HW 8

1a) Equilibrium factors of the population purchasing the good

Given:

R(x) = 1-x

F(z)= 4z

Price p = 0.75

Reservation price for consumer x: $r(x)f(z) = (1-x)(4z)$. At equilibrium, $x = z$, so:

$$(1-z)(4z) = 0.75$$

Expand and simplify:

$$4z - 4z^2 = 0.75 \quad \Rightarrow \quad 4z^2 - 4z + 0.75 = 0$$

Divide by 4:

$$z^2 - z + 0.1875 = 0$$

Solve the quadratic equation:

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(0.1875)}}{2(1)} = \frac{1 \pm \sqrt{0.25}}{2}$$

$$z = \frac{1 \pm 0.5}{2}$$

Thus:

$$z = 0.75 \quad \text{or} \quad z = 0.25$$

Answer: $z = 0.25$ and $z = 0.75$.

b) stability of equilibria

To determine stability:

- $z = 0.25$: If z increases slightly, $f(z) = 4z > 0.75$, leading to further adoption. $z = 0.25$ is **unstable**.
- $z = 0.75$: If z decreases slightly, $f(z) < 0.75$, leading to fewer users. $z = 0.75$ is **stable**.

Answer: $z = 0.75$ is stable; $z = 0.25$ is unstable.

c)trend if z = 0.3 initially

At $z = 0.3$:

$$f(z) = 4z = 4(0.3) = 1.2 \quad \text{(greater than 0.75)}.$$

This means adoption will increase. The product becomes more popular, trending toward $z = 0.75$.

Answer: The product becomes more popular over time, trending toward $z = 0.75$.

d) trend if z = 0.5 initially

at z = 0.5:

f(z) = 4z = 4(0.5) = 2 (greater than 0.75)

adoption will continue rising toward z = 0.75

the product becomes increasingly popular , trending toward z = 0.75

2)

a) Is z = ¼ an equilibrium?

At z = ¼ , f(z) = 4z = 1:
R(z)f(z) = (1-z)f(z) = (1-1/4)(1) = ¾
Since ¾ > 7/16 , z = ¼ isn't an equilibrium because more people will adopt the product.
Thus, z = ¼ isn't an equilibrium

b) Equilibrium fractions lesser than ¼

For z < ¼ , f(z) = 4z . At equilibrium:
(1-z)(4z) = 7/16
4z – 4z^2 = 7/16 => 4z^2 – 4z +7/16 = 0
Z^2 -z + 7/64 = 0

Solve:

$$z = \frac{1 \pm \sqrt{1 - 4(1)(7/64)}}{2} = \frac{1 \pm \sqrt{0.5625}}{2}$$

$$z = \frac{1 \pm 0.75}{2}$$

Thus:

$$z = 0.875 \quad \text{(invalid as it exceeds 1)} \quad \text{or} \quad z = 0.125$$

Answer: $z = 0.125$ is the only equilibrium less than $1/4$.

c) Equilibrium fractions greater than ¼

For $z > 1/4$, $f(z) = 1$. At equilibrium:

$$(1 - z) = 7/16 \quad \Rightarrow \quad z = 9/16$$

Answer: $z = 9/16$ is the equilibrium greater than $1/4$.

D) trend if z=1/4 initially
Since z = ¼ isn't an equilibrium ; adoption will increase toward z = 9/16
Ans: adoption increases to z = 9/16
3) given budget of $50k split into x (reducing contacts) and y (reducing infection probability)

1. Current spending ($x = y = 25,000$):
   - Contact reduction: $20 - 25,000/5,000 = 15$
   - Infection probability: $0.1 - 25,000/2,500,000 = 0.09$
2. Reallocating budget entirely to x:
   - Contacts: $20 - 50,000/5,000 = 10$
   - Probability remains 0.1.
3. Reallocating budget entirely to y:
   - Probability: $0.1 - 50,000/2,500,000 = 0.08$
   - Contacts remain 20.

Compare basic reproduction number (R_0):

$$R_0 = \text{Contacts} \times \text{Infection Probability}$$

- $x = y = 25,000$: $R_0 = 15 \times 0.09 = 1.35$
- All to x: $R_0 = 10 \times 0.1 = 1.0$
- All to y: $R_0 = 20 \times 0.08 = 1.6$

Answer: Allocating all $\$50,000$ to x is the optimal strategy.

4)

Question 4:

(a) Expected outcome

- **Network 1:** Calculate q for each node:
  - Central node: $q = 1 - (1-p)^4 = 1 - (1-0.5)^4 = 0.9375$
  - Peripheral nodes: $q = 0.5$.

  Average $q = (0.9375 + 4 \times 0.5)/5 = 0.5875$.
- **Network 2:** Symmetric connections:
  - Each node: $q = 1 - (1-p)^2 = 1 - (1-0.5)^2 = 0.75$.

  Average $q = 0.75$.

Answer: Network 1: $q = 0.5875$, Network 2: $q = 0.75$.

(b) Worst-case outcome

- **Network 1:** Worst-case node is the central one, $q = 0.9375$.
- **Network 2:** All nodes have the same $q = 0.75$.

Answer: Network 1: $q = 0.9375$, Network 2: $q = 0.75$.