

Domain wall Quasiparticles  $\leftrightarrow$  Flipped spin  
Quasiparticles.

$$H = -t \sum_{\langle j, k \rangle} (c_j^\dagger c_k + c_k^\dagger c_j) + V \sum_{\langle j, k \rangle} n_j n_k$$

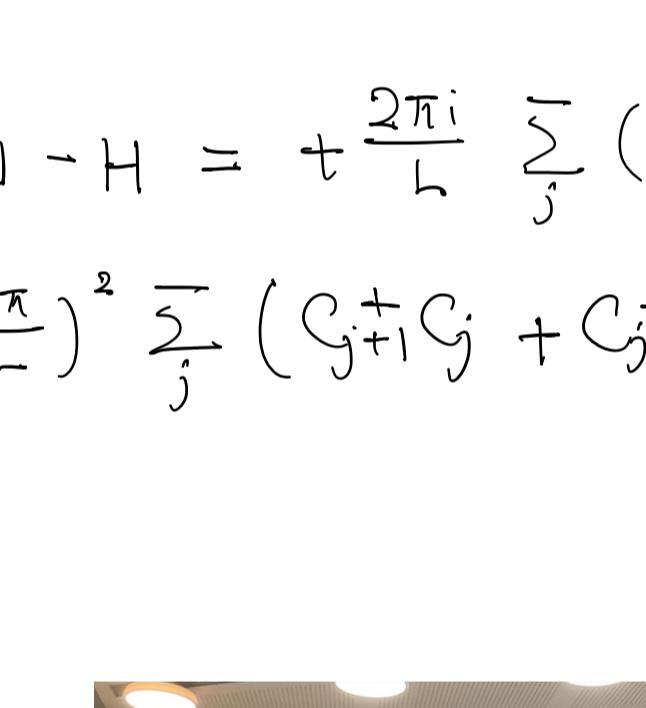
$T(1)$  symmetry:  $c_j \rightarrow e^{i\theta} c_j$   
 $c_j^\dagger \rightarrow e^{-i\theta} c_j^\dagger$

Nambu Goldstone theorem  $\rightarrow$  gapless excitation

There are many systems without SSB.

However, any other mechanism for gaplessness?

Yes, if there is a lattice with translational invariance.

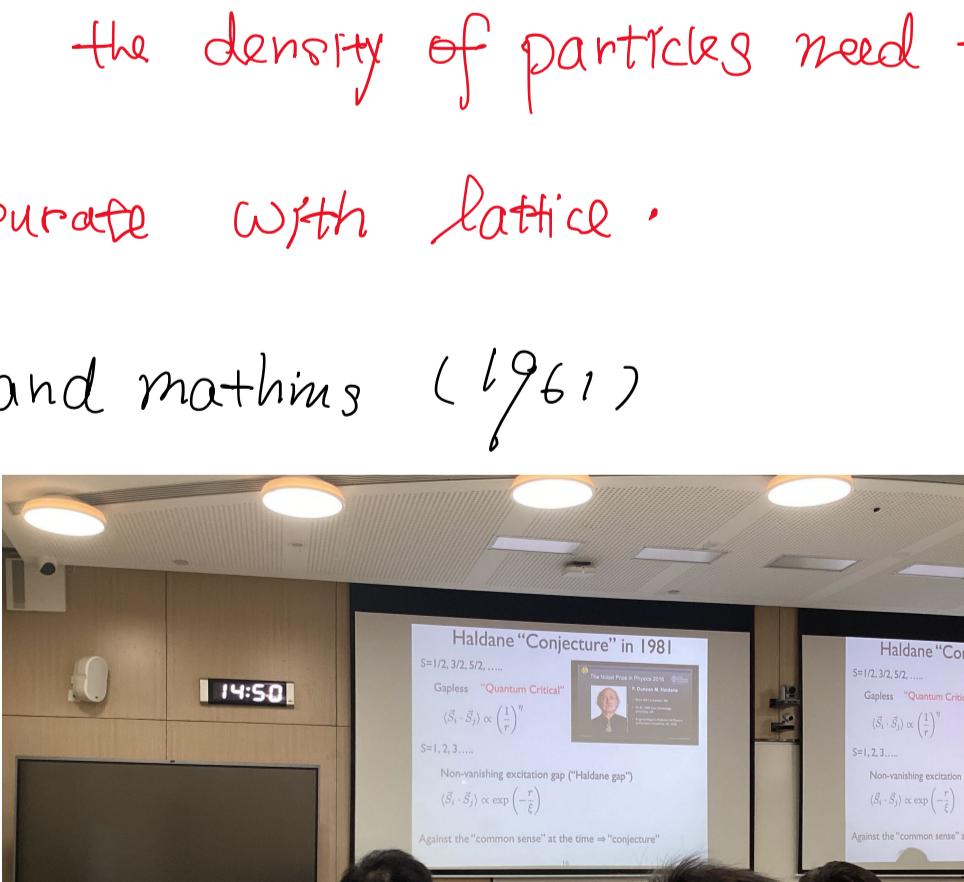


What is the difference between two systems, one with gapped excitation spectrum, the other with gapless excitation spectrum?

LSM:  $T(1) \rightarrow$  Lattice translational symmetry + spatial inversion or time reversal

$T(1)$  Generator:  $e^{i\theta N} = e^{i\theta \sum c_i^\dagger c_i}$

$$T^\dagger c_j T \rightarrow e^{i\theta j} c_j$$



$T|\Psi_0\rangle$  could be identical to  $|\Psi_0\rangle$

$$T|\Psi_0\rangle = e^{i\theta_0} |\Psi_0\rangle$$

$$T U T^\dagger |\Psi_0\rangle = e^{i(\theta_0 + i\nu n)} |\Psi_0\rangle$$

$\nu \neq \text{integer}$

Haldane half-odd integer  $\rightarrow$  gapless

Yamakata  $\rightarrow$  MD-Affleck (1997)

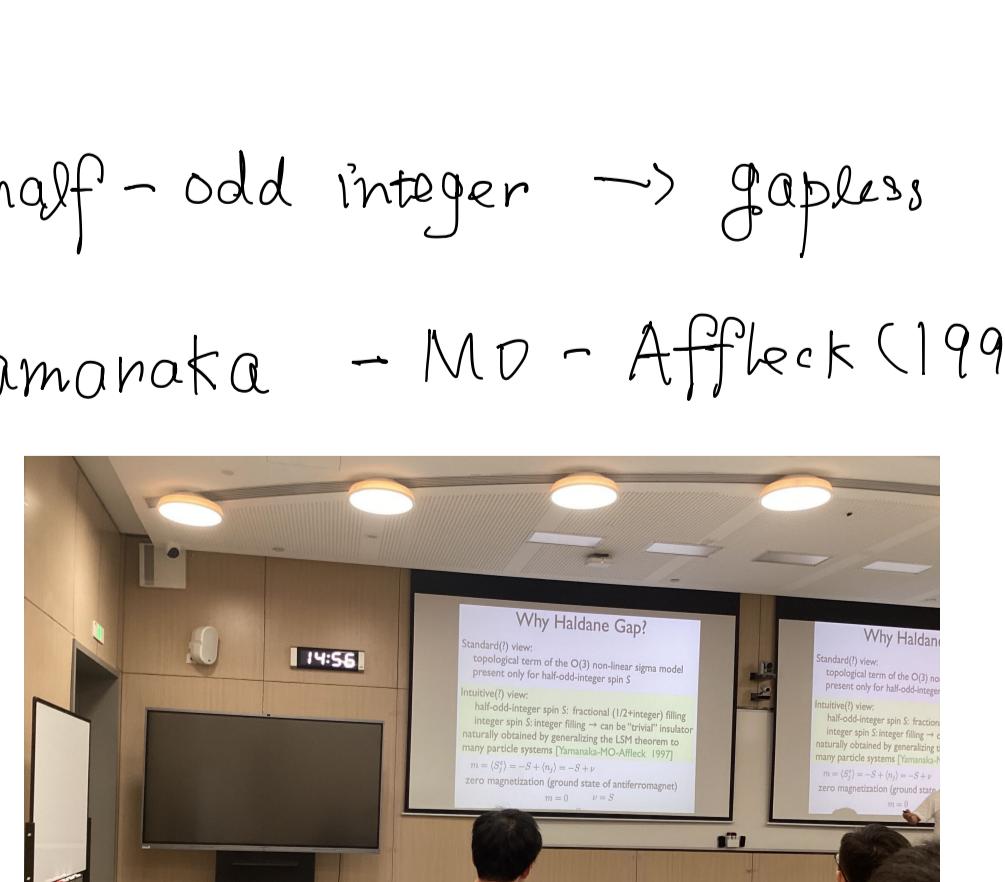


$\frac{2\pi i}{L} \sum_j (c_{j+1}^\dagger c_j - c_j^\dagger c_{j+1})$

$$+ \left(\frac{2\pi}{L}\right)^2 \sum_j (c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1}) + \mathcal{O}\left(\frac{1}{L^2}\right)$$

Gapped phase needs the particles to be locked, the density of particles need to be commensurate with lattice.

Lieb and Mattis (1961)



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Hofst anomaly

