Spacecraft Formation Flying Using Sliding Mode Control

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Outline

- Introduction to Formation Flying
- Introduction to Sliding Mode Control
- Objective of Project
- Dynamics Model
- Classical Sliding Surface
- Alternative Sliding Surfaces
- Nonsingular Terminal Sliding Surface
- Simulations
- Conclusions
- Experiment

Outline

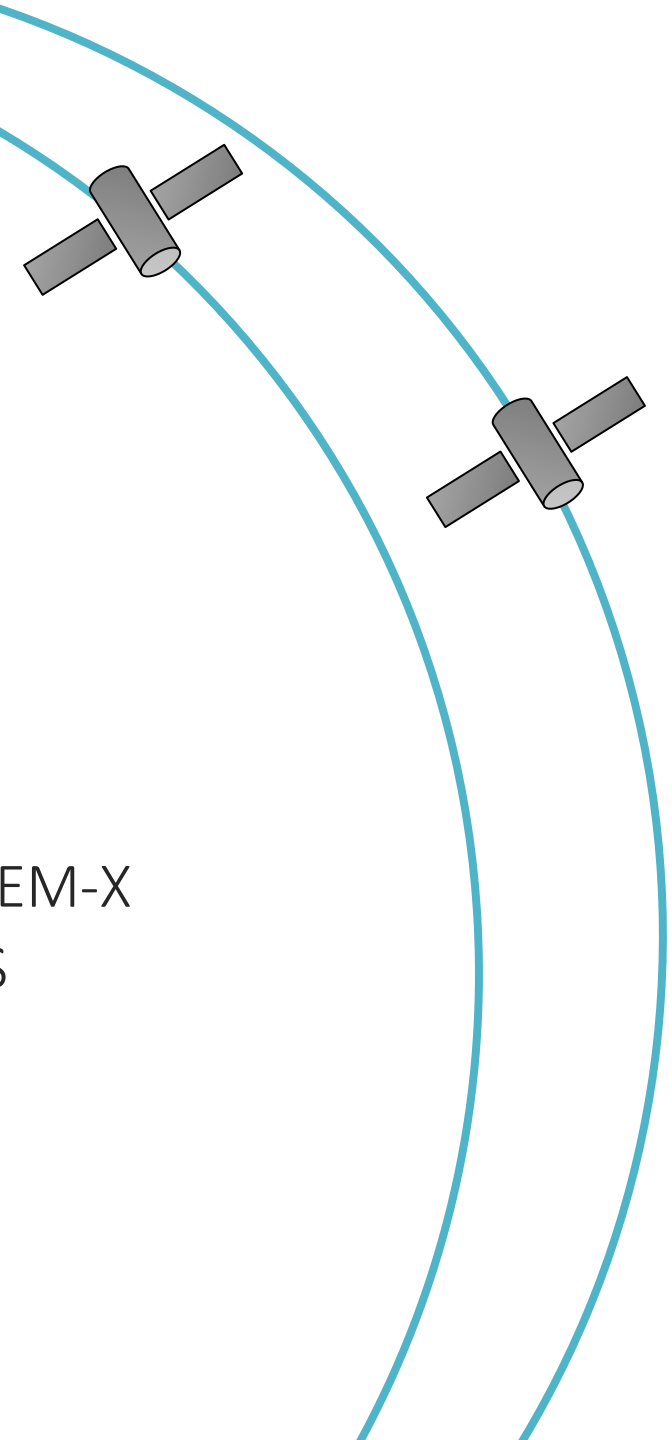
Formation Flying

Benefits:

- Robustness to faults
- Versatility
- Cost

Past and planned missions using formation flying:

- Earth science missions: CLUSTER, GRACE, and TanDEM-X
- Space science missions: MMS, Stellar Imager, EXO-S
- Technology demonstration missions: PRISMA



Robust Control for Formation Flying

Multiple satellite system dynamics models can be complex

Simplified models are less accurate

Thrust and parametric uncertainties due to long-term operations or hazardous conditions

Outline

Sliding Mode Control

Popular due to design simplicity and robustness to disturbances

2 steps:

- Design of sliding variable
- Choice of control to drive dynamics to sliding surface

Many sliding variable and control technique pairs exist

Outline

Introduction to Formation Flying

Introduction to Sliding Mode Control

Objective of Project

Dynamics Model

Classical Sliding Surface

Alternative Sliding Surfaces

Nonsingular Terminal Sliding Surface

Simulations

Conclusions

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Objective

Apply novel nonsingular terminal sliding mode controller to spacecraft formation flying

Prove robustness to uncertainty and disturbance

Present simulation and experiment results

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Dynamics

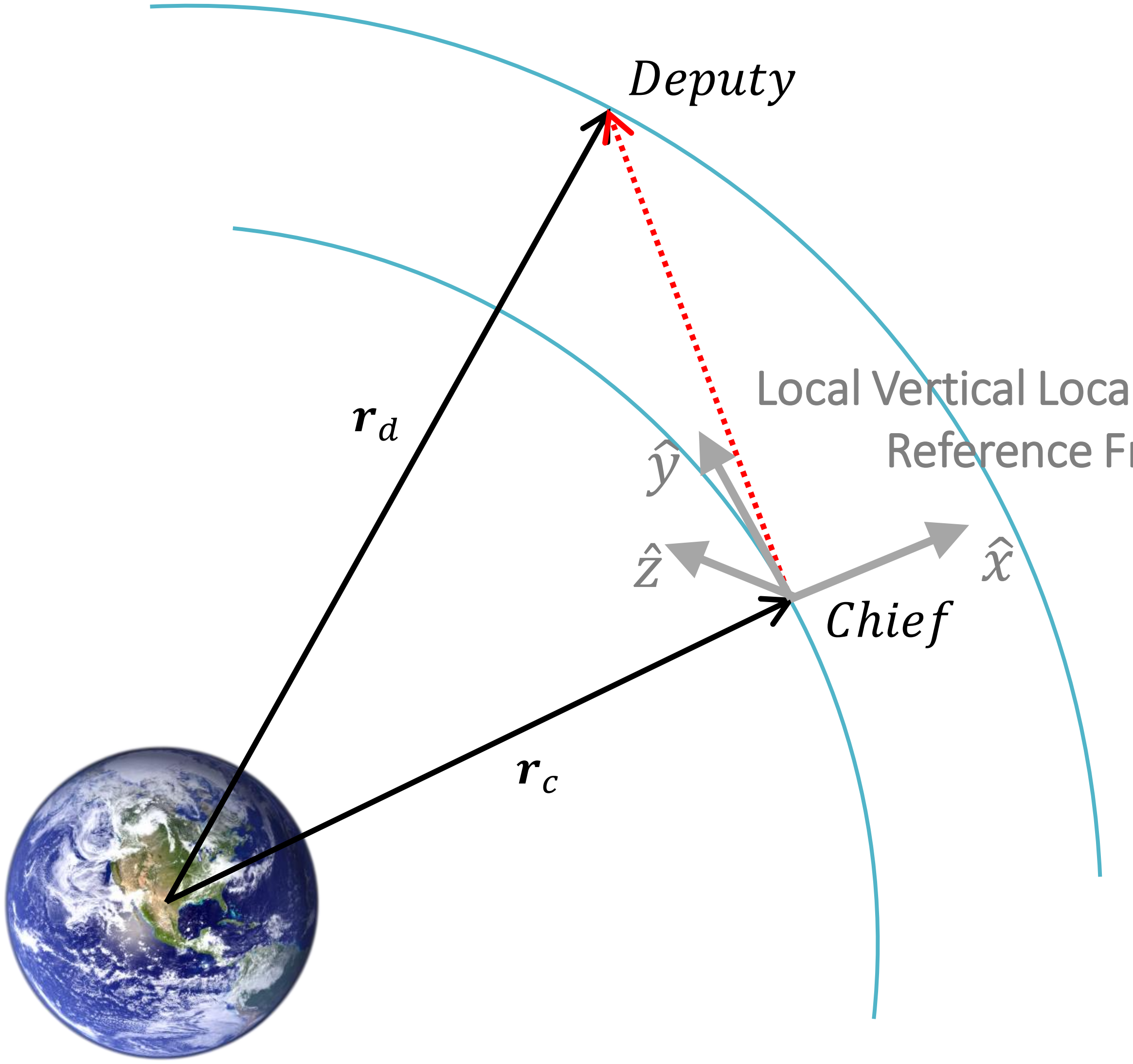
2 satellites, called “chief” and “deputy”

Assumptions: circular chief orbit and small separation distance between satellites

Clohessy-Wiltshire equations can be used to describe the relative motion

$$\ddot{x} - 2n\dot{y} - 3n^2x = u_x + d_x$$
$$\ddot{y} + 2n\dot{x} = u_y + d_y$$
$$\ddot{z} + n^2z = u_z + d_z$$

Dynamics



$Deputy$
$\boldsymbol{r}_d$
Local Vertical Local Horizontal Reference Frame
$\hat{y}$
$\hat{z}$
$\hat{x}$
$Chief$
$\boldsymbol{r}_c$

Dynamics

$$\dot{X} = f(X) + U + D$$

where $f(X) = AX \in \mathbb{R}^{6\times 1}$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_x \\ d_y \\ d_z \end{bmatrix} \quad .$$

Dynamics

Assumptions:

u_x, u_y, u_z can be written as $u_i = b_i u_{i0} + \delta u_i$ where u_{i0} are commanded control inputs, b_i are multiplicative control errors ≈ 1, and δu_i have known upper bound less than u_{i0}

d_x, d_y, d_z have known upper bounds

$f(X)$ can be written as $f(X) = f_0(X) + \delta f(X)$ where $f_0(X)$ is the known nominal vector and $\delta f(X)$ has a known upper bound less than $\overline{\delta f}(X)$

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Classical Sliding Surface

Sliding variable:

$$\sigma_x = \dot{\varepsilon}_x + \alpha \varepsilon_x, \text{ where } \alpha > 0$$

Time derivative of σ:

$$\dot{\sigma}_x = \ddot{\varepsilon}_x + \alpha \dot{\varepsilon}_x$$

Where

$\varepsilon_x(t) = x_{ref}(t) - x$

$\dot{\varepsilon}_x(t) = \dot{x}_{ref}(t) - \dot{x}$

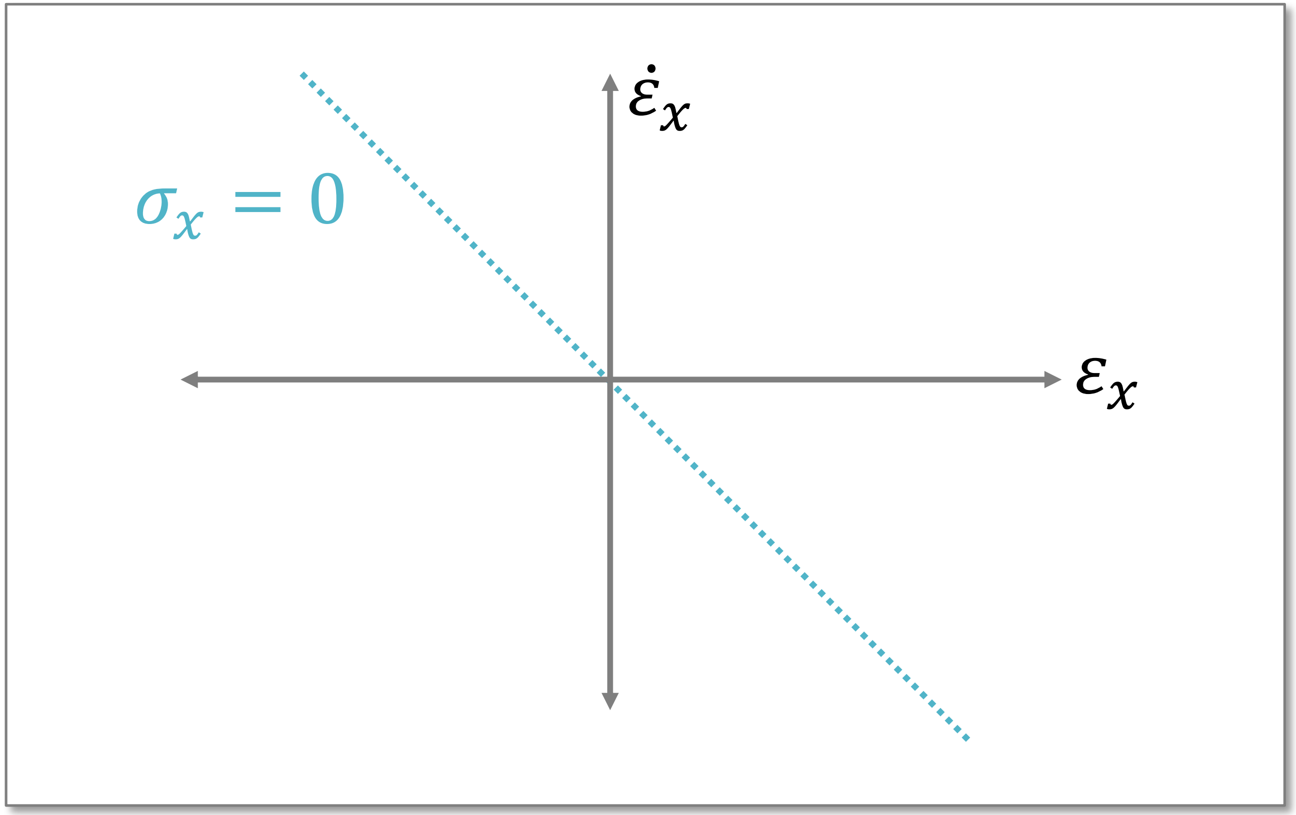
$\ddot{\varepsilon}_x(t) = \ddot{x}_{ref}(t) - \ddot{x}$

Classical Sliding Surface

Assumptions:

Desired trajectory and its first and second derivatives are bounded by known positive constants $\left(x_{ref}, \dot{x}_{ref}, \ddot{x}_{ref}\right) < (\bar{x}_{ref}, \bar{\dot{x}}_{ref}, \bar{\ddot{x}}_{ref})$

Classical Sliding Surface



$\sigma_x = \dot{\varepsilon}_x + \alpha\varepsilon_x = 0$

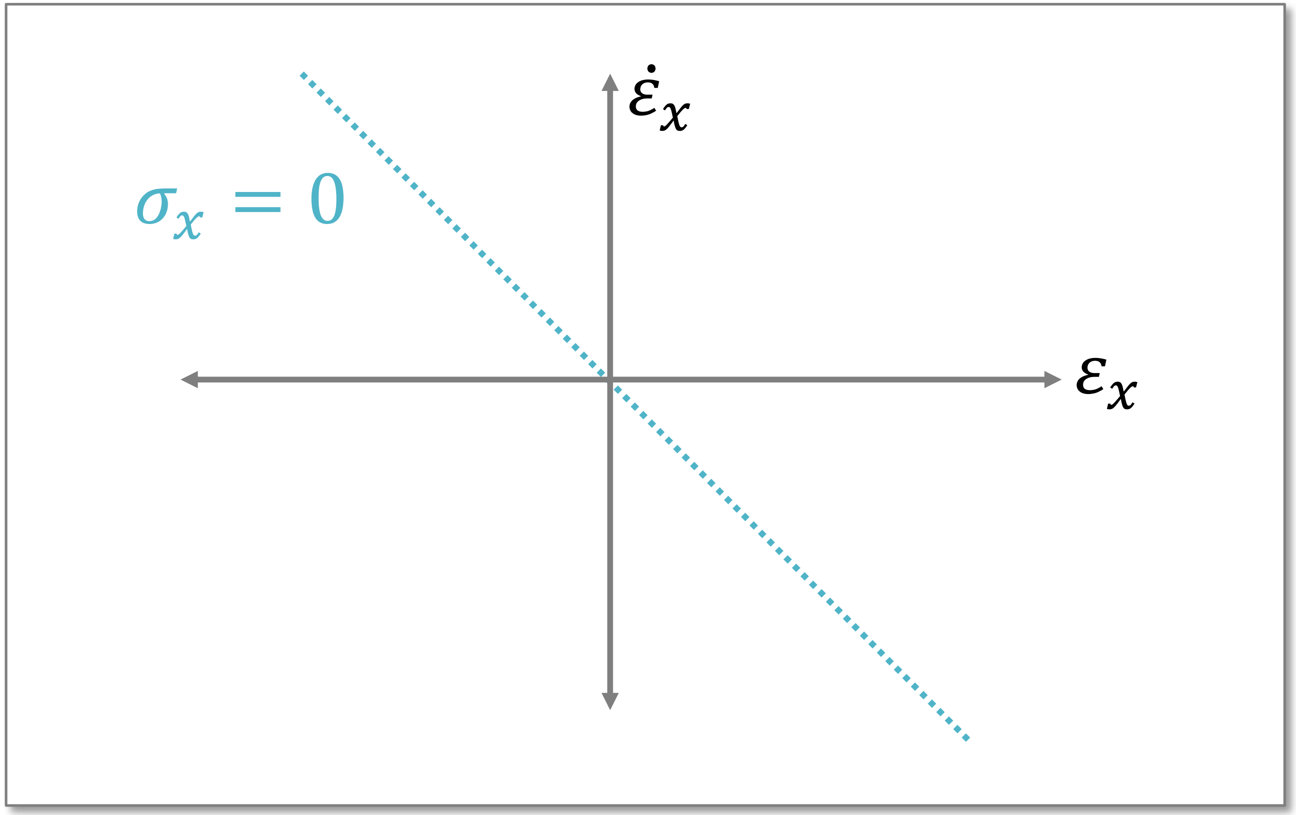
$\dot{\varepsilon}_x = -\alpha\varepsilon_x$

Solution:

$\varepsilon_x = C\, e^{-\alpha t}$

When dynamics slide on surface
$\sigma_x = 0$, $\varepsilon_x \to 0$ exponentially

Classical Sliding Surface



To guarantee convergence of $\varepsilon_x(t)$ and $\dot{\varepsilon}_x(t)$ to zero, $\sigma_x \to 0$ and remain at 0

When $\sigma_x > 0, \dot{\sigma}_x < 0$

When $\sigma_x < 0, \dot{\sigma}_x > 0$

Classical Sliding Surface

First control choice:

$$u_{x0} = k_x \, sign(\sigma_x) + \alpha \dot{\varepsilon}_x - f_{x0}(X)$$

Again, $f_0(X) = AX$

Where $A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$

Therefore, $\boldsymbol{f_{x0}(X) = 3n^2 x + 2n\dot{y}}$

Classical Sliding Surface

$$\dot{\sigma}_x = \ddot{x}_{ref} - \ddot{x} + \alpha\dot{\varepsilon}_x$$
$$= \ddot{x}_{ref} - \delta f_x(X) - \delta u_x - d_x - b_x k_x\, sign(\sigma_x)$$

$$\boxed{\dot{\sigma}_x = -b_x k_x sign(\sigma_x) + F_x}$$

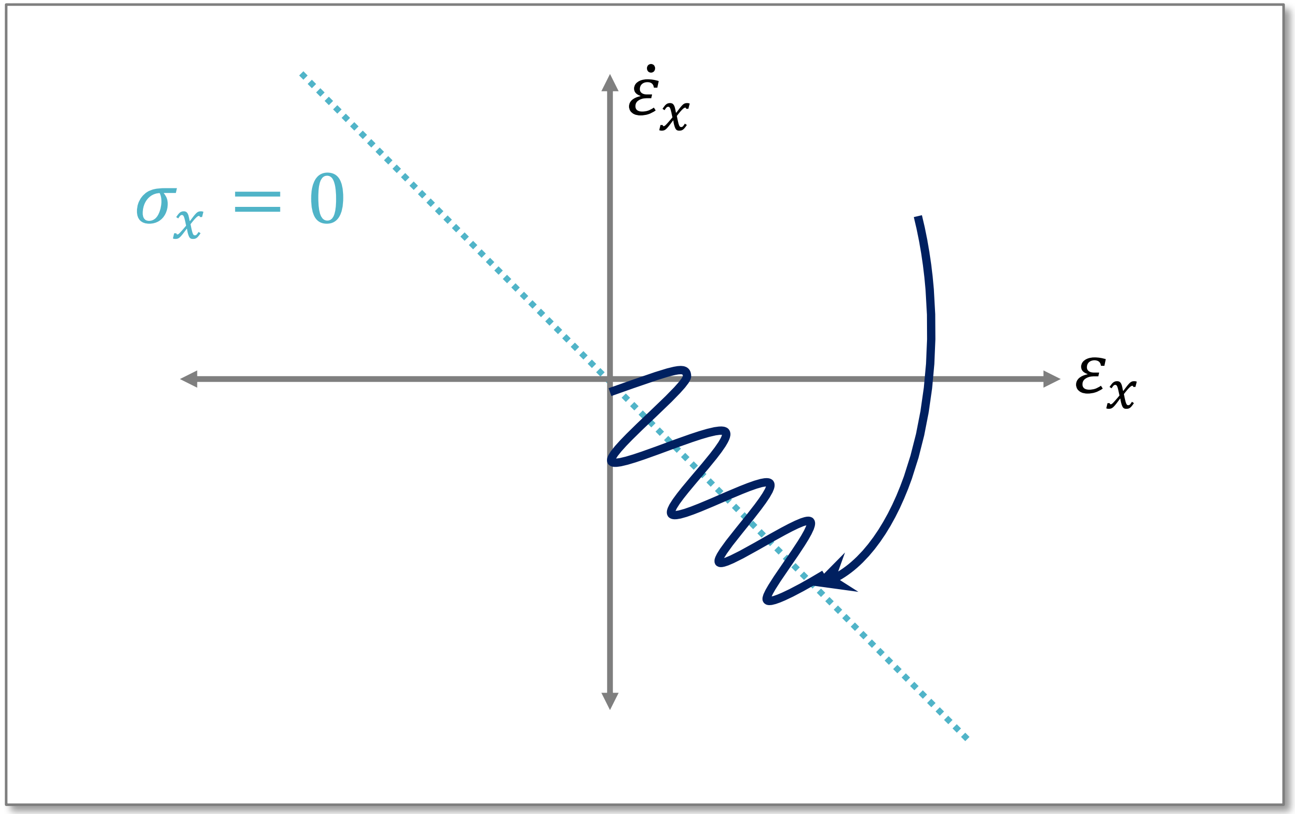
Where $F_x = \ddot{x}_{ref} - \delta f_x(X) - \delta u_x - d_x$

Based on assumptions, F_x is bounded above as $F_x < \overline{F_x}$

Gain k_x is designed to satisfy the inequality $k_x > \overline{F_x}$

Classical Sliding Surface

Potential trajectory:



Classical Sliding Surface

Previous control structure contains $\dot{\varepsilon}_x$ term, potentially increasing required control magnitude larger than the necessary value

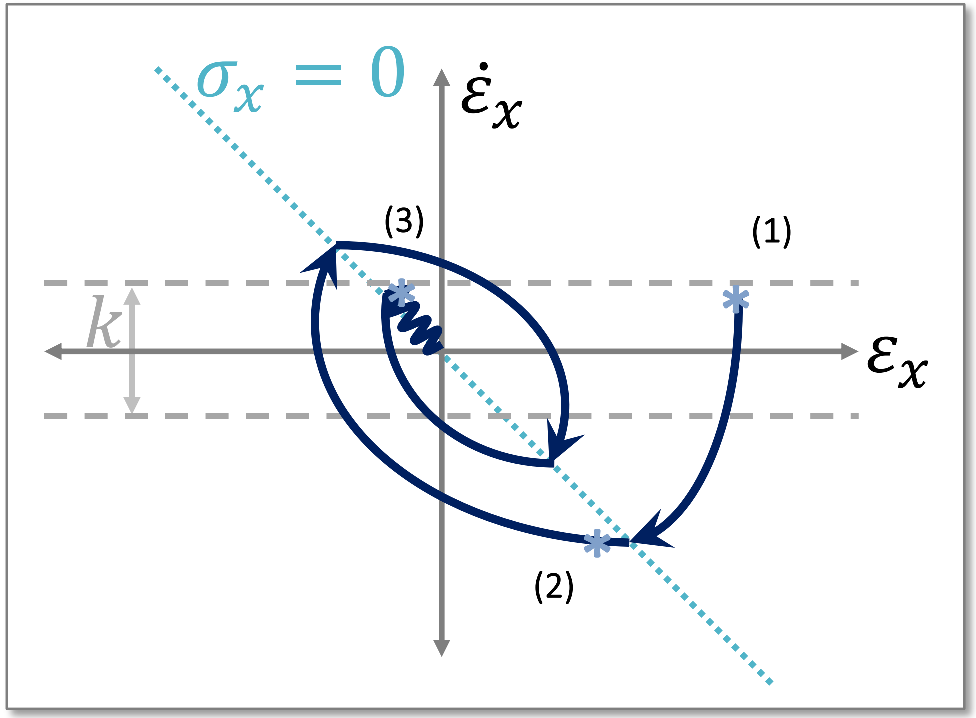
$$u_{x0} = k_x \, sign(\sigma_x) + \alpha\dot{\varepsilon}_x - f_{x0}(X)$$

Alternative control choice:

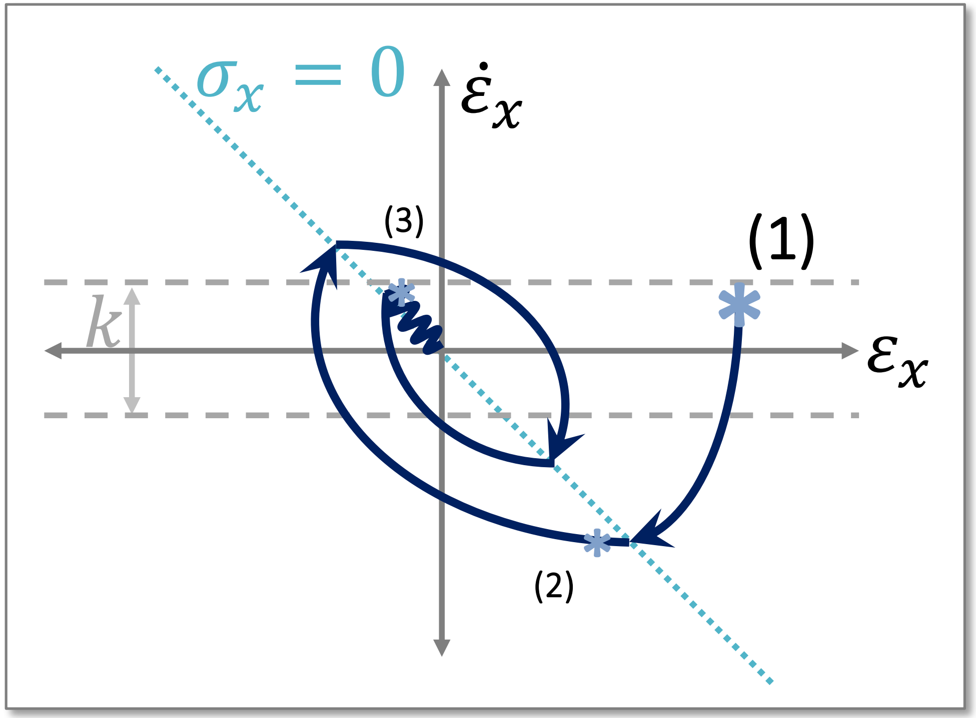
$$u_{x0} = k_x \, sign(\sigma_x) - f_{x0}(X)$$

Classical Sliding Surface

Potential Trajectory:



Classical Sliding Surface



(1)

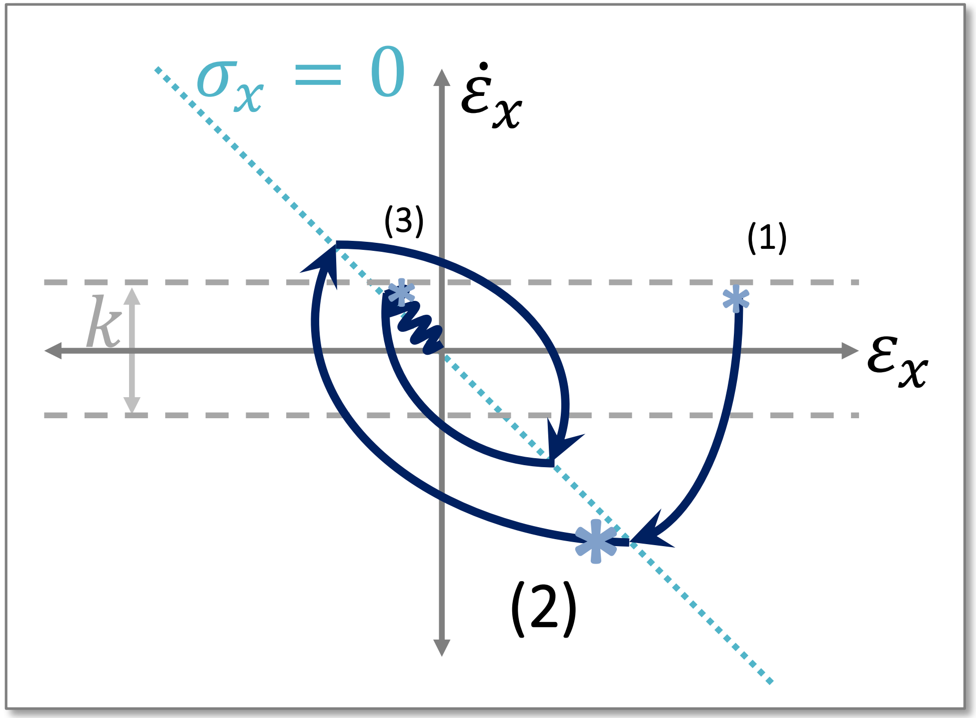
$\sigma_x > 0$, so $u_{x0} = k_x - f_{x0}(X)$

$\ddot{\varepsilon}_x = \ddot{x}_{ref} - \ddot{x} = -b_x k_x + F_x < 0$

until surface is reached

if undisturbed, will slide on surface

Classical Sliding Surface



(2)

negative disturbance forces system away from surface

$\dot{\sigma}_x = b_x k_x + F_x + \alpha \dot{\varepsilon}_x.$

k_x may be too small to compensate for F_x and $\alpha \dot{\varepsilon}_x$

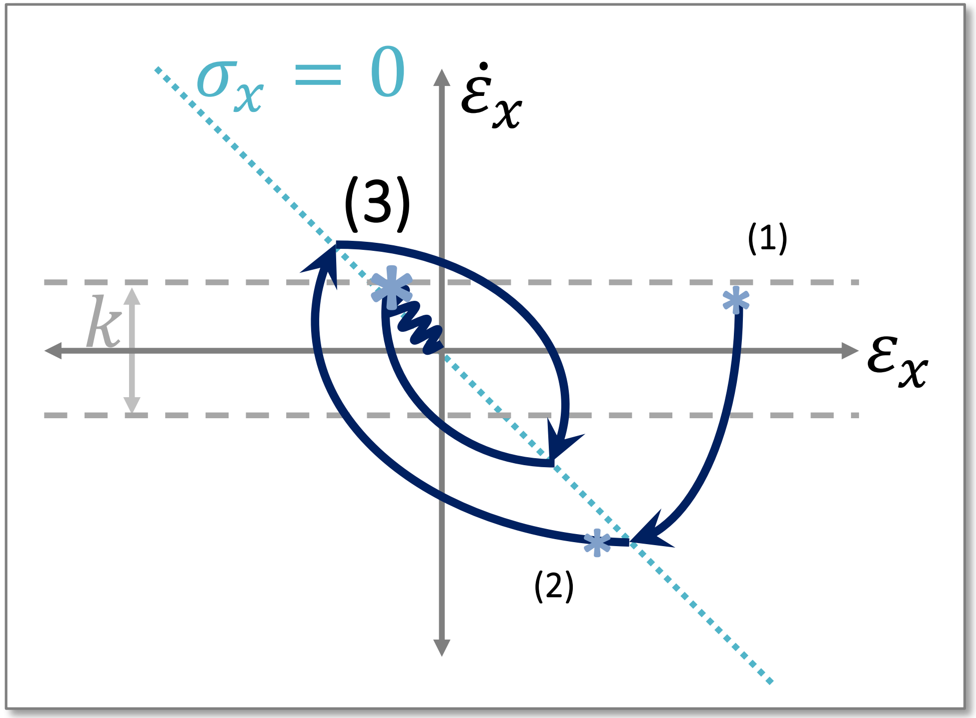
If $\dot{\sigma}_x < 0$, push the trajectory further from surface

$\sigma_x < 0 \rightarrow u_{x0} = -k_x - f_{x0}(X)$

$\ddot{\varepsilon}_x = b_x k_x + F_x > 0$

trajectory will eventually impact the surface again

Classical Sliding Surface



(3)

In region where $k_x > |F_x + \alpha\dot{\varepsilon}_x|$

Oscillate to origin as in previous method

Classical Sliding Surface

Advantages:

- Does not require $\dot{\varepsilon}_x$ in control structure (potentially lower magnitude)
- $(\varepsilon_x, \dot{\varepsilon}_x)$ does converge to σ_x in finite time → exponential convergence of ε_x

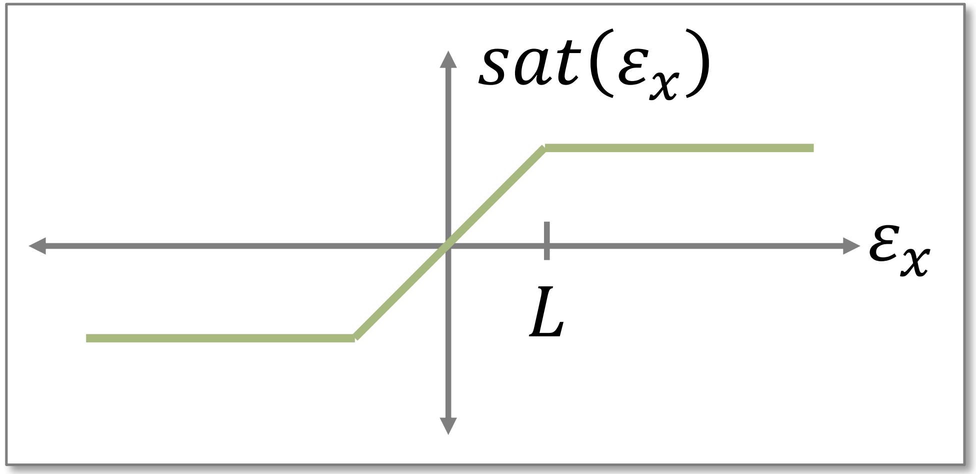
Disadvantages:

- Potentially takes longer to converge

Outline

Alternative Sliding Surfaces

$$\sigma_x = \dot{\varepsilon}_x + \alpha\, sat(\varepsilon_x)$$



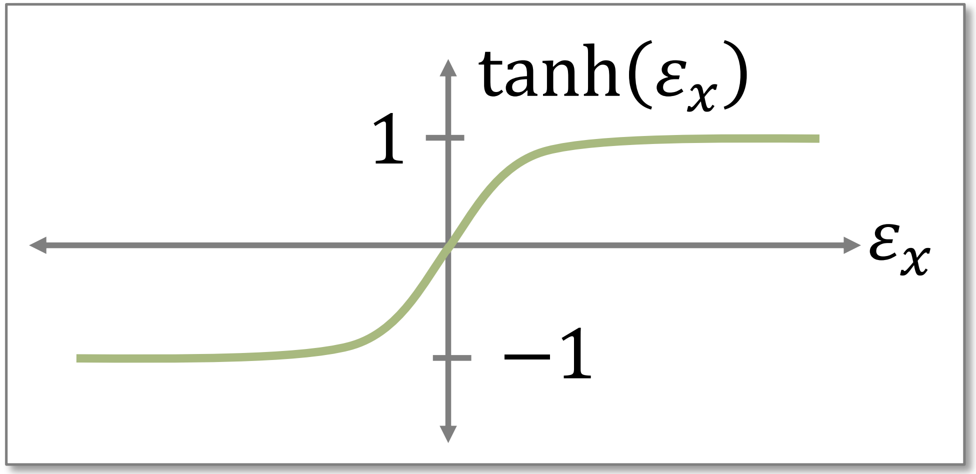
When $|\varepsilon_x| > L$,

$$\dot{\sigma}_x = \ddot{\varepsilon}_x = -b_x k_x sign(\sigma_x) + F_x$$

Discontinuity leads to $\dot{\sigma}_x(\varepsilon_x = L) = \infty$

Alternative Sliding Surfaces

$$\sigma_x = \dot{\varepsilon}_x + \alpha \tanh(\varepsilon_x)$$



Eliminates discontinuity

Non-exponential convergence

Outline

Nonsingular Terminal Sliding Surface

$$\sigma_x = e^{\frac{c_2 \varepsilon_x^2}{2}} \dot{\varepsilon}_x^{\frac{m_2}{m_1}} + c_3 \varepsilon_x$$

$m_1, m_2 > 0$ and odd integers

$1 < \frac{m_2}{m_1} < 2$

$c_2, c_3 > 0$

$$\dot{\sigma}_x = g(\dot{\varepsilon}_x)\ddot{\varepsilon}_x + c_3 \dot{\varepsilon}_x$$

$$g(\dot{\varepsilon}_x) = \left(c_2 \dot{\varepsilon}_x^{\frac{m_1+m_2}{m_1}} + \frac{m_2}{m_1} \dot{\varepsilon}_x^{\frac{m_2-m_1}{m_1}} \right) e^{\frac{c_2 \dot{\varepsilon}_x^2}{2}}$$

Nonsingular Terminal Sliding Surface

$$g(\dot{\varepsilon}_x) = \left(c_2 \dot{\varepsilon}_x^{\frac{m_1+m_2}{m_1}} + \frac{m_2}{m_1} \dot{\varepsilon}_x^{\frac{m_2-m_1}{m_1}} \right) e^{\frac{c_2 \dot{\varepsilon}_x^2}{2}}$$

$$\dot{\varepsilon}_x^{\frac{m_1+m_2}{m_1}} = \left(\dot{\varepsilon}_x^{m_1+m_2}\right)^{1/m_1} = (+\mathrm{Re})^{1/\mathrm{m}_1}$$

$$\dot{\varepsilon}_x^{\frac{m_2-m_1}{m_1}} = \left(\dot{\varepsilon}_x^{m_2-m_1}\right)^{1/m_1} = (+\mathrm{Re})^{1/\mathrm{m}_1}$$

Nonsingular Terminal Sliding Surface

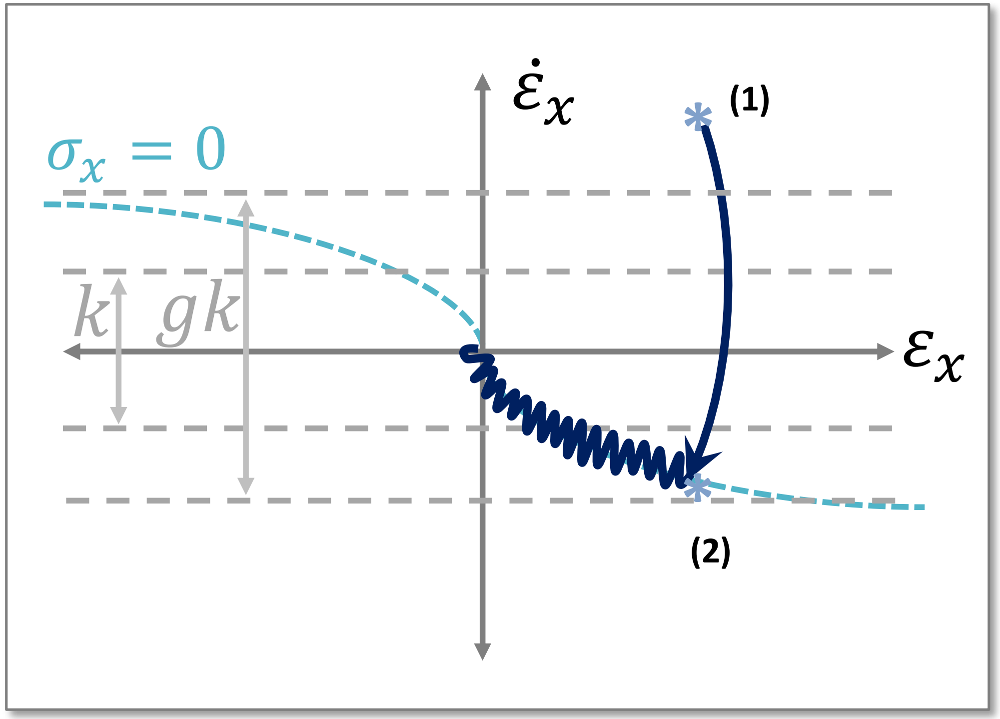
Control input:

$$u_{x0} = k_1\, sign(\sigma_x) - f_{x0}(X)$$

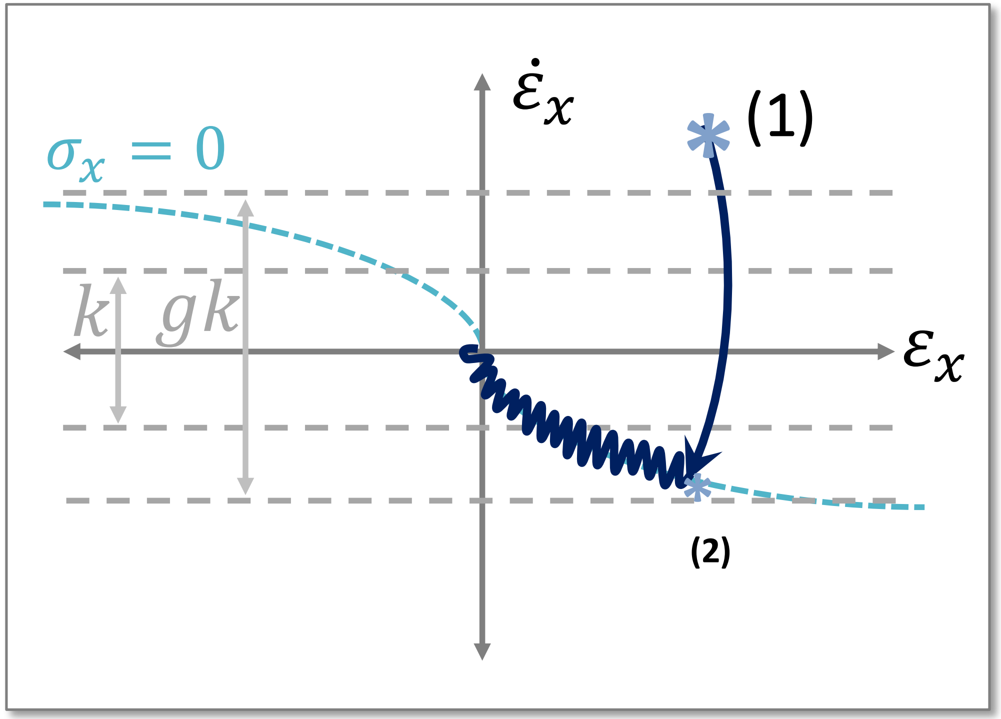
Does not require subtraction of a $\dot{\varepsilon}_x$ term

Nonsingular Terminal Sliding Surface

Potential Trajectory:



Nonsingular Terminal Sliding Surface



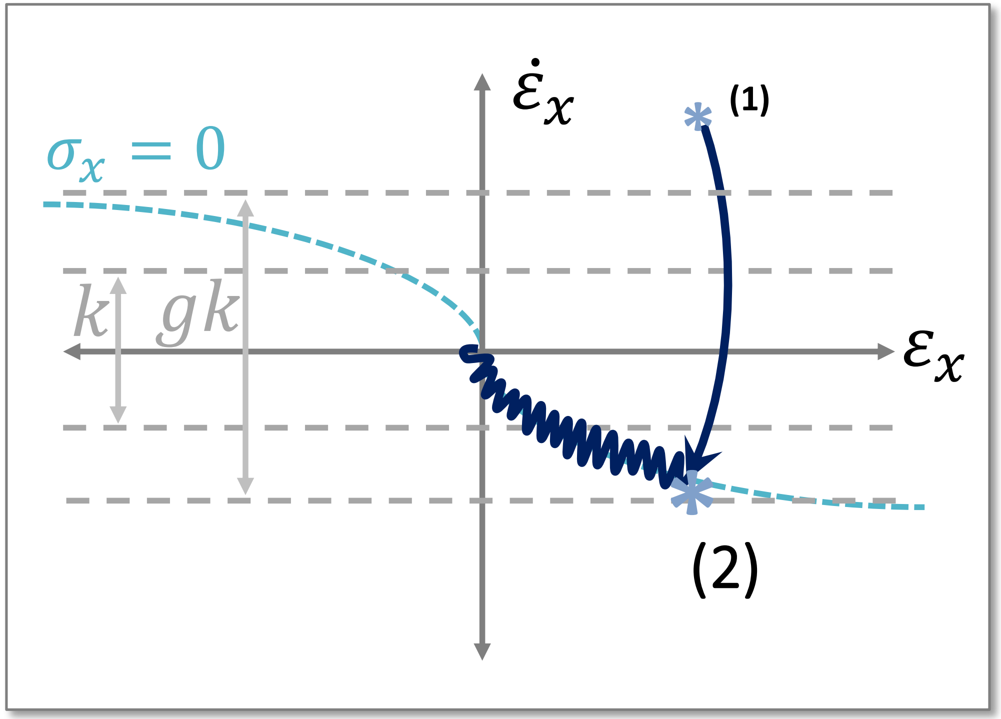
(1)

$\sigma_x > 0$, so $u_{x0} = k_1 - f_x(X)$

$\ddot{\varepsilon}_x = -b_x k_1 + F_x < 0$

until surface is reached

Nonsingular Terminal Sliding Surface



(2)

negative disturbance forces system away from surface

$\dot{\sigma}_x = g(\dot{\varepsilon}_x)(-b_x k_1 sign(\sigma_x) + F_x) + c_3 \dot{\varepsilon}_x$

$g(\dot{\varepsilon}_x)$ can be designed such that $g(\dot{\varepsilon}_x)k_1$ immediately dominates

system will slide on the surface, ensuring that $\varepsilon_x \to 0$

Nonsingular Terminal Sliding Surface

Additional benefits:

- Finite time convergence of ε_x due to selection of $\frac{m_2}{m_1}$
- Four constants (i.e. $m_1, m_2, c_2,$ and c_3) can be tuned to yield desired performance characteristics
- Use control that does not subtract $\dot{\varepsilon}_x$ term
- No singularity as introduced in other attempts to avoid subtracting $\dot{\varepsilon}_x$ term

Outline

Simulations

- Propagated 2 satellites’ nonlinear equations of motion separately
  - In Earth centered inertial (ECI) frame
  - Included J2 perturbation
- Calculated x, y, and z components at each time step for computation of control inputs

Simulations

Initial conditions and parameters

Chief initial orbital elements:

a (km)	e (dim)	i (dim)	$\boldsymbol{\Omega}$ (rad)	$\boldsymbol{\omega}$ (rad)	$\boldsymbol{M_0}$ (rad)
R_{Earth}+300	0.1	0	$\pi/4$	$\pi/6$	0

Initial relative state:

$\boldsymbol{x_0}$ (km)	$\boldsymbol{y_0}$ (km)	$\boldsymbol{z_0}$ (km)	$\boldsymbol{\dot{x}_0}$ (km/s)	$\boldsymbol{\dot{y}_0}$ (km/s)	$\boldsymbol{\dot{z}_0}$ (km/s)
10	5	5	0	0	0

Control parameters:

$\boldsymbol{c_2, c_3}$ (dim)	$\boldsymbol{m_1}$ (dim)	$\boldsymbol{m_2}$ (dim)	$\boldsymbol{k_i}$ (N)
0.1	3	5	1.0

Simulations

Error magnitudes:

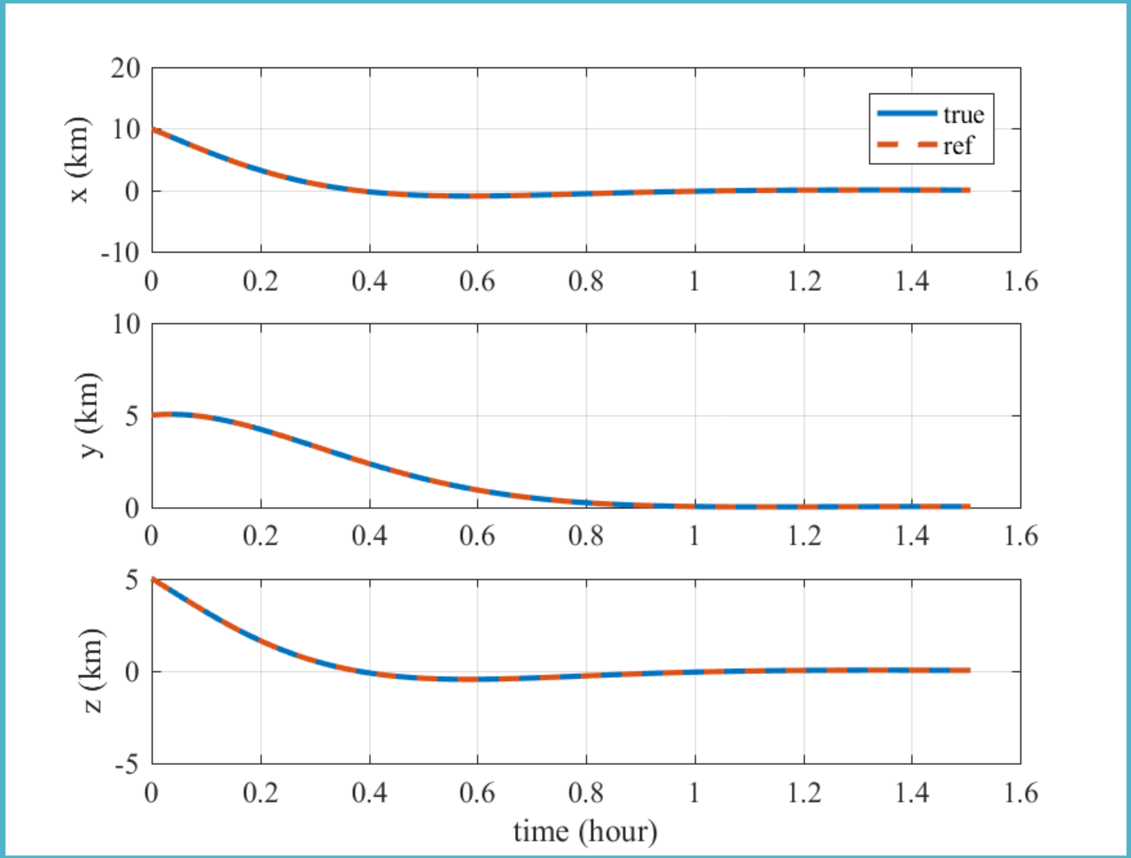
Disturbances: d_i

Thrust error: multiplicative (b_i) and noise (δu_i)

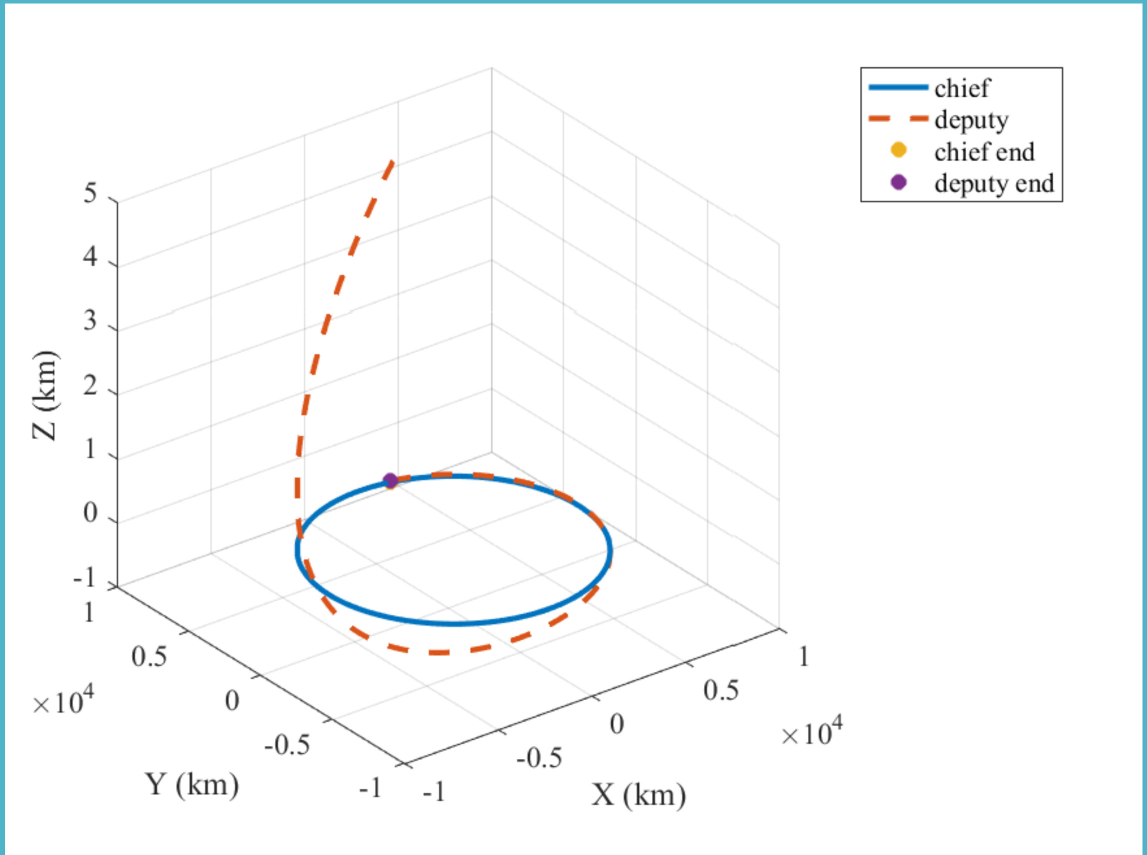
$$u_i = b_i u_{i0} + \delta u_i$$

Disturbance $(\boldsymbol{m/s^2})$	$\boldsymbol{b_i}$ (dim)	$\boldsymbol{\delta u_i}$ (N)
0.01	0.9	0.1

Simulations

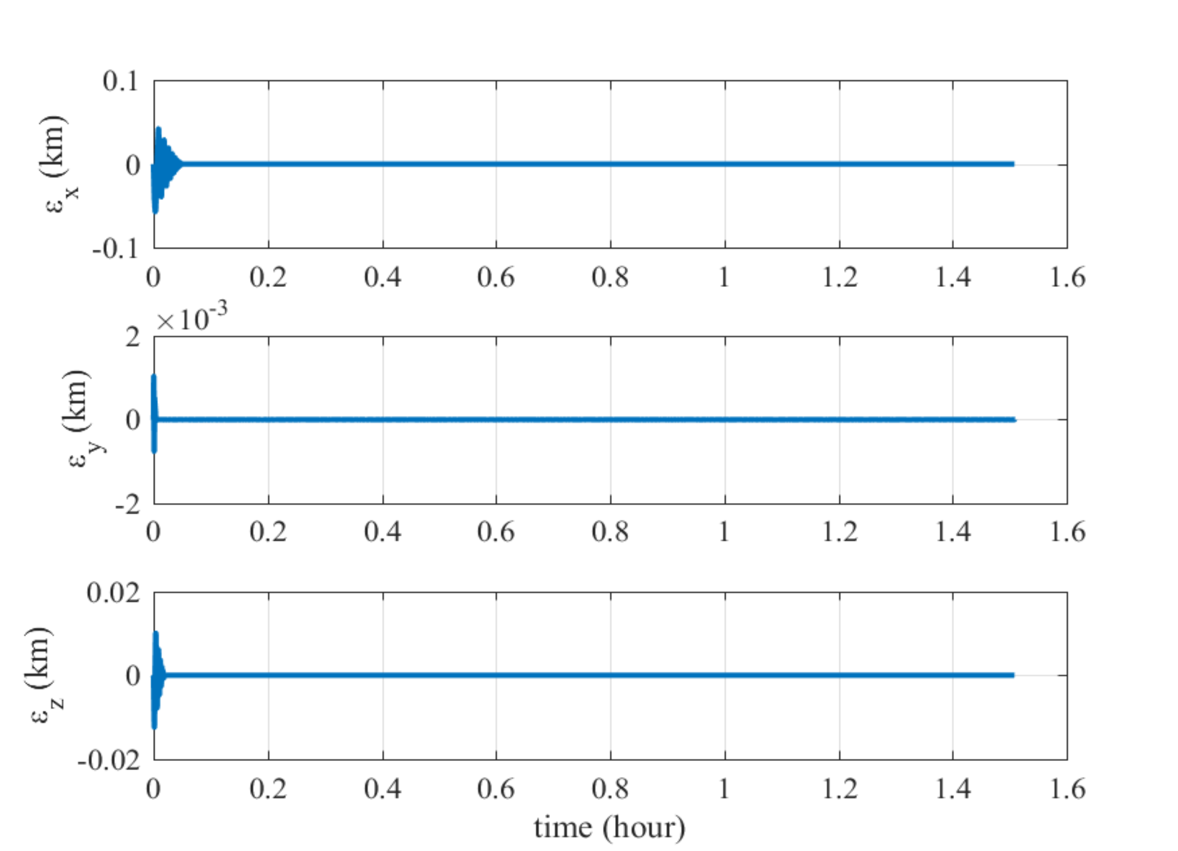


Relative orbit components

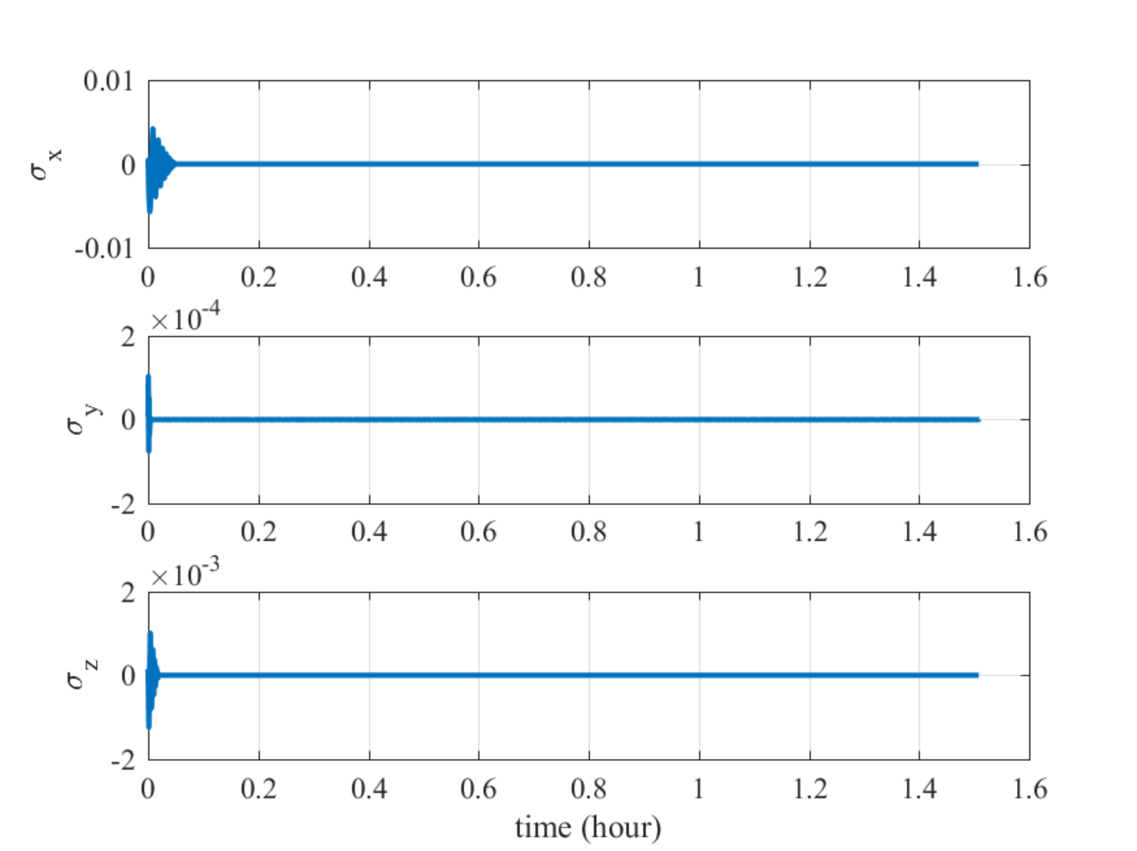


Satellite trajectories in ECI frame

Simulations

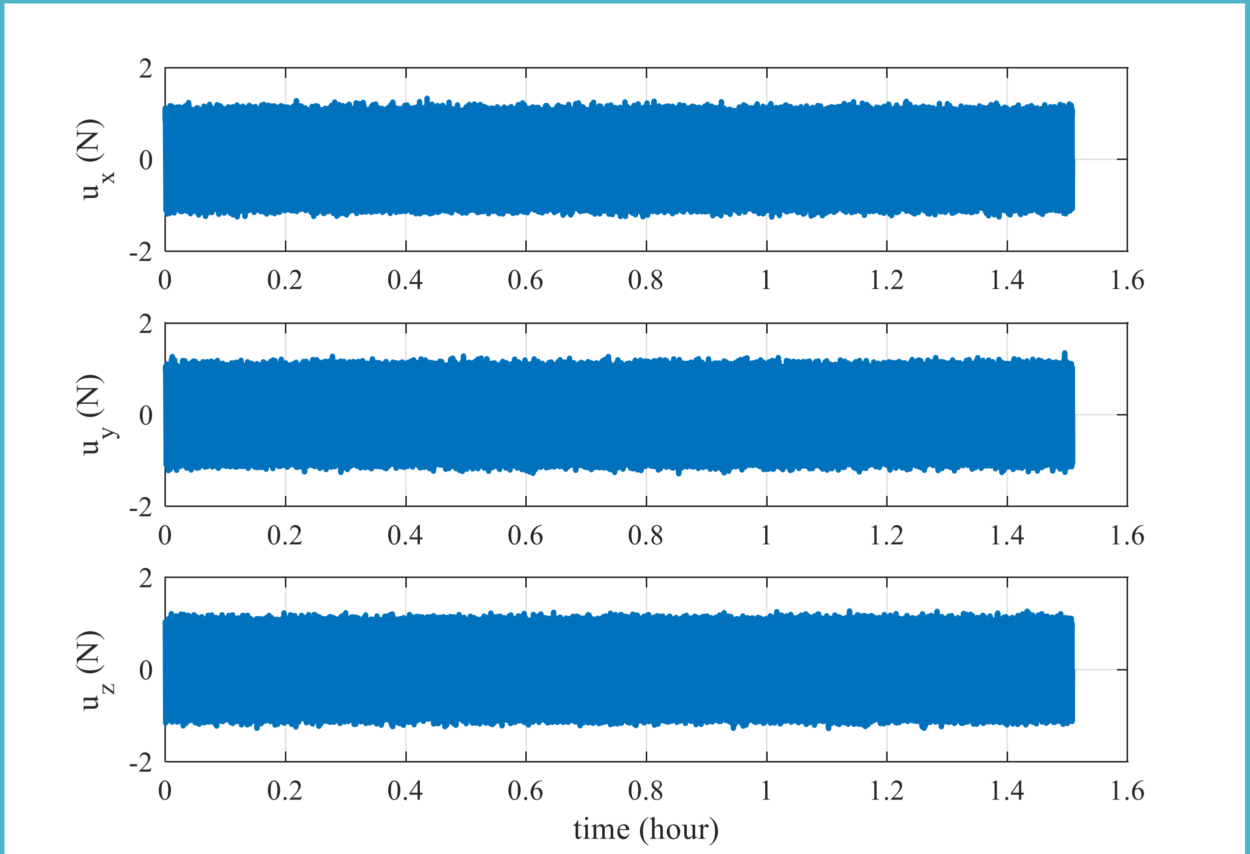


Error in relative orbit components, ε

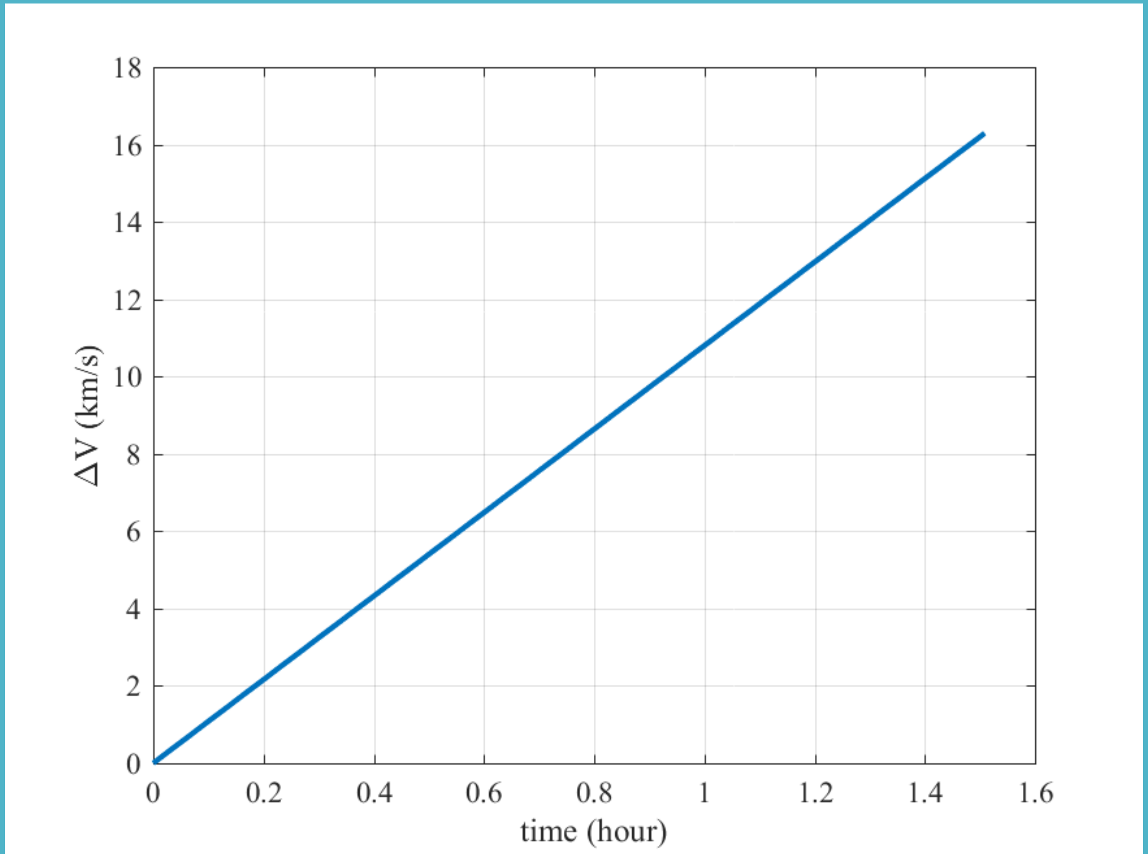


Sliding variable components, σ

Simulations



Control input components, u



Total ΔV over one orbit

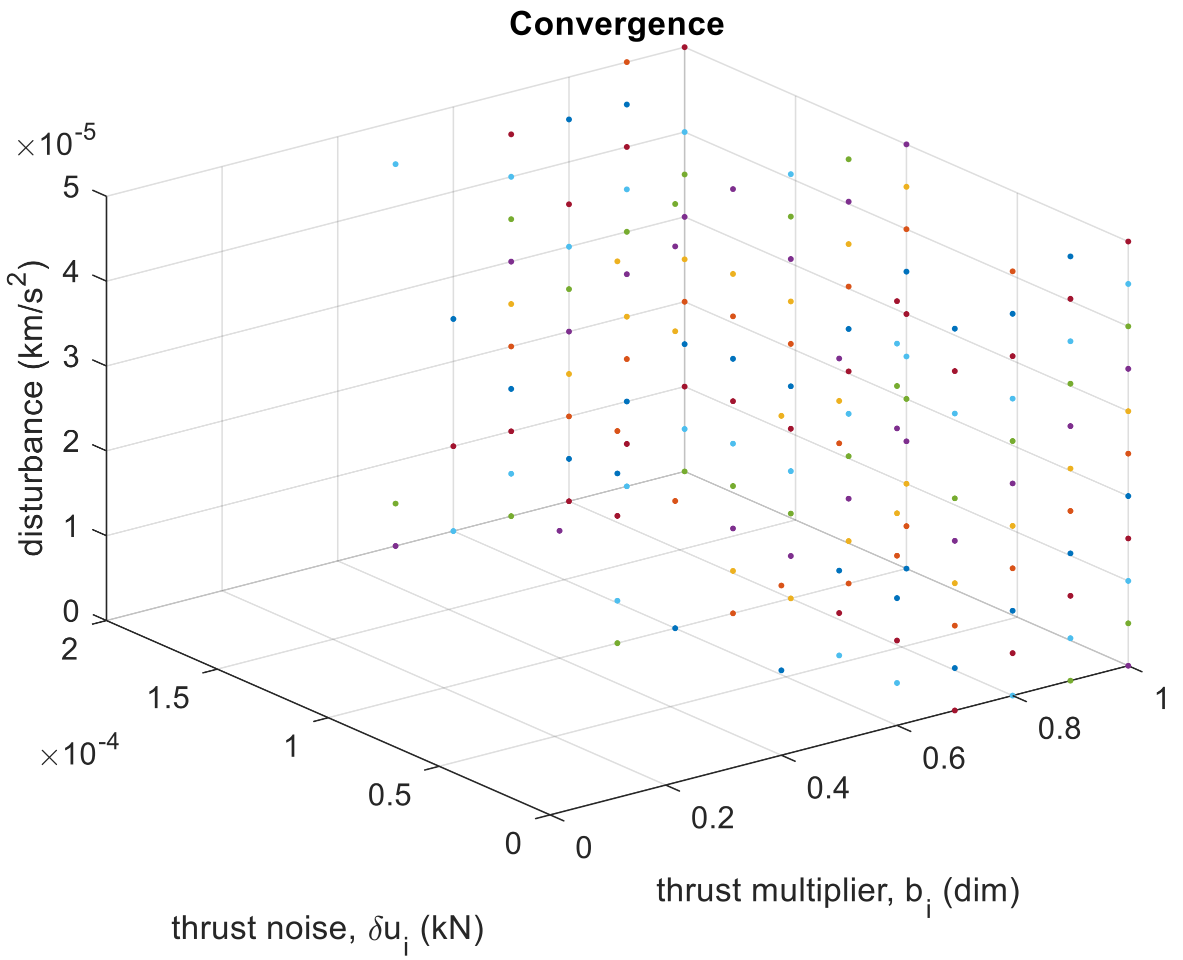
Simulations

Error magnitude was then varied to determine the range of values that can be compensated by the controller

Maximum error for each error type still resulting in convergence of x, y, and z:

Disturbance (m/s^2)	b_i (dim)	δu_i (N)
0 – 0.05	0.75 - 1	0 - 0.2

Simulations



Convergence of sampled error combinations

When errors are combined, convergence is achieved over a smaller range of error magnitudes

Outline

Conclusions

Nonsingular terminal sliding mode control scheme is effective for control of a formation flying satellite system with a range of disturbances and uncertainties

Simulation results demonstrate successful performance

Discontinuous control function is difficult and expensive for onboard implementation

Thank you. Questions?

Maggie Lawn

Outline

Experiment

Simulate orbital motion of the spacecraft in the $x - y$ plane

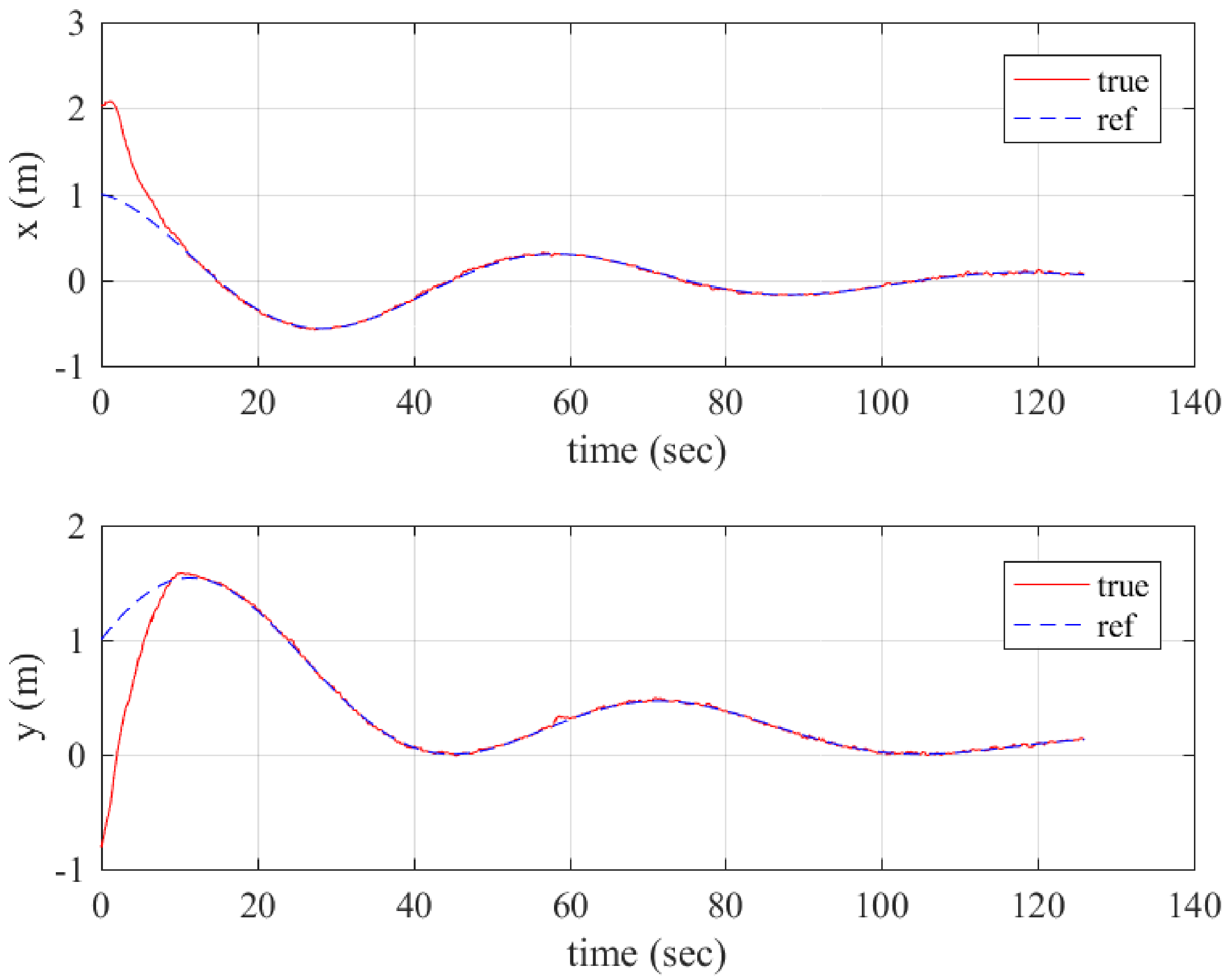
Dynamics were scaled down to the lab frame and used to obtain the wheel velocity and command inputs

- 1 orbital period in the spacecraft frame = 60s in the lab frame
- 1km in the spacecraft frame = 1m in the lab frame

Control parameters:

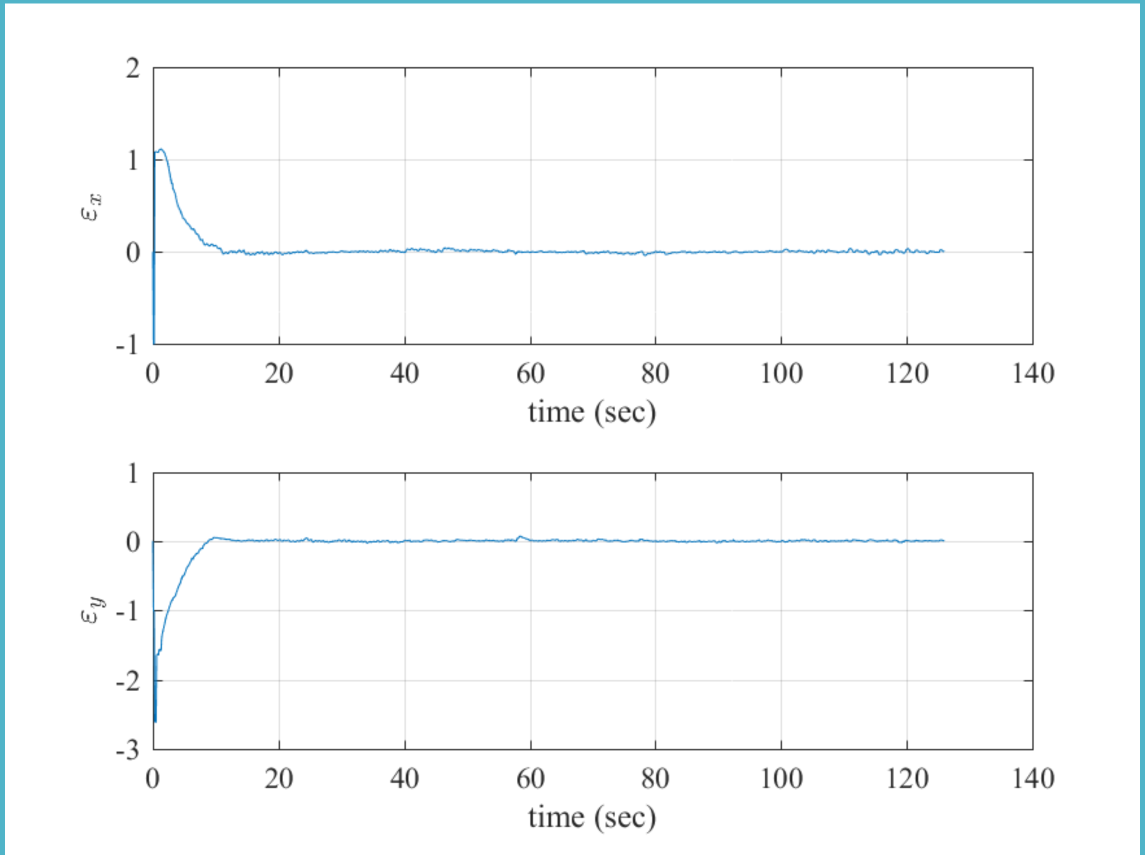
$\boldsymbol{c_2, c_3}$ (dim)	$\boldsymbol{m_1}$ (dim)	$\boldsymbol{m_2}$ (dim)	$\boldsymbol{k_i}$(N)
0.1	3	5	1.0

Experiment

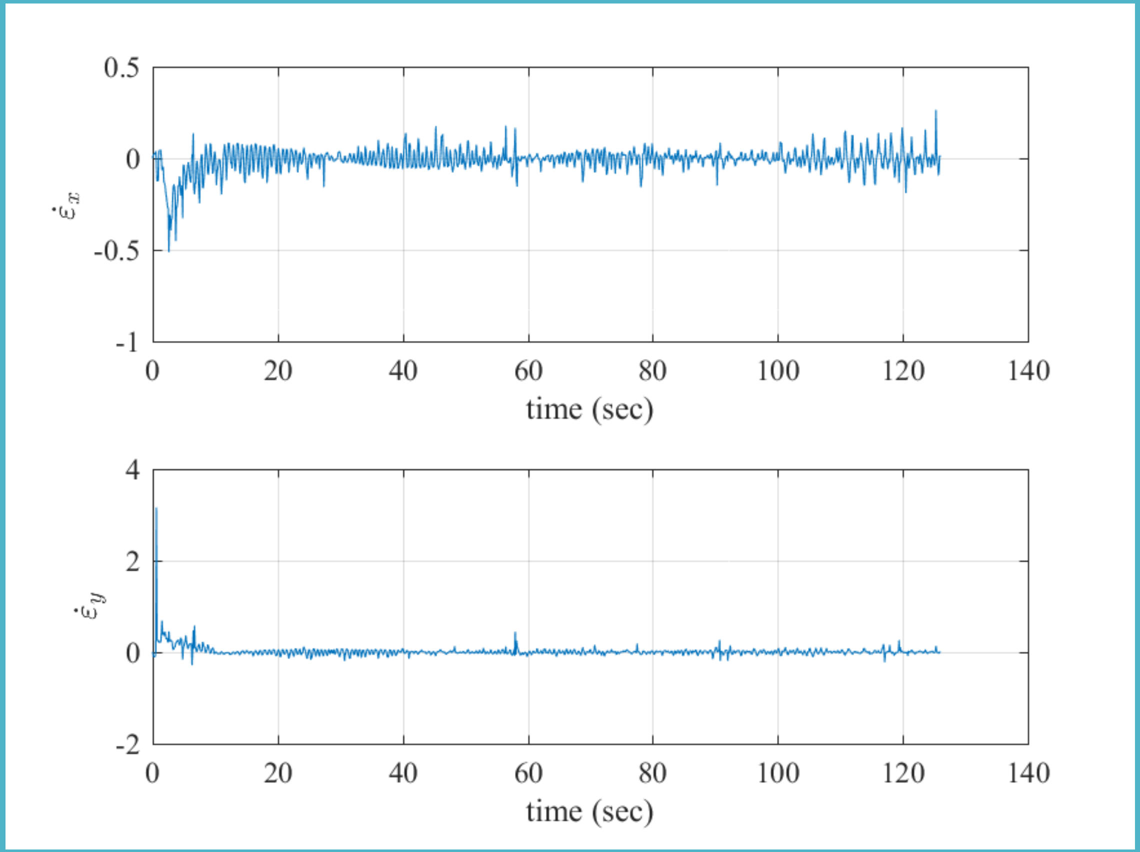


Robot’s trajectory with desired trajectory

Experiment

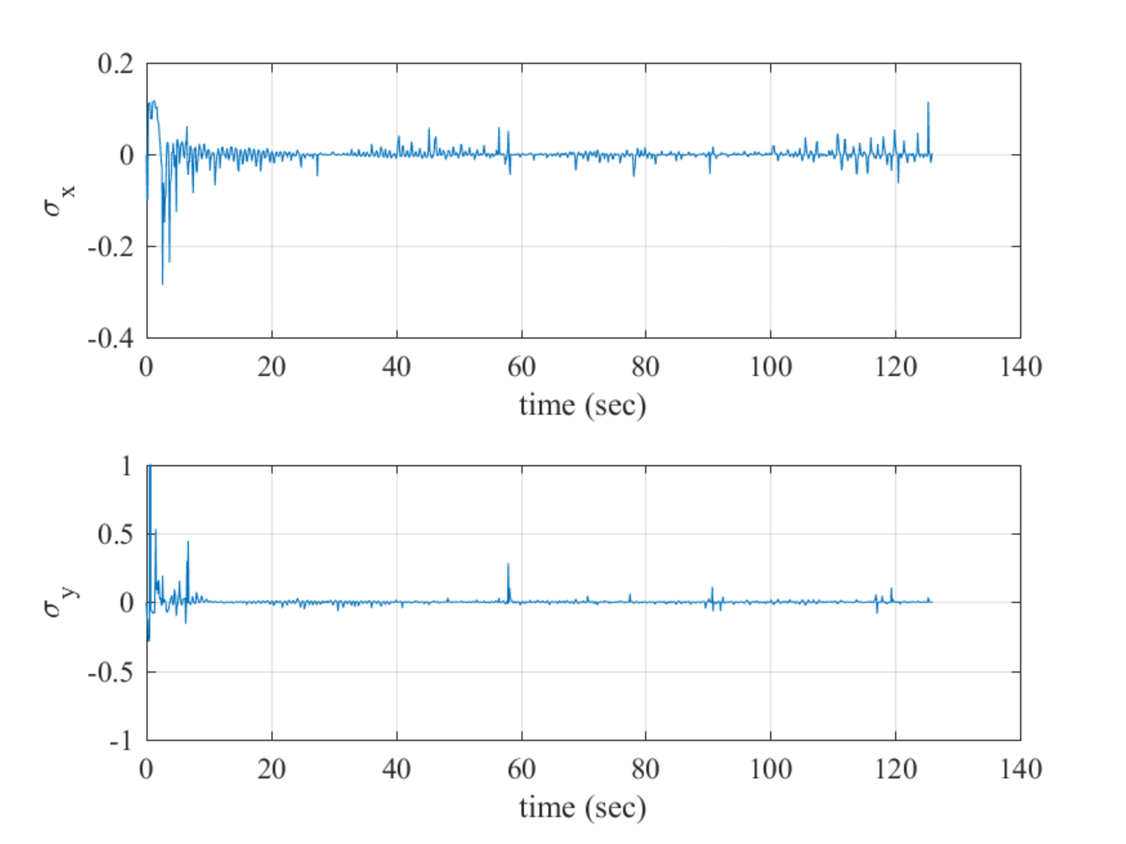


Robot's position error

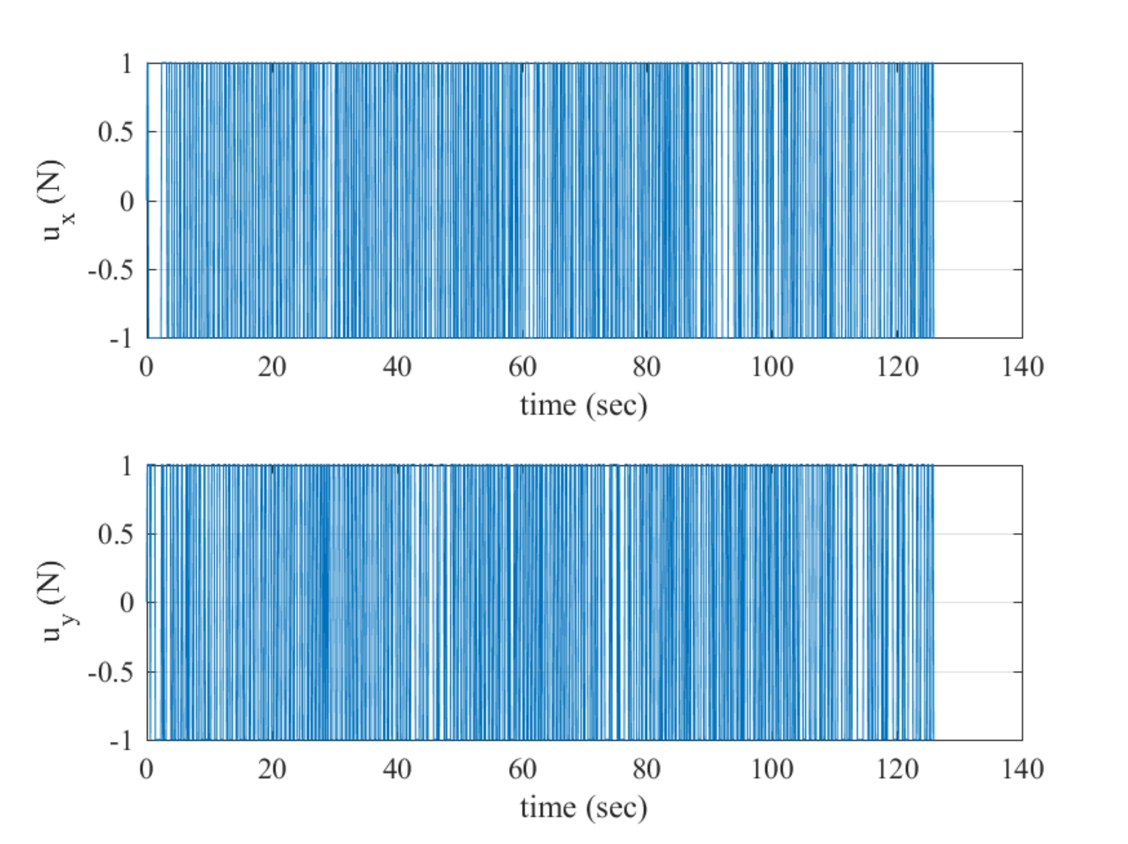


Robot's velocity error

Experiment



Sliding variable evolution over time



Robot’s commanded control input

Thank you. Questions?

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