HW 7

1)a) The probability of selecting from urn X when a red ball is picked. If $P(X|R) > ½$, indicates X will choose urn X. When $p>b/(b+a)$, it is large enough to always select X.

$P(X|R) = P(R|X)P(X) /(P(R|X)P(X) + p(R|Y)P(Y)) > ½$

$= ap/(ap+b(1-p)) > ½$

$Ap > ½ (ap+b(1-p))$

$½ bp + ½ ap > ½ b$

$Bp + ap > b$

$b/(b+a) < p <=1$

b) Player 2 encounters a situation similar to Player 1. With limited information, Player 1 makes a guess based on a calculated probability, p. Knowing Player 1's guess but not the marble's color, Player 2 also uses this probability and chooses X when p is sufficiently large to always guess for X.

c) Given p = 0.5 , a=0.5, and b = 0.75:

If Player 1 chose Urn X, it suggests he picked a blue marble. If he drew a red marble, he would have guessed Urn Y as Y has greater probability for red marbles.

Player 2’s choice of Urn Y implies picking a red marble.

If Player 3 draws a blue marble, they will choose Urn X since $8/11 > ½$

$P(X|BRB) = P(BRB|X)P(X)/(P(BRB|X)P(X)+P(BRB|Y)P(Y))$

$= (1-a)^2 * ap/((1-a)^2* ap + (1-b)^2 * b* (1-p))$

$= 0.5^4 /(0.5^4 +0.25^2 * 0.75 * 0.5)$

$=8/11 = 0.727$

d) Let’s determine the color of the marble chosen by P1:

$P(X|R) = P(R|X)P(X)/P(R)$

$= P(R|X) * P(X)/((P(R|X)P(X) + P(R|Y)P(Y))$

$= 0.25 * 0.6 / (0.15 + 0.75 * 0.4)$

$= 1/3 = 0.33$

$1/3 < ½$, suggests X chose blue marble.

$P(X|BB) = P(BB|X)P(X)/(P(BB|X)P(X) + P(BB|Y)P(Y))$

$= (1-a)^2 * p / ((1-a)^2 * p + (1-b)^2 * (1-p))$

$= 0.75^2 * 0.6 / (0.75^2 * 0.6 + 0.25^2 * 0.4)$

= 27/29 = 0.931

27/29 > ½ , means Y chose urn X.

2) a) Starting from Node 1:

1st step: Both Nodes 2 & 9 switch to A , as 50% of their neighbours use A, exceeds threshold of 0.4

2nd step: Both nodes 3 & 8 switch to A , as 50% of their neighbors use A, exceeds threshold of 0.4.

3rd step: Both nodes 4 & 7 switch to A , as 50% of their neighbours use A , exceeds threshold of 0.4

4th step: Both nodes 5 & 6 switch to A, as 50% of their neighbours use A , exceeds threshold of 0.4

All nodes will eventually adopt A

b) Start from Node 2:

Step 1: 1 switch to A , as 50% of its neighbors use A, which exceeds threshold of 0.4

Step 2: 9 switch to A , as 67% of its neighbors use A, which exceeds threshold of 0.4

Nodes 1,2 & 9 eventually adopt A.

c) No, as this graph seems to be symmetric, 3 & 9 can be taken as an example. In this case, 1 ,2 , 9 , 3, 4 and 5 will eventually adopt A. But, spreading stops at nodes 6 & 8 , since only 1/3 of their neighbors are adopting A, which is less than threshold of 0.4.

Using cluster density, cluster density is 2/3 for each small triangle, which is > 1-q . Means there won't be any cascades.

Q3

a)

Starting from node 3:

- In the first step, nodes 1, 2, 6, and 7 switch to state A:
  - Node 1: 33% of its neighbors have adopted A.
  - Node 2: 33% of its neighbors have adopted A.
  - Node 6: 50% of its neighbors have adopted A.
  - Node 7: 33% of its neighbors have adopted A.
  - Node 4 does not switch to A, as only 25% of its neighbors have adopted A.

This indicates that the threshold value for switching should lie between 25% and 33%.

- In the second step, nodes 4, 5, and 8 switch to state A:
  - Node 4: 75% of its neighbors have adopted A.
  - Node 5: 100% of its neighbors have adopted A.
  - Node 8: 50% of its neighbors have adopted A.

Thus, the suggested threshold value here is 1/3.

b)

Starting from node 3:

- In the first step, node 6 switches to A:
  - Node 6: 50% of its neighbors have adopted A.

The following nodes do not switch to A:

- Node 1: 33% of its neighbors have adopted A.
- Node 2: 33% of its neighbors have adopted A.
- Node 7: 33% of its neighbors have adopted A.
- Node 4: 25% of its neighbors have adopted A.
- In the second step, node 5 switches to A:
  - Node 5: 50% of its neighbors have adopted A.

No other nodes switch to A, suggesting a threshold value of 1/2.

c)

- In the third step, node 2 switches to A, as 66% of its neighbors have adopted A, exceeding the threshold of 0.5.
- In the fourth step, node 1 switches to A, as 66% of its neighbors have adopted A, also exceeding the threshold of 0.5.
- In the fifth step, node 4 switches to A, as 50% of its neighbors have adopted A, matching the threshold of 0.5.
- In the sixth step, nodes 7 and 8 switch to A, with 66% and 50% of their neighbors, respectively, having adopted A.

4) Q4

a)

Assuming total number of co-workers is d.

Payoff of A = d(3x+2(1-x))

Payoff of B = d(2x+5(1-x))

If A should be chosen, Payoff of A must be greater than payoff of B.

d(3x+2(1-x))> d(2x+5(1-x))

x>3/4

b)

Payoff of A = d(3x+0)

Payoff of B = d(0+5(1-x))

Payoff of A > Payoff of B

$d(3x+0) > d(0+5(1-x))$

$x>5/8$

Yes. q in (b) is smaller than q in (a), suggesting that less co-workers has to be working on system A now in b as compared to a, in order to let A be chosen in the future.