Info 204 Hw-7

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Q1

a)

To approach this problem, start by considering the probability of selecting from urn X when a red ball is picked. If P(X|R) exceeds 1/2, it indicates X will choose urn X.

When $p > \frac{b}{b+a}$, it is sufficient to always select X.

b)

P2 encounters a scenario similar to P1. Without any additional information, P1 makes a guess based on the calculated p. P2, who knows P1's guess but not the color of the marble, also applies this calculation and opts for X when p is sufficiently large.

c)

Given p = 0.5, a = 0.5, and b = 0.75:

If Player 1 chose Urn X, it suggests he picked a blue marble. Had he drawn a red marble, he would have guessed Urn Y, as Y has a higher probability for red marbles.

Player 2's choice of Urn Y implies picking a red marble.

If Player 3 draws a blue marble, they will choose Urn X since $\frac{8}{11} > \frac{1}{2}$.

d)

To identify the color of the marble chosen by P1:

$\frac{1}{3} < \frac{1}{2}$, suggesting X chose a blue marble.

$\frac{27}{29} > \frac{1}{2}$, meaning Y will select urn X.

Q2

a)

Starting from node 1:

- Step 1: Nodes 2 and 9 transition to A as over 50% of their neighbors adopt A, surpassing the threshold of 0.4.
- Step 2: Nodes 3 and 8 adopt A under similar conditions.
- Step 3: Nodes 4 and 7 switch to A.
- Step 4: Nodes 5 and 6 change to A.

Eventually, all nodes adopt A.

b)

Starting from node 2:

- Step 1: Node 1 adopts A as 50% of its neighbors meet the 0.4 threshold.
- Step 2: Node 9 switches to A with 67% adoption.

Nodes 1, 2, and 9 will ultimately adopt A.

c)

No. With the graph appearing symmetric, nodes like 3 and 9 illustrate this. Nodes 1, 2, 3, 4, 5, and 9 will adopt A, but adoption halts at nodes 6 and 8 as only 33% of their neighbors meet the threshold (less than 0.4).

The cluster density of 2/3 in small triangles, being greater than 1 - q, indicates no cascades will occur.

Q3

a)

Starting from node 3:

- Step 1: Nodes 1, 2, 6, and 7 transition to A.
  - Node 1: 33% neighbor adoption.
  - Node 2: 33% neighbor adoption.
  - Node 6: 50% neighbor adoption.
  - Node 7: 33% neighbor adoption.
  - Node 4 remains at 25% adoption.

The threshold lies between 25% and 33%.

- Step 2: Nodes 4, 5, and 8 adopt A.
  - Node 4: 75% adoption.
  - Node 5: 100% adoption.
  - Node 8: 50% adoption.

A threshold of 1/3 is suggested.

b)

Starting from node 3:

- Step 1: Node 6 adopts A with 50% neighbor adoption.
- Remaining nodes:
  - Node 1: 33%.
  - Node 2: 33%.
  - Node 7: 33%.
  - Node 4: 25%.

- Step 2: Node 5 adopts A at 50%.

No further nodes adopt A. A threshold of ½ is suggested.

c)

- Step 3: Node 2 transitions to A with 66% adoption.
- Step 4: Node 1 switches to A with 66% adoption.
- Step 5: Node 4 follows with 50% adoption.
- Step 6: Nodes 7 and 8 adopt A at 66% and 50%, respectively.

Q4

a)

Assume there are d total coworkers.

- Payoff for A: $d(3x + 2(1 - x))$.
- Payoff for B: $d(2x + 5(1 - x))$.

For A to be chosen: $d(3x + 2(1 - x)) > d(2x + 5(1 - x))$.

This simplifies to $x > \frac{3}{4}$.

b)

- Payoff for A: $d(3x)$.
- Payoff for B: $d(5(1-x))$.

Condition for A: $d(3x) > d(5(1-x))$.

Solving yields $x > \frac{5}{8}$.

Yes, q in (b) is lower than in (a), indicating that fewer coworkers need to adopt system A in (b) to ensure it is chosen in the future.