar{a}_2 b_2 (\bar{a}_3 \oplus b_3) \oplus \bar{a}_3 b_3}$$

③ Convert boolean expression to circuit.

$F = Z + V \cdot X$

Convert to Exclusive-Sum-of-Products for quantum circuit.

Formula: $a + b = a \oplus ab \oplus b$

↳ $F = Z \oplus ZVX \oplus VX$

$= Z \oplus VX(Z \oplus 1)$

$= Z \oplus \bar{Z}VX$