Peer-to-Peer Systems

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1 Introduction

The task of the project was to define DECCEN, a decentralized algorithm to compute stress centrality indices in a network based on the ideas introduced in [8], and study its behavior using the PeerSim simulator [10]. Additionally – given the high cost of the algorithm – approximation techniques could be proposed in order to reduce the communication overhead and memory requirements, and the algorithm could be extended to also compute the closeness and betweenness centrality indices.

This report defines a version of DECCEN capable of computing all the three centrality measures mentioned. Furthermore, a different decentralized algorithm is defined, based on the previous works of Brandes, and Eppstein and Wang [1, 3, 2], to approximate centrality values by sampling a limited amount of network nodes. This second algorithm, referred to as MULTI-BFS, has a significantly smaller communication overhead than DECCEN.

This document is organized as follows: in section 2 some preliminary definitions are given, while DECCEN is defined in section 3 and MULTI-BFS is defined in section 4. Experimental results are shown in section 5: the aims of the experiments were to compare the performance of the two algorithms, and to evaluate the quality of the estimates obtained with MULTI-BFS. Follow a small overview of the most relevant choices made in the development of the project code, and the proofs of some results introduced in section 4.

2 Preliminary definitions and assumptions

The task is to compute centrality indices for a given undirected graph $G = (V, E)$, which is assumed to be connected. Unless otherwise stated, n denotes the number of nodes $|V|$ and m the number of edges $|E|$. Each vertex $v \in V$ represents a network node with some given computational power, that can only communicate with its direct neighbors $N_v = \{u \in V : \{u, v\} \in E\}$. The terms *vertex* and *node* will be used interchangeably.

A *path* of *length* k from a source $s \in V$ to a destination $t \in V$ is a sequence $\langle v_0, v_1, \dots, v_k \rangle$ of vertices such that $s = v_0$, $t = v_k$ and $\{v_{i-1}, v_i\} \in E$ for $i = 1, 2, \dots, k$. The *distance* $d(u, v)$ between two vertices is the length of the shortest path that connects them (with $d(u, u) = 0$) while the diameter Δ is the maximum distance between any pair of vertices. Note that $d(u, v) = d(v, u)$ since the graph is undirected. A vertex v is a *predecessor* of w with respect to s if $\{v, w\} \in E$ and $d(s, v) + 1 = d(s, w)$. The *predecessor set* $P_s(w)$ of a vertex w is the set of all predecessors of w with respect to s.

The number of different shortest paths that connect two vertices $s, t \in V$ is denoted by σ_{st}, while the quantity $\sigma_{st}(v)$ is the number of shortest paths between s and t that pass through v (this means that if $v = s$ or $v = t$ then $\sigma_{st}(v)$ is always zero).

The centrality indices relevant to this document are the following:

Closeness centrality. The *closeness* centrality $C_C(v)$ of a vertex $v \in V$ is

$$C_C(v) = \frac{\sum_{u \in V} d(u, v)}{n - 1} \tag{1}$$

Stress centrality. The *stress* centrality $S_C(v)$ of a vertex $v \in V$ is

$$S_C(v) = \sum_{s \in V} \sum_{t \in V} \sigma_{st}(v) \tag{2}$$

Betweenness centrality. The *betweenness* centrality $B_C(v)$ of a vertex $v \in V$ is

$$B_C(v) = \sum_{s \in V} \sum_{t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{3}$$

The algorithms described in this report all assume an underlying synchronous communication model where the computation evolves in steps: at each step all the network nodes perform their computations independently and autonomously, and the messages they send at step t are delivered to the destinations and processed at step $t + 1$.

3 The DECCEN algorithm

The specification of DECCEN is based on the algorithmic scheme outlined in [8]. Initially, each node broadcasts itself on the network. Exploiting the synchronous model, after k steps any node $t \in V$ will know all the nodes $s \in V$ such that $d(s, t) = k$ and the number of shortest paths σ_{st} that links it to each of them. This information is stored locally and also reported in broadcast to allow other nodes $v \in V$ to compute the quantity $\sigma_{st}(u)$ necessary to compute the betweenness and stress centrality indices. The value $\sigma_{st}(u)$ can be determined by exploiting the following lemma.

Lemma (Bellman conditions). *A node $v \in V$ lies on a shortest path from $s \in V$ to $t \in V$ if and only if $d(s,t) = d(v,s) + d(v,t)$.*

The synchronous model ensures that if a node v lies on a shortest path between s and t and receives a report for such a pair, it has already computed σ_{vs} and σ_{vt}. Then, according to the above conditions $\sigma_{st}(v) = \sigma_{vs} \cdot \sigma_{vt}$.

3.1 Message types

DISCOVERY$\langle s, u, d, \sigma_{su} \rangle$ These messages are used to track distances and the number of shortest paths from a specific origin. They contain the source $s \in V$ of the broadcast, the sender $u \in V$, the distance $d = d(s,u)$ of u from s and the number of shortest path that connect u to s.

REPORT$\langle (s,t), \sigma_{st}, d_{st} \rangle$ These messages are broadcast by t after having determined the number of shortest paths to s and the distance from it.

3.2 Node state

Each node v maintains three accumulators C_C, C_B and C_S for closeness, betweenness and stress centrality (the closeness centrality will be retrieved as $1/C_C$), a set R of node pairs for which a REPORT message has already been received, and two dictionaries D and S that associate each node $s \in V$ with the discovered distance $d(s,v)$ and the number of shortest paths σ_{sv} respectively.

3.3 Protocol initialization

Each node $v \in V$ initializes the accumulators C_C, C_B and C_S to 0, the set R to the empty set, the dictionary D so that it only contains the entry $(v, 0)$ and S so that it only contains the entry $(v, 1)$. Furthermore, it sends to all its neighbors a DISCOVERY$\langle v, v, 0, 1 \rangle$ message.

3.4 Step actions

The processing performed by each node v at each step is the following:

1. All the DISCOVERY messages having a source s for which the dictionary D contain no mapping are grouped together. (These will be the nodes "discovered" at this step).

2. Each group of messages is processed independently. For each group, let s be the source and d be the distance of all the DISCOVERY$\langle s, u, d, \sigma_{su} \rangle$ messages in it (these will be the same for all the messages), then:

    2.1. Add the entry $(s, d+1)$ to the dictionary D, so that $D[s] = d + 1$.

2.2. Let $\sigma_{sv} = \sum_u \sigma_{su}$ and add the entry (s, σ_{sv}) to S. (The number of shortest paths from s to v is the sum of the number of shortest paths from s to all the predecessors of v).

2.3. Send a REPORT$\langle (s, v), \sigma_{sv}, d + 1 \rangle$ message to each neighbor node.

3. For each REPORT$\langle (s, t), \sigma_{st}, d_{st} \rangle$ message such that $(s, t) \notin R$:

3.1. If $s = v$ then $C_C \leftarrow C_C + d_{st}$.

3.2. If $d_{st} = D[s] + D[t]$ let $\sigma_{st}(v) = \sigma_{sv} \cdot \sigma_{tv}$, then $C_B \leftarrow \frac{\sigma_{st}(v)}{\sigma_{st}}$ and $C_S \leftarrow \sigma_{st}(v)$.

3.3. Add the pair (s, t) to the set R, and forward the REPORT message to all the neighbors.

3.5 Cost analysis

The broadcast of a DISCOVERY or a REPORT requires $O(m)$ messages. Since each nodes starts a DISCOVERY and generates $n - 1$ reports the total number of messages exchanged is $O(nm + n^2m)$, with REPORT messages inducing the dominant factor n^2m. An optimization that can be performed by a node v at step 3.3 is to avoid the propagation of a (s, t) REPORT if $\sigma_{st}(v) = 0$. In this case the report is irrelevant to v and any neighbor that may have needed it will have received it earlier from the broadcast of nodes that lie on shortest st-paths.

In terms of memory consumption, each node will add $O(n)$ entries each of the two dictionaries and $O(n^2)$ pairs to the set R.

4 Approximation of centrality indices

The main issue with DECCEN is its high cost in terms of number of messages exchanged and the requirement of a quadratic space data structure, which make its use impractical for networks of reasonable size.

Eppstein and Wang introduced in [3] an approximation algorithm for the computation of closeness centrality. Their approach is to sample the contributions of a subset of nodes to the centrality indices and extrapolate an approximate value from those contributions; the contribution of a node s to any v is taken to be the distance $d(s, v)$ and can be computed by solving a Single-Source-Shortest-Path problem with s as source. Brandes extended this scheme in [2] to the approximation of betweenness centrality: his method relies on reformulating the betweenness in terms of contributions from other nodes and employs an augmented SSSP to compute those contributions recursively, as originally proposed in [1]. This approach can be applied in a decentralized way (by adapting the SSSP problem, which in this case becomes a Breadth-First Search) and extended to the estimation of stress centrality, leading to the definition of a different decentralized algorithm for the computation (and estimation) of centrality indices.

4.1 Reformulating centrality indices

The first step is to write each centrality index of a node v as a sum of terms where each term denotes the contribution of a second node s (referred to as the *source* of the contribution) to the index value.

Contribution of a source to Closeness centrality As already mentioned, the contribution of a source node s to the closeness centrality of v is the distance $d(s,v)$. The closeness of a node is simply the reciprocal of the sum of the contributions of all the other nodes to its centrality.

Contribution of a source to Betweenness centrality The contribution of a source s to the betweenness centrality of v is the *dependency* of s on v introduced in [1]:

$$\delta(s|v) = \sum_{t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}, \tag{4}$$

that allows to rewrite the betweenness centrality of v as

$$BC(v) = \sum_{s \in V} \delta(s|v). \tag{5}$$

Contribution of a source to Stress centrality The contribution to stress centrality is analogous to the betweenness centrality:

$$\sigma(s|v) = \sum_{t \in V} \sigma_{st}(v), \tag{6}$$

and the stress centrality of v is rewritten as

$$SC(v) = \sum_{s \in V} \sigma(s|v). \tag{7}$$

4.2 Computing contributions from a single source

A decentralized Breadth First Search can be adapted to compute the contribution of a source to any of the three centrality indices considered.

The closeness centrality contribution is simply the distance from the source to the node and it can be computed directly during the visit.

For betweenness centrality, Brandes proved in [1] that dependencies of a source obey a recursive relation expressed in terms of predecessors set:

Theorem 1 (Brandes, 2001)**.** *The dependency of $s \in V$ on any $v \in V$ obeys*

$$\delta(s|v) = \sum_{w : v \in P_s(w)} \frac{\sigma_{sv}}{\sigma_{sw}} \cdot (1 + \delta(s|w)). \tag{8}$$

For stress centrality contributions a similar relation holds (the proof is reported in the appendix):

Theorem 2. *The stress centrality contribution of $s \in V$ on any $v \in V$ obeys*

$$\sigma(s|v) = \sum_{w:v \in P_s(w)} \sigma_{sv} \cdot \left(1 + \frac{\sigma(s|w)}{\sigma_{sw}}\right). \tag{9}$$

The predecessor sets can be easily discovered by the BFS algorithm during the network exploration, while contributions are computed while walking back to the source s from the frontier of the BF-Tree .

4.3 Random sampling of source nodes

To let the algorithm operate in a decentralized way, each node independently initiates a visit with a given probability p, which reflects the fraction of the network sampled. Even if the number of samples is not known beforehand, eventually all the nodes become aware of the number of sources that started a visit by taking part in each of them (let it be k).

To estimate the value of centrality indices, first the result of sampling from a source node v_i at a node u is modeled with the following random variables that rely on the contributions of v_i to the centrality values of u:

$$X_i(u) = \frac{n}{n-1} \cdot d(v_i, u), \qquad Y_i(u) = n \cdot \delta(v_i|u), \qquad Z_i(u) = n \cdot \sigma(v_i|u).$$

Then, centrality indices are approximated with the following estimators:

$$\widetilde{C}_C(u) = \sum_{i=1}^{k} \frac{X_i(u)}{k}, \qquad \widetilde{C}_B(u) = \sum_{i=1}^{k} \frac{Y_i(u)}{k}, \qquad \widetilde{C}_S(u) = \sum_{i=1}^{k} \frac{Z_i(u)}{k}.$$

The derivations for the next theorem – which ensures the estimators are unbiased – are reported in the appendix.

Theorem 3 (Unbiased estimators)**.** *The expected values of the centrality estimators are the actual centrality indices:*

(a) $\mathbf{E}[\widetilde{C}_C(u)] = \widehat{C}_C(u)$,

(b) $\mathbf{E}[\widetilde{C}_S(u)] = C_S(u)$,

(c) $\mathbf{E}[\widetilde{C}_B(u)] = C_B(u)$.

4.4 MULTI-BFS algorithm specification

The only parameter of the algorithm is the probability p of a node becoming the source of a decentralized BFS and having its centrality contributions being sampled at every location. The network size n is assumed to be known in advance to all the nodes, otherwise it can be easily computed during the backtracking phase of the augmented BFS and broadcast by one or more sources.

Note that if this algorithm is executed with $p = 1$ it is basically the decentralized version of the algorithm described in [1], and at the end of the execution the estimates are the exact centrality values.

4.4.1 Message types

DISCOVERY$\langle s, u, d, \sigma_{su} \rangle$ Messages of this type are used during the descent from a source to build the predecessors sets at each node. Relevant fields are the source s of the visit, the sender u, the distance d of the sender to the source (that is, $d = d(s, u)$) and the number of shortest path from the source to the sender σ_{su}.

REPORT$\langle s, v, \delta(s|v), \sigma(s|v), \sigma_{sv} \rangle$ These messages are sent by a node v as part of the backtracking phase to inform its predecessors of the computed contributions and to allow them to compute their own by applying the recursive relations introduced in section 4.2.

4.4.2 Visit states

Visit states are parametric with respect to the discovery from a source $s \in V$. A node v is in state:

WAITING(s) if it has not yet received any DISCOVERY having s as source.

ACTIVE(s) if it has received one or more DISCOVERY messages with source s and has not yet computed the contributions of s to its centrality indices.

COMPLETED(s) if it has computed the contributions of s to its centrality indices, updated them accordingly, and reported the contributions to each predecessor in $P_s(v)$ by sending the appropriate REPORT messages.

4.4.3 Node state

Each node v maintains three centrality accumulators C_C, C_B and C_S like in DECCEN, and a counter k to track the number of sample nodes involved in the protocol. Furthermore, while a node is in state ACTIVE(s) when dealing with a visit from s it will need to partition the set of neighbors N_v in three subsets: the set of predecessors $P_s(v)$, the set of siblings $S_s(v)$ and the set of children $C_s(v)$, as well as track contributions of s to its centrality scores with three parametric accumulators $C_C^{(s)}$, $C_B^{(s)}$ and $C_S^{(s)}$.

4.4.4 Protocol initialization

Upon initialization, a node v clears its centrality accumulators and the counter k, and enters state WAITING(s) for all $s \in V$. Then, with probability p initiates a visit by entering state ACTIVE(v), sending a DISCOVERY$\langle v, v, 0, 1 \rangle$ to every neighbor and letting $P_v(v) = \emptyset$.

4.4.5 Step actions

The actions performed by a node v at each step are the following:

1. The messages received at the current step are divided by type and then DISCOVERY messages are grouped by source.
2. For each group of DISCOVERY$\langle s, u, d, \sigma_{su}\rangle$ messages having s as a common source (at distance d), with v in state WAITING(s):
   - 2.1. Change state to ACTIVE(s) and initialize $C_C^{(s)} \leftarrow d+1$, $C_B^{(s)} \leftarrow 0$ and $C_S^{(s)} \leftarrow 0$; then collect all the senders u in the predecessor set $P_s(v)$ and compute $\sigma_{sv} = \sum_{u \in P_s(v)} \sigma_{su}$.
   - 2.2. If $P_s(v) = N_v$ send a REPORT$\langle s, v, 0, 0, \sigma_{sv}\rangle$ message to all the predecessors $u \in P_s(v)$ and change state to COMPLETED(s).
   - 2.3. Otherwise, send to all $w \in N_v \setminus P_s(v)$ a DISCOVERY$\langle s, v, d+1, \sigma_{sv}\rangle$ message and change state to ACTIVE(s).
3. For each group of DISCOVERY$\langle s, u, d, \sigma_{su}\rangle$ messages having source s and distance d with v in state ACTIVE(s):
   - 3.1. Under the synchronous model assumption v can only receive such messages from nodes u such that $d(s,u) = d(s,v)$. Collect these nodes in the set $S_s(v)$ of the siblings of v with respect to s. (Note that these messages are received exactly one step after v has been contacted by its predecessors).
4. For each REPORT$\langle s, w, \delta(s|w), \sigma(s|w), \sigma_{sw}\rangle$ message received with v in state ACTIVE(s):
   - 4.1. Add w to the children set $C_s(v)$.
   - 4.2. Update $C_B^{(s)} \leftarrow C_B^{(s)} + \frac{\sigma_{sv}}{\sigma_{sw}} \cdot (1 + \delta(s|w))$.
   - 4.3. Update $C_S^{(s)} \leftarrow C_S^{(s)} + \sigma_{sw} \cdot (1 + \frac{\sigma(s|w)}{\sigma_{sw}})$.
5. For any $s \in V$ with v is in state ACTIVE(s), if $P_s(v) \cup S_s(v) \cup C_s(v) = N_v$ then:
   - 5.1. If $s \neq v$ update the accumulators:
     - $C_C \leftarrow C_C + C_C^{(s)}$
     - $C_B \leftarrow C_B + C_B^{(s)}$
     - $C_S \leftarrow C_S + C_S^{(s)}$

     and send a REPORT$\langle s, v, \delta(s|v), \sigma(s|v), \sigma_{sv}\rangle$ message to all the predecessors contained in $P_s(v)$.
   - 5.2. Increment the counter k and change state to COMPLETED(s).

A node v can obtain the value of the estimators in the following way:

$$\widetilde{C}_C(v) = \frac{nC_C}{k(n-1)}, \qquad \widetilde{C}_B(v) = \frac{nC_B}{k}, \qquad \widetilde{C}_S(v) = \frac{nC_S}{k}.$$

4.5 Cost analysis

Each independent search requires $O(m)$ messages for the DISCOVERY phase and $O(m)$ messages to backtrack with REPORT messages. The parameter p determines the fraction of nodes pn that initiate a DISCOVERY search, so the total number of messages is $O(\lceil pn \rceil m)$.

In terms of memory consumption, note that for each $v \in V$ there are at most $\lceil pn \rceil$ other nodes s for which v is in state ACTIVE(s) and needs to partition its neighbor set N_v in the three subsets $P_s(v)$, $S_s(v)$ and $C_s(v)$, so the cost is $O(\lceil pn \rceil N_v)$.

5 Experiments

Simulations were performed to evaluate the performances of the algorithms, and to assess the quality of the estimations obtained by running MULTI-BFS with different values of the parameter p.

The networks used in the experiments (taken from the KONECT repository [7]) are:

`dolphins` This is a social network of bottlenose dolphins. The nodes are the bottlenose dolphins (genus *Tursiops*) of a bottlenose dolphin community living off Doubtful Sound, a fjord in New Zealand. An edge indicates a frequent association. The dolphins were observed between 1994 and 2001. ($n = 62$, $m = 159$, source [9]).

`surf` This network contains interpersonal contacts between windsurfers in southern California during the fall of 1986. ($n = 62$, $m = 336$, source [4]).

`macaques` This directed network contains dominance behaviour in a colony of 62 adult female Japanese macaques. An undirected version of the network (where edges are made symmetric) was used to run the experiments. ($n = 62$, $m = 1167$, source [11]).

`train` This network contains contacts between suspected terrorists involved in the train bombing of Madrid on March 11, 2004 as reconstructed from newspapers. ($n = 62$, $m = 243$, source [6]).

`email` This is the email communication network at the University Rovira i Virgili in Tarragona in the south of Catalonia in Spain. Edges represent contacts between users. ($n = 1133$, $m = 5451$, source [5]).

				DECCEN		MULTI-BFS	
Network	n	m	Δ	Steps	Messages	Steps	Messages
surf	43	336	3	6	181374	7	28896
dolphins	62	159	8	16	140051	17	19716
macaques	62	1167	2	4	1458368	5	144708
train	64	243	6	12	272673	13	31104

Table 1: Steps required to complete and number of exchanged messages by DECCEN and MULTI-BFS executed with $p = 1$.

powergrid This network is the high-voltage power grid in the Western States of the United States of America. The nodes are transformers, substations, and generators, and the ties are high-voltage transmission lines. ($n = 4941$, $m = 6594$, source [12]).

5.1 Performance analysis

The performance metrics used to evaluate the algorithms were the number of steps required to complete the computation and the number of messages that the agents needed to generate to do so. Note that due to the high memory requirements of DECCEN, these simulations were performed on small networks.

Table 1 reports the results obtained by running DECCEN and MULTI-BFS with $p = 1$ in order to compute the exact centrality values. As expected, in both cases the number of steps required to complete is exactly twice the diameter Δ of the networks, since each "discovery" phase takes at most Δ steps to reach any destination from a given source, and the same also holds for the report phase of DECCEN or the backtracking in MULTI-BFS. However, the number of messages required by DECCEN is significantly larger. This follows from the different way in which the report phase evolves in the two algorithms: while in DECCEN any node must generate a report for each source and broadcast it to every other node in the network, in MULTI-BFS reports are routed back to the source by signaling only the predecessors at each step.

5.2 MULTI-BFS approximations

The quality of the estimations yielded by MULTI-BFS was evaluated both in terms of the numerical error introduced by the estimators, and the difference in the ranking of the nodes induced by the centrality values which could prove to be accurate even in presence of non-negligible error in the estimates.

Figure 1 reports the average relative error ϵ_r yielded by the centrality estimators for increasing values of the parameter p in the dolphins, email and powergrid networks. The estimation of Closeness centrality is much more accurate than the estimation of Stress and Betweenness centrality, which behave roughly the same.

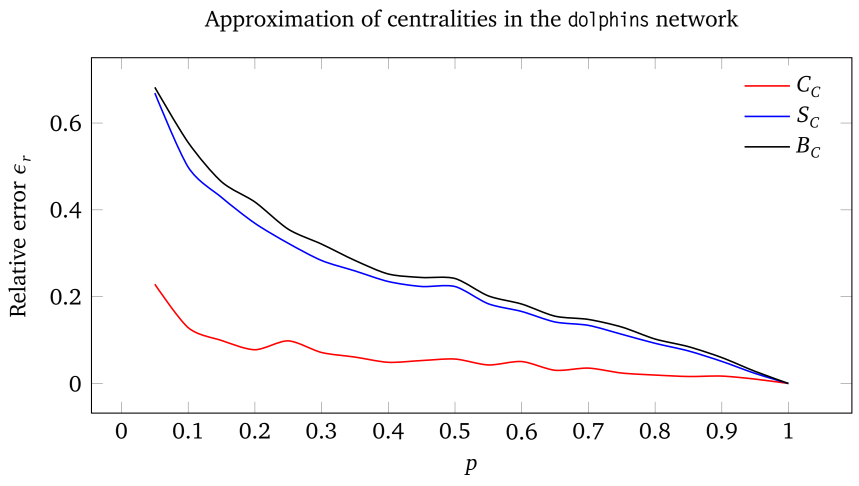
The accuracy of the ranking in presence of approximated indices is measured by counting among all the pair of nodes, the fraction of pairs in which the nodes are wrongly ordered with respect to the ranking induced by the exact centrality values. Results are reported in figure 2. In this case, even for small values of p the fraction of pairs of nodes in wrong rank order is relatively small. This is encouraging if the estimated indices are used locally, to make decisions based on the values computed at a node and at its neighbors.

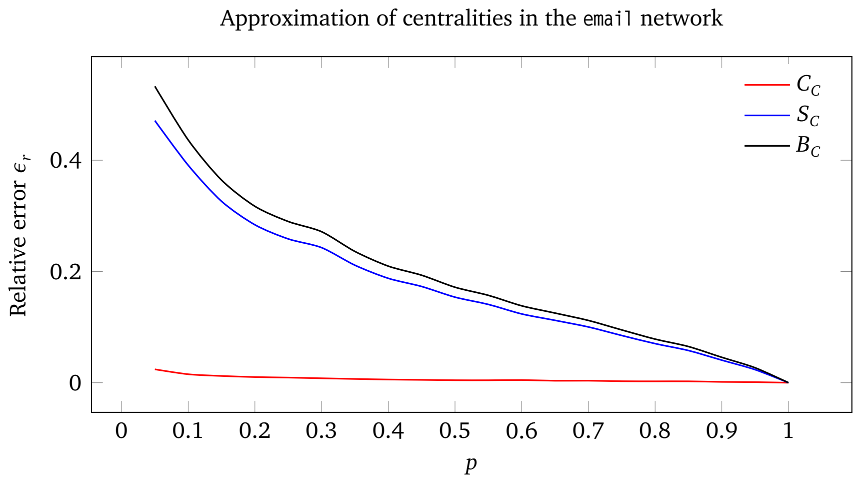
6 Project code overview

Simulations are based on the *cycle-driven* engine of PeerSim. This seems a natural choice given the synchronous communication model assumption, but the way in which the engine handles the execution of the protocol code required some adjustments. When dealing with cycle-driven protocols, PeerSim executes each simulation cycle sequentially by iterating through the `Node` instances that form the network and invoking the `nextCycle` method that the protocol implements. If precautions are not taken, this execution policy clashes with the synchronous model assumption: if a node n has two neighbors that at the current step need to interact with it (for example to communicate the number of shortest paths) and one of them is executed after n in the iteration order, the information will be available to n only at the following cycle.

Rather than adapting the protocol code to be mindful of this potential situation, the choice was to include the synchronous communication model in the simulation. A synchronous protocol is a `CDProtocol` that can send messages to other synchronous protocols with the guarantee that those will be received after the `nextCycle` method will be invoked by the simulator on the receiver at the current step, but before the invocation at the following step. This allows to execute each simulation cycle at each node in isolation, only using information generated in the previous cycle exactly as the synchronous communication model assumption requires. This is achieved by implementing the `CycleBasedTransportSupport` interface:

```
public interface CycleBasedTransportSupport<T> {
    interface SendQueueEntry<T> {
        T getMessage();
        CycleBasedTransportSupport<T> getDestination();
    }
    void addToSendQueue(T message, CycleBasedTransportSupport<T> destination);
    boolean hasOutgoingMessages();
    void addToIncoming(T message);
    Iterator<T> getIncomingMessagesIterator();
```





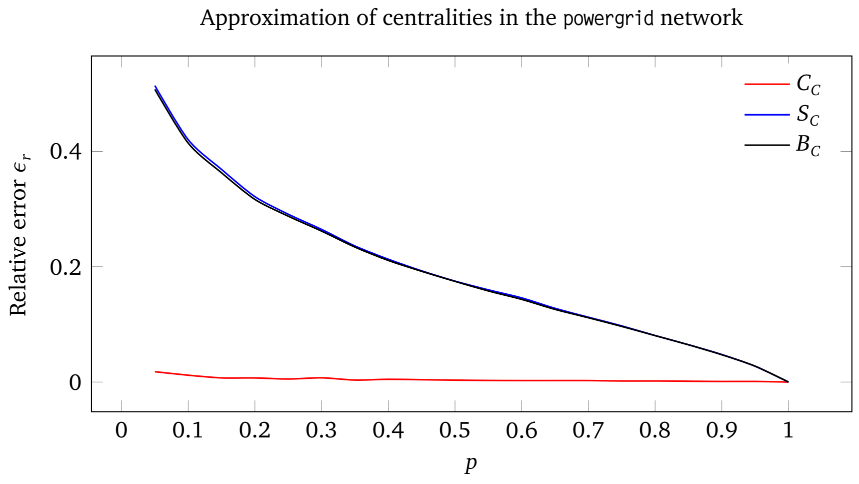


Figure 1: Approximation error in the estimation of centrality indices.

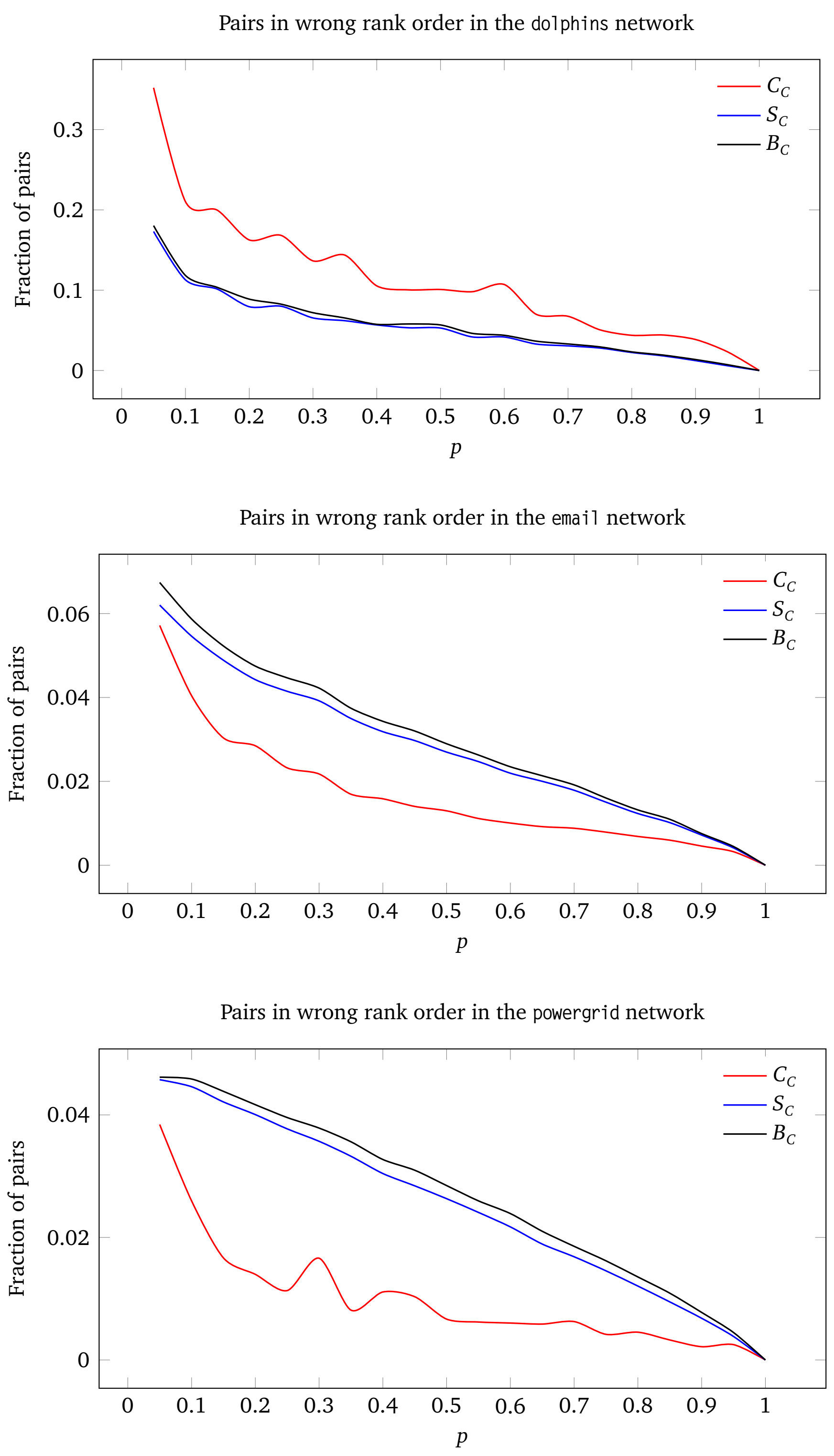


Figure 2: Pairs in wrong rank order with approximated centralities

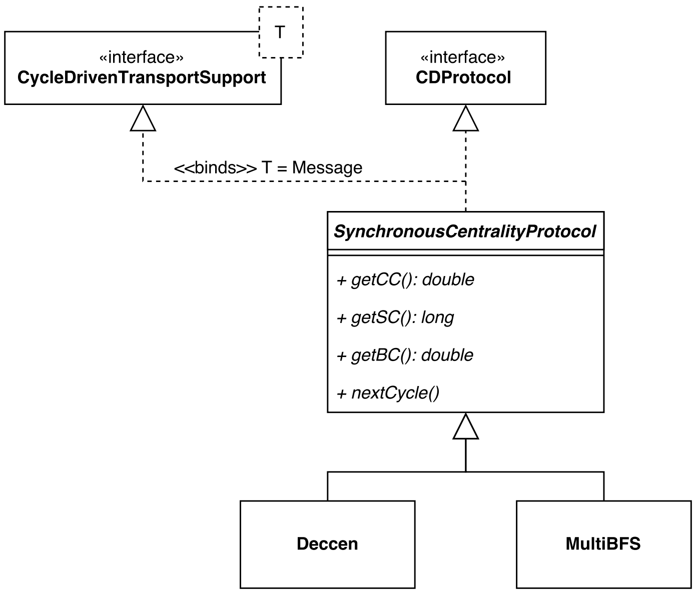


Figure 3: Class diagram of the implemented protocols.

```
    Iterator<SendQueueEntry<T>> getSendQueueIterator();
}
```

This generic interface offers facilities to synchronously transfer objects between protocols that implement it. The `addToSendQueue` method should temporarily store the message and the destination in a send queue, which is then iterated by a `CycleBasedTransport` control at the end of a cycle in order to deliver each message to the appropriate destination. A protocol can then call the `getIncomingMessageIterator` method in the next cycle and iterate through the received messages.

6.1 Messages

The messages exchanged by the protocols during the simulation are instances of the `Message` class. This class provides factory methods to generate DISCOVERY and REPORT message objects that follow the conventions used in the definition of the algorithms.

6.2 Protocol classes

Protocol classes are organized in the hierarchy shown in figure 3. The abstract class `SynchronousCentralityProtocol` implements the `CycleBasedTransportSupport<Message>` interface, while the `nextCycle` method of the PeerSim interface `CDProtocol` is declared `abstract`.

6.3 DECCEN protocol implementation

The Deccen class implements a DECCEN node in the simulator. A node state consists of the shortest path information it collects and of the reports it has handled during the execution:

```
public class Deccen extends SynchronousCentralityProtocol {
    private static class ShortestPathData {
        public final int count;
        public final int length;

        public ShortestPathData(int count, int length) {
            this.count = count;
            this.length = length;
        }
    }

    private static class OrderedPair<T1,T2> {
        public final T1 first;
        public final T2 second;

        public OrderedPair(T1 first, T2 second) {
            this.first = first;
            this.second = second;
        }
        ...
    }
    ...
    private Map<Node,ShortestPathData> shortestPathMap;
    private Set<OrderedPair<Long,Long>> handledReports;
    ...
    public void nextCycle(Node self, int protocolID) { ... }
    ...
}
```

Whenever new DISCOVERY messages are received, information about the number and length of shortest paths toward a particular source become available: this information is stored in the shortestPathMap structure by adding a new ShortestPathData entry paired with the appropriate source. Since the algorithm computes all the centrality indices, it stores both the distance from the source and the number of different paths counted. Reports about a source–destination pair (s, t) are tracked by storing the handledReports set an OrderedPair of Node IDs (which are long integers in PeerSim).

The implementation of nextCycle simply executes the actions described in section 3.4:

```
public void nextCycle(Node self, int protocolID) {
    Map<Node,List<Message>> discoveryMap = new HashMap<Node,List<Message>>();
    List<Message> reportList = new LinkedList<Message>();
    parseIncomingMessages(discoveryMap, reportList);
    if (!discoveryMap.isEmpty())
        processDiscoveryMessages(self, protocolID, discoveryMap);
    if (!reportList.isEmpty())
        processReportMessages(self, protocolID, reportList);
}
```

6.4 MULTI-BFS protocol implementation

The state of a MULTI-BFS node consists of a `Map` to keep track of the various visits that are performed during the algorithm execution and that encounter the node itself:

```
public class MultiBFS extends SynchronousCentralityProtocol {
    private static class VisitState {
        ...
    }
    ...
    private Map<Node, VisitState> activeVisits;
    private Set<Node> completed;
    ...
    public void nextCycle(Node self, int protocolID) { ... }
    ...
    public boolean isWaiting(Node source) { ... }
    public boolean isActive(Node source) { ... }
    public boolean isCompleted(Node source) { ... }
}
```

Recall that the state of a node is parametric with respect to each source of a visit. The state is WAITING(s) if the `Node` instance `s` does not appear as key in the `activeVisits` map and neither is contained in the `completed` set; this is the initial state for any source since both structures are empty at the start of the protocol. When a node is in state ACTIVE(s) an entry with key `s` is present in the `activeVisits` map. After a node has reported back to all the predecessors, it changes state COMPLETED(s) by removing the mapping from `activeVisits` and inserting `s` in the `completed` set.

Entries in the `activeVisits` map are used to store data while a node is in the "active" phase. A `VisitState` object is used to keep track of the predecessors and children sets, and to incrementally compute the values to be reported to the predecessor nodes:

```
private static class VisitState {
    public Set<Node> predecessors;
    public Set<Node> siblings;
    public Set<Node> children;
    public int distanceFromSource;
    public int timestamp;
    public int sigma;
    public long contributionSC;
    public double contributionBC;
    ...
    public void accumulate(Node child, double bcc, long scc, int numSP) {
        ...
    }
}
```

The `accumulate` method updates the local contributions of the source by applying the recursive relations (8) and (9), and is invoked whenever a child node reports back.

Finally, the `nextCycle` method implementation which again follows the specification given in section 4.4.5:

```
public void nextCycle(Node self, int protocolID) {
    Map<Node,List<Message>> discoveryMap = new HashMap<Node,List<Message>>();
    List<Message> reportList = new LinkedList<Message>();
    parseIncomingMessages(discoveryMap, reportList);
    if (!discoveryMap.isEmpty())
        processDiscoveryMessages(self, protocolID, discoveryMap);
    if (!reportList.isEmpty())
        processReportMessages(reportList);
    reportNewCompleted(self, protocolID);
}
```

Messages are parsed and handled by type, then active visits for which all child nodes have reported are finalized with the `reportNewCompleted` method by integrating the contributions in the centrality accumulators and reporting the relevant data to each predecessor.

Appendix: Theorem proofs

Theorem 2. *The stress centrality contribution of $s \in V$ on any $v \in V$ obeys*

$$\sigma(s|v) = \sum_{w:v\in P_s(w)} \sigma_{sv} \cdot \left(1 + \frac{\sigma(s|w)}{\sigma_{sw}}\right). \tag{9}$$

Proof. The proof is analogous to the one provided by Brandes in [1] to prove Theorem 1.

Let $\sigma_{st}(v,e)$ be the number of shortest paths between s and t that pass through v and across edge e. Observe that if a shortest path from s to t travels through v, then after v it must immediately reach some other node w that has v in its predecessor set $P_s(w)$, so equation (6) can be rewritten as

$$\begin{aligned}\sigma(s|v) &= \sum_{t\in V}\sigma_{st}(v) = \sum_{t\in V}\sum_{w:v\in P_s(w)}\sigma_{st}(v,\{v,w\})\\ &= \sum_{w:v\in P_s(w)}\sum_{t\in V}\sigma_{st}(v,\{v,w\}).\end{aligned}$$

Let w be any node with $v\in P_s(w)$, then

$$\sigma_{st}(v,\{v,w\}) = \begin{cases}\sigma_{sv} & \text{if } t=w\\ \frac{\sigma_{sv}}{\sigma_{sw}}\cdot\sigma_{st}(w) & \text{if } t\neq w\end{cases}$$

and substituting it in the previous expression yields

$$\begin{aligned}\sigma(s|v) &= \sum_{w:v\in P_s(w)}\sum_{t\in V}\sigma_{st}(v,\{v,w\})\\ &= \sum_{w:v\in P_s(w)}\left(\sigma_{sv}+\sum_{t\neq w}\frac{\sigma_{sv}}{\sigma_{sw}}\cdot\sigma_{st}(w)\right)\\ &= \sum_{w:v\in P_s(w)}\sigma_{sv}\cdot\left(1+\frac{\sigma(s|w)}{\sigma_{sw}}\right).\end{aligned}$$ □

Theorem 3 (Unbiased estimators)**.** *The expected values of the centrality estimators are the actual centrality indices:*

(a) $\mathbf{E}[\widetilde{C}_C(u)] = C_C(u)$,

(b) $\mathbf{E}[\widetilde{C}_S(u)] = C_S(u)$,

(c) $\mathbf{E}[\widetilde{C}_B(u)] = C_B(u)$.

Proof. (a) Recall that for the estimation of closeness centrality, the result of sampling from a source v_i at a node u was modeled with the random variable $X_i(u) = \frac{n}{n-1}\cdot d(v_i,u)$. The derivation exploits the linearity of the expected value operator $\mathbf{E}$ and the fact that source nodes are random (that is, each node has equal probability $1/n$ of being a source).

$$\begin{aligned}
\mathbf{E}[\widetilde{C}_C(u)] &= \mathbf{E}\left[\sum_{i=1}^{k}\frac{X_i(u)}{k}\right] = \mathbf{E}\left[\sum_{i=1}^{k}\frac{n\cdot d(v_i,u)}{k(n-1)}\right] \\
&= \frac{n}{k(n-1)}\sum_{i=1}^{k}\mathbf{E}\left[d(v_i,u)\right] && \text{(by linearity of expectation)} \\
&= \frac{n}{k(n-1)}\cdot k\cdot\frac{1}{n}\sum_{v\in V}d(v,u) && \text{(random source selection)} \\
&= \frac{\sum_{v\in V}d(v,u)}{n-1} = C_C(u)
\end{aligned}$$

(b) To estimate stress centrality, the result of sampling from v_i is modeled at u with the random variable $Y_i(u) = n\cdot\sigma(v_i|u)$. The derivation relies again on the linearity of **E** and the uniform distribution of source nodes.

$$\begin{aligned}
\mathbf{E}[\widetilde{C}_S(u)] &= \mathbf{E}\left[\sum_{i=1}^{k}\frac{Y_i(u)}{k}\right] = \mathbf{E}\left[\sum_{i=1}^{k}\frac{n\cdot\sigma(v_i|u)}{k}\right] \\
&= \frac{n}{k}\sum_{i=1}^{k}\mathbf{E}\left[\sigma(v_i|u)\right] = \frac{n}{k}\cdot k\cdot\frac{1}{n}\sum_{v\in V}\sigma(v|u) = C_S(u)
\end{aligned}$$

(c) The proof is the same as (b) with the substitution of $Z_i(u) = n\cdot\delta(v_i|u)$ for $Y_i(u)$. $\square$

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