Homework II, Fall 2024

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All the exercises will be based on the following model. There is a representative household with preferences over private consumption, c_t, and labor l_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} 0.97^t \left(\log c_t - \frac{l_t^2}{2} \right)$$

The household consumes, saves, and works with a budget constraint (notation is self-explanatory):

$$c_t + i_t = w_t l_t + r_t k_t$$

There is a production function:

$$c_t + i_t = e^{z_t} k_t^{0.33} l_t^{0.67}$$

with a law of motion for capital:

$$k_{t+1} = 0.9 k_t + i_t$$

and a technology level z_t that follows an AR(1) process:

$$z_t = 0.95 z_{t-1} + 0.007 \varepsilon_t$$

where $\varepsilon_t \sim N(0, 1)$.

Please answer the following exercises for a total of 50 points.

1. Chebyshev (10 points)

Compute the solution to the model when you use six Chebyshev polynomials on capital and a three-point finite approximation to z_t using Tauchen's method.

2. Finite Elements (10 points)

Compute the solution to the model when you use eight finite elements on capital and a three-point finite approximation to z_t using Tauchen's method. Use a Galerkin weighting scheme.

3. Perturbation (10 points)

Find a third-order perturbation solution to the model.

4. Deep Learning (10 points)

Compute the solution to the model when you use a neural network with three layers and 25 nodes per layer on capital and a three-point finite approximation to z_t using Tauchen's method.

5. Comparison (10 points)

Compare all the previous solutions.