

## Requirements

1. Credits are given to typed homework only.
2. Submit a word or pdf file on <http://compass2g.illinois.edu> before 1pm on Wednesday 3/16/2016. Append your codes. You homework must be finished independently. It will go through plagiarism screening.
3. Submit a hardcopy without codes at the beginning of the class on Wednesday 3/16/2016.

(20 points)

In future works, you should use the L'Ecuyer algorithm (Fig 2.3) you implemented in HW3. Consider two asset-or-nothing call options in the BSM model with  $S_0 = 2000$ ,  $r = 0.5\%$ ,  $q = 2\%$ ,  $\sigma = 0.3$ ,  $T = \frac{1}{12}$ ,  $K_1 = 1900$ ,  $K_2 = 2200$ .

1. (4 points) Derive and compute the exact prices for the two options.
2. (4 points) Use Monte Carlo simulation to estimate the prices (direct approach). Report your estimates, absolute errors, standard errors and total computational times for an increasing sequence of sample sizes.
3. (5 points) Repeat 2 with the underlying asset as a control using the same sample sizes. Report the above quantities. For which option does the control variate approach work better? Explain.
4. (5 points) Repeat 2 using importance sampling using the same sample sizes. For each option, try and find  $\hat{\mu}$  that achieves the smallest standard error. Report the same quantities as in 2 using the best  $\hat{\mu}$  you find. For which option does the importance sampling approach work better? Explain.
5. (2 points) For each option, compare the three methods (direct, control variate, importance sampling) in terms of efficiency.