

# Maggie's Summer Summary

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## 1 The Summary List

1. Reviewed last summers codes
2. Generalized Deflation - grumpy face and happy face
  - Reviewed Patrick Farrell's papers
  - Worked out how the process could be made general (happy face and grumpy face)
  - Implemented and debugged the automatic deflation
3. Lot's of flowcharts that were updated when I changed the structure of my 1D coupling of codes
4. Tolerance,  $\epsilon$ , and  $N$  tests. Here I learned that I was oversolving the problem and could use a  $tol = 1 \times 10^{-8}$  instead of  $1 \times 10^{-12}$ .
5. Explored ascent directions. In particular, I read Dennis and Schnabel's book about how to perturb your Newton step in the event of a descent direction
6. Implemented a code to perturb the Newton step if needed (if problem became ill-conditioned leading to an ascent direction)
7. Did some Deflation Parameter testing in Colab ( $\rho$  and  $\alpha$ ). These tests became too time consuming for Colab.
8. Revisited and finalized the 1D Linear Singularly Perturbed Problem (the initial example for the MP-iteration). I used Dr. MacLachlan's problem which has a known analytical solution that can be used to track the error.
9. Experimented with the choice of Mesh Density Function. In particular, I compared the different choices in the HR Book. I ended up choosing the Optimal Curvature Mesh Density function because it resolved the solution features nicely (the Arclength Mesh Density Function could lead to spikes for small  $\epsilon$ ).

10. Once I had things working in 1D I focused on solution discovery. I created a solution discovery code which coupled all of my methods to try and find as many solutions as possible. I was channeling my inner Patrick Farrell
11. I implemented parameter continuation
12. I researched other forms of continuation (the PseudoArclength continuation in AUTO)
13. Learned about remote access and Git
14. Transitioned my codes to Dr. MacLachlan's computer
15. Updated the structure of solution discovery. I wanted to make use of every guess to find as many solutions as possible. Now I was really channeling my inner Patrick Farrell
16. Wrote a summary LaTeX document of my work in 1D
17. Learned about Finite Element Methods with a focus on the weak formulations (Mark Gockenbach, Graeme Fairweather)
18. FireDrake tutorials and setup
19. Solved simple PDE's with FireDrake
20. Learned about Adaptive Meshing in 2D (HR Book)
21. Simple mesh solve - create a uniform mesh (x,y)
22. L - Shaped Mesh exercise
23. Tried to nail down the  $(x, y)$  mesh problem ( use  $M$  in the PDE, solve on the background mesh  $(\xi, \eta)$ , the resulting  $x$  and  $y$  give your mesh directly, may require us to solve a nonlinear problem ) verses the  $(\xi, \eta)$  mesh problem ( $\frac{1}{M}$  in your PDE, the  $\xi$  and  $\eta$  solution needs to be interpolated to obtain the adapted  $(x, y)$  mesh (need the  $p^{ij}$  in R. Hagmeijer's paper) values in , often allows us to solve a linear instead of nonlinear problem).
24. Worked on the MP-Iteration
  - The basic setup in FireDrake
  - Switching between the  $x(\xi, \eta)$  problem and the  $\xi(x, y)$  problem.
  - Computing the  $p^{ij}$  values for the  $\xi(x, y)$  problem. I did not finish this work.
  - Worked on the classic MP - Iteration (P solve then M solve and repeat)
  - Read Niall Madden's paper and worked on their version of the MP - Iteration (P solve and then an M iteration)
  - Started working on their Singularly Perturbed Example.

## 2 Future Goals

- Need to finish the MP-iteration work and start solving the desired singularly perturbed problem
- Deflation in 2D
- Solution discovery, line search, interpolation, etc in 2D
- Paper write up.

## 3 Important Notes

- Alan Lindsay is working on this same problem which is scary. He is aware that I have been working on it but that will not stop him. He did suggest I work on Carrier's Problem for an example.