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1 Definitions

Consider an agent who makes a choice over $x \in \mathbb{R}^N_+$, that is a distribution of the resources to N players including herself. Let $x = (x_s, x_O)$, where $x_s \in \mathbb{R}_+$ determines player's own payoff and $x_O \in \mathbb{R}^{N-1}_+$ is the payoff to all other players. We assume that a player is choosing from the budget set (or the set of available actions) denoted by B. Given the $\geq$ partial order over $\mathbb{R}^N_+$ denote by

$$B_{\geq} = \{y : \text{ there is } x \in B \text{ such that } y \geq x\}$$

the downward closure of the budget and denote by $B_{>}$ the interior of the downward closure.

Furthermore, we observe only finite set of choices. Hence, dataset $D = \{(x^t, B^t)\}$ is the collection of choices from budgets. In the general case we assume the budgets just to be compact. In addition, if we consider a uniformly distributed decision. That is every dollar is equally distributed among all other players, who may have different initial endowments. We just need the expenditures to be distributed

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at the equal rate.[1] We refer to such experiment as to *uniform experiment*. This type of situation for instance to the labor supply subject to re-distributive taxation, or choosing the optimal tax rates.

Further, we are looking for the utility function which rationalizes the observed choices and consistent with the particular theory. That is chosen point is at least as good as every choice in the budget. Next, we provide detailed definitions for every theory considered.

1.1 Altruistic Preferences In this case player gets utility about her own payoff and the payoff for every other player. We explicitly refer to these preferences as to altruistic since we do not require in general the budget to be equal to it's downward closure. If it is the case, then the assumption of monotonicity of the utility function can be reduced to the standard notion of other-regarding preferences. That is, utility function is just locally non-satiated.

Definition 1. *A dataset $D = \{(x^t, B^t)\}$ is rationalizable with altruistic preferences if there is a continuous and monotone utility function $u : \mathbb{R}^N_+ \to \mathbb{R}$ such that $u(x^t) \in \underset{x \in B^t}{\operatorname{argmax}}\, u(x)$.*

1.2 Justice Seeking Preferences We assume that a person gets utility from her own payoff and the justice in the society. Here we use general notion of justice consistent with the definition from e.g. Becker et al. (2013). That is the notion of justice is some social welfare function, which aggregates the payoffs the entire group receives. For instance, some players can be utilitarian while other can be egalitarian. The definition of the notion of justice we use allows for both (and much more) cases.

Before we proceed with the formal definition of the notion of justice let us introduce some more notation. Denote by S_n the space of all possible permutations of the vector of length n and by $\sigma \in S_n$ a permutation.

[1] In more general setting we can allow the money to be distributed at constant pace for only sub-population. Moreover, it can be possible that money are distributed with different weights to the different parts of the population, however, the player herself can only decide on the total amount of money to be distributed, but not the detailed distribution to each member.

Definition 2. *A continuous function* $f : \mathbb{R}^{N-1} \to \mathbb{R}$ *is said to be a justice function if it satisfies*

- *monotonicity:* $x \geq (>)x'$ *implies* $f(x) \geq (>)f(x')$ *for every* $x, x' \in \mathbb{R}_+^{N-1}$, *and*
- *symmetric:* $f(x) = f(\sigma(x))$ *for every* $\sigma \in S_{N-1}$.

Notion of justice is only required to satisfy the Pareto ordering and to be anonymous. That is quite general specification which is satisfied by most social welfare functions. Although, let us note that the notion of justice does not depend on the payoff player receives herself. At the same time the interdependence between the distribution of payoffs in the society and the payoff the player receives is done via the utility function.[2]

Definition 3. $f : \mathbb{R}^{N-1} \to \mathbb{R}$ *be a justice function. A dataset* $D = \{(x^t, B^t)\}$ *is rationalizable with justice seeking preferences if there is a continuous and monotone utility function* $u(x_s, f(x_O))$ *such that* $u(x^t) \in \underset{x \in B^t}{\operatorname{argmax}}\, u(x)$.

1.3 Inequality Averse Preferences Next, we consider a person with inequality averse preferences. In this case we require the notion of justice to be inequality averse. That is if we transfer some money from reach person to the poor person (such that ordering of people is preserved) the inequality reduces. This property is usually referred in the literature on inequality measures as Pigou-Dalton (transfer) principle. Hence, we can define the inequality measure.

Definition 4. *A continuous function* $f : \mathbb{R}^{N-1} \to \mathbb{R}$ *is said to be an inequality measure if it satisfies*

- *inequality aversion:* x *can be obtained (in relative income terms) from* x' *by a sequence of Pigou-Dalton transfers implies* $f(x) \geq (>)f(x')$ *for every* $x, x' \in \mathbb{R}_+^{N-1}$, *and*
- *symmetric:* $f(x) = f(\sigma(x))$ *for every* $\sigma \in S_{N-1}$.

[2] If we include x_s into the justice function, all the further results would still be necessary, being also sufficient for special cases. For more details on this case see Becker et al. (2013)

Note that inequality measure does not take into account the possible dominance relation over the income distributions. Although, the both properties can be merged upon necessity (see Appendix X for more details). However, this would deliver the theory which is weaker than both inequality aversion and justice seeking preferences.

Definition 5. *$f : \mathbb{R}^{N-1} \to \mathbb{R}$ be an inequality measure. A dataset $D = \{(x^t, B^t)\}$ is rationalizable with inequality averse preferences if there is a continuous and monotone utility function $u(x_s, f(x_O))$ such that $u(x^t) \in \operatorname*{argmax}_{x \in B^t} u(x)$.*

2 Results

Next we present the revealed preference tests for the three theories of social preferences listed above. We start with presenting the test for altruistic preferences, next we present the test for the justice seeking preferences and finally the test for the inequality averse preferences.

2.1 Altruistic Preferences Altruistic preferences has been studied by various previous studies and the revealed preference criteria follows from the classical results from Afriat (1967) and more recent result from Nishimura et al. (2017). Hence, the rationalization with altruistic preferences is equivalent to the Generalized Axiom of Revealed Preferences (introduced by Varian, 1982).

Definition 6. *A dataset satisfies GARP if for every sequence $x^{t_1}, \ldots, x^{t_n}$ such that $x^{t_j} \in B^{t_{j+1}}_{\geq}$ for every $j \in \{1, \ldots, n-1\}$ then $x^{t_n} \notin B^{t_1}_{>_{t_1}}$.*

Theorem 1 (Afriat (1967); Varian (1982))**.** *A dataset is rationalizable with altruistic preferences if and only if it satisfes GARP.*

2.2 Justice Seeking Preferences

Definition 7. *A dataset satisfies Sy-GARP if for every sequence $x^{t_1}, \ldots, x^{t_n}$ such that $(x_s^{t_j}, \sigma(x_O^{t_j})) \in B^{t_{j+1}}_{\geq}$ for some $\sigma \in S_{N-1}$ and for every $j \in \{1, \ldots, n-1\}$ then there is no $\sigma \in S_{N-1}$ such that $(x_s^{t_n}, \sigma(x_O^{t_n})) \notin B^{t_1}_{>_{t_1}}$.*

Theorem 2. *If an experiment is rationalizable with justice seeking preferences, then it satisfies Sy-GARP. Moreover, if data comes from the uniform experiment, then the reverse is also true.*

2.3 Inequality Averse Preferences Denote by $L(u)$ the Lorenz curve, where $u \in [0,1]$ are the incomes in the normalized (by the maximum income) terms. We refer to the L to *dominate* L' if $L(u) \geq L'(u)$ for every $u \in [0,1]$.

Definition 8. *A dataset satisfies LD-GARP if for every sequence $x^{t_1}, \ldots, x^{t_n}$ such that $x_s^{t_j} \geq x_s^{t_{j+1}}$ and $L^{t_j}(u)$ dominates $L^{t_{j+1}}(u)$ for some $\sigma \in S_{N-1}$ and for every $j \in \{1, \ldots, n-1\}$ then $x_s^{t_1} \leq x_s^{t_n}$ or/and $L^{t_1}(u)$ not dominates $L^{t_n}(u)$.*

Theorem 3. *If an experiment is rationalizable with inequality averse preferences, then it satisfies LD-GARP. Moreover, if data comes from the uniform experiment, then the reverse is also true.*

References

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