

Requirements

1. Credits are given to typed homework only.
2. Submit a word or pdf file on <http://compass2g.illinois.edu> before 1pm on Wednesday 2/17/2016. Append your codes. You homework must be finished independently. It will go through plagiarism screening.
3. Submit a hardcopy without codes at the beginning of the class on Wednesday 2/17/2016.

(10 points) Consider the European vanilla call option on SP500 with strike price $K = 1870$ and maturity $T = 1/52$ (which is one week). Assume that the index follows a geometric Brownian motion in the risk neutral world:

$$S_T = S_0 \exp \left(\left(r - q - \frac{1}{2} \sigma^2 \right) T + \sigma B_T \right).$$

The current SP500 value is $S_0 = 1868.99$. The risk free interest rate is $r = 0.3866\%$. The dividend yield of SP500 is $q = 2.32\%$. The volatility is $\sigma = 29.79\%$. Using the Black-Scholes formula, the call price can be computed to be 29.9558.

1. (4 points) Read lecture notes 2. Write a C++ program to compute the call price using Monte Carlo simulation with antithetic variates.
2. (4 points) Construct a table comparing the standard approach in HW1 and the antithetic approach. Your table must contain the following:
 - An increasing sequence of sample sizes.
 - For each sample size, the call price computed using the antithetic approach, the estimated standard error, the 95% CI, the total computational time in seconds, the efficiency measure $(\text{standard error})^2 \times \text{computational time}$. Report the same for the standard approach.
3. (2 point) Which method is more efficient? Describe what you have done to improve the speed of your implementation.

Notes:

1. Please make sure you report all the quantities that are asked for. In HW1, many people just didn't report all the quantities I asked for in their tables.
2. For each normal variate, you generate one call payoff in the standard approach and two call payoffs in the antithetic approach
3. The largest sample size you use should be such that the 95% CI has a width of at most two cents for the antithetic approach (the CI for the standard approach would be wider)
4. Think about what you can do to improve the speed of your implementation
5. To check accuracy of your implementation, compare your estimates to the benchmark price
6. Read lecture notes 1 and check your codes for the standard approach again. Many people reported prices that do not converge to the Black-Scholes price. Other people reported standard errors that are obviously wrong.