BEN 580: Computational Methods in Biomedical Engineering
Assignment One

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1 Parachutist Problem Excel

The objective of the parachute problem is to solve for the velocity of a parachutist, with some given initial conditions. There are a variety of ways to solve this problem, including using numerical and analytical solutions in Excel, as well as using a MATLAB code to do the same. First, the problem was attacked using Excel. In order to do this, some connections must be made using knowledge from physics and calculus to find analytical and numerical equations.

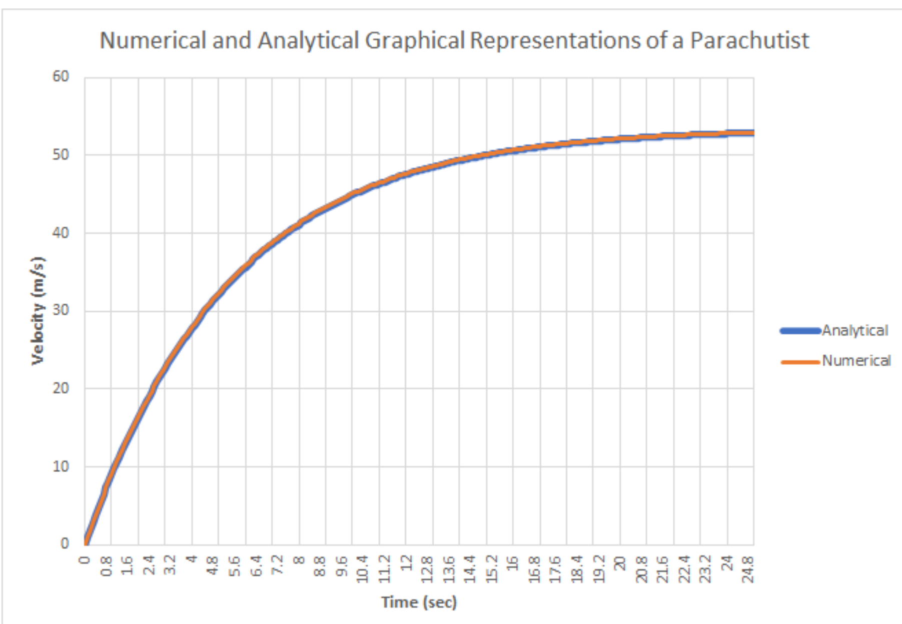

Figure 1: Results Graph from Excel Parachutist Exercise

In Figure 1, it can be seen that the numerical and analytical solutions to this problem given the initial conditions are relatively similar. Finding the difference between the analytical value and the numerical value at one time interval can give insight to where the largest error occurs and how it behaves, when graphed against time.

2 Parachutist Problem MATLAB

The next method taken into consideration for this problem was to use the same analytical and numerical equations, but this time in MATLAB. In order to do this, a simple code was created that essentially lists the constants (g, m, c, t, and dt), states the analytical equation, creates a for loop to iterate over the numerical equation, and plots the two results as a representation of velocity vs. time. This can be seen in the attached MATLAB script. After running the script, a figure box pops up and gives results that look almost identical to the results from Excel.

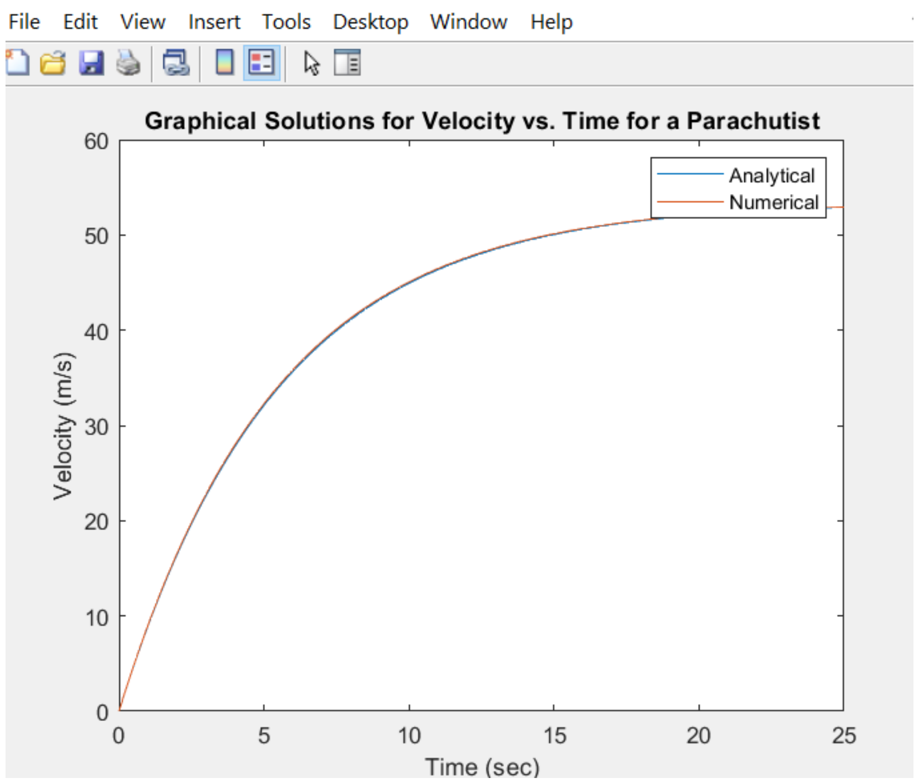

Figure 2: Results Graph from MATLAB Parachutist Exercise

3 Problem 1.9

Problem 1.9 requires the use of Euler's method to solve for the depth, y, from $t = 0$ to $10d$ with a step of $0.5d$. Some parameter values are given, including that $A = 1250m^2$, $Q = 250m^3/d$, and the initial condition of $y = 0$. Below is the equation for depth, given that the surface area, A, is constant. The second equation, which is the solution to the ODE, is then modeled in MATLAB to solve for the depth at time $t = 0$ to $10d$.

$$\int \frac{dy}{dt} = \int \left(\frac{3Q}{A} \sin^2 t - \frac{Q}{A} \right) dt$$

$$y(t) = \frac{3Q}{A} \left(\frac{1}{2}t - \frac{1}{4} \sin 2t \right)$$

The MATLAB code to solve for this is simple. Like above, the constants are defined, the equation is inputted, then a graph is plotted from these results. Conceptually, it makes sense for the slope to be positive due to the fact that at y = 0, the tank is half full.

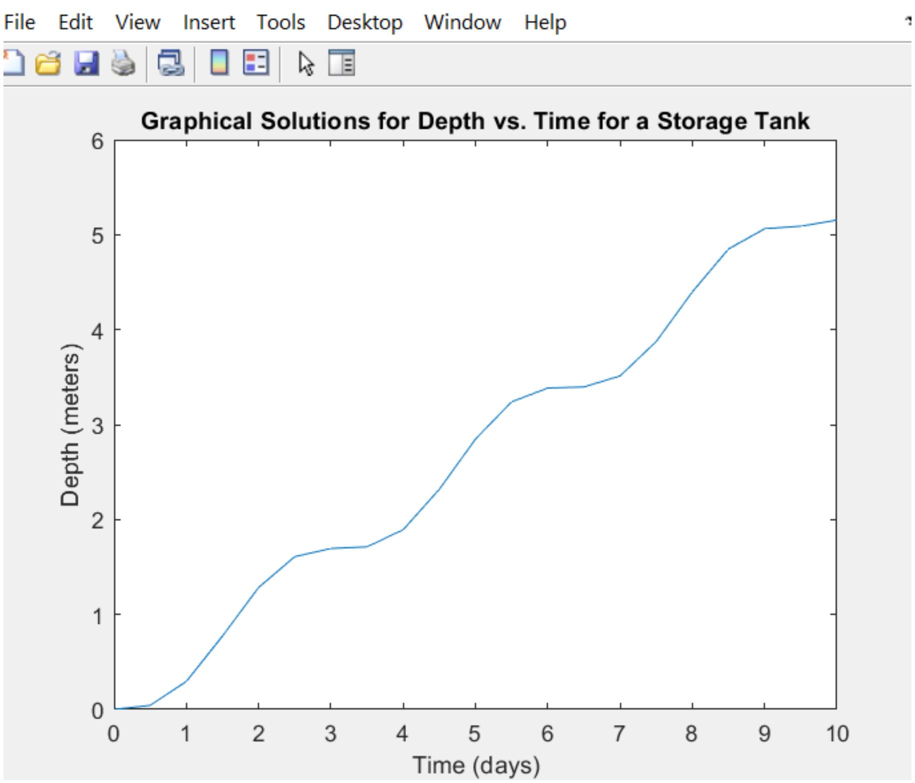

Figure 3: Results Graph from MATLAB for Problem 1.9

4 Problem 1.16

Problem 1.16 states that cancer cells grow exponentially with a doubling time of $20h$ when they have unlimited nutrient supply, however, as the cells start to form a solid spherical tumor (without blood supply), growth becomes limited and the cell begins apoptosis. The answers to questions a, b, and c will be listed below and can be confirmed by the MATLAB code that will also be submitted.

a. Exponential growth of these cancer cells can be modeled through the ordinary differential equation given ($\frac{dN}{dt} = \mu N$, where $N(0) = 1$ and $N(20) = 2N(0)$). Some simple computations are done on this ODE to calculate the value of the growth rate of the cells.

$$\frac{dN}{dt} = \mu N \Rightarrow \frac{1}{N}dN = \mu dt$$

$$\int \frac{1}{N}dN = \int \mu dt \Rightarrow N(t) = N_0 e^{\mu t}$$

$$N(20) = e^{20\mu} \Rightarrow \mu = \frac{\ln 2}{20}$$

Using this information, it can be concluded that the growth rate of these tumor cells is uniformly modeled by $\mu = \frac{\ln 2}{20}$

b. The next step in solving this problem is to write an equation that describes the rate of change of tumor volume during exponential growth, given that the initial diameter of a cell is 20 microns.

$r_{cell} = 10 * 10^{-6}$. Volume of one cancer cell $= \frac{4}{3}\pi {r_{cell}}^3$.

$$V(t) = V_{cell}N(t) \Rightarrow \frac{dV}{dt} = \mu V_{cell}N(t)$$
$$\frac{dV}{dt} = \frac{\ln 2}{20}V_{cell}N(t) \Rightarrow \frac{dV}{dt} = \frac{\ln 2}{20}V(t)$$

c. Finally, it can be determined how long it will take for the tumor to exceed the critical size of 500 microns, which is the point at which the cells at the center of the tumor die (but continue to take up space in the tumor).

$$V(t_0) = \frac{4}{3}(250 * 10^{-6})^3\pi \Rightarrow V(t_0) = \left(\frac{500}{20}\right)^3 V_{cell}$$

By plugging this expression into the original solution from the ODE from above, the V_{cell} component of the $V(t_0)$ expression, gets cancelled and leaves $N(t_0) = 15625$.

$$N(t_0) = e^{\mu t_0}$$
$$\ln 15625 = \frac{\ln 2}{20}t_0$$

$$t_0 = \frac{20 \ln 15625}{\ln 2}$$

Using this result, the amount of time it takes for the tumor to reach a critical diameter of 500 microns, is 12 days.