The physics of ski jumping

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1 Introduction

The Olympian ideal of going faster, jumping further and leaping higher than the opposition is central to competitive sports. Winning or losing in sport is related to a number of factors, and biomechanics, anthropometrics, and aerodynamics play a major role in many sports. This lecture focuses on ski jumping.

Performance in ski jumping is determined not only by the motor abilities of the athlete, but also to a large extent by the aerodynamic features of the equipment used and by a low body weight. Many ski jumpers were extremely underweight to the point of having a body mass index (BMI = m/h^2 of 16.4 kgm^{-2} (height h = 1.73m, body mass m = 49 kg).

Severe eating disorders (e.g., anorexia nervosa, bulimia; [1]) were health problems of major concern in this sport. Strategies for improving the health, fairness, and safety of the athletes by modifying the regulations have been developed by the lecturer and his research team in close co-operation with the International Ski Federation (FIS), the International Olympic Committee (IOC), and the Austrian Research Funds (FWF). Based on our scientific studies the FIS has passed changes to the ski jumping regulations which relate relative body weight (in terms of BMI) to the maximum ski length permitted. Shorter skis (i.e., 'smaller wings') compensate for the advantage of very low weight and thus it is not attractive for the athletes to be underweight any more [2].

Our analyses of contemporary ski jumping employ field studies during World Cup and Olympic Games competitions, wind tunnel measurements, computational fluid dynamic (CFD) modelling of aerodynamic forces and torques, computer simulations of the flight trajectory, and computer-modelling-based design of jumping hills [3–7].

2 The dynamics of ski jumping: a brief description

Ski jumping puts high demands on the athlete's ability to control posture and movement. During the in-run the athlete tries to maximize acceleration by minimizing both the friction between skis and snow and the aerodynamic drag in order to obtain a maximum in-run speed v_0, which has a high degree of influence on the jump length. The friction between skis and snow is not well understood. The physics text book solutions to this problem do not reflect reality. The theoretical as well as the empirical basis for these complex problems are not sufficiently developed. The reduction of aerodynamic drag in the in-run phase is primarily a question of the athlete's posture and his dress. Owing to the curved form of the in-run just before the ramp, the athlete has to counteract the centrifugal force acting on him (as seen from the athlete's point of view) and this phase is immediately followed by the athlete's acceleration perpendicular to the ramp due to the muscular forces exerted. During this decisive phase of approximately 0.3 s duration the athlete has to produce a maximal momentum mv_{p0} perpendicular to the ramp (m: mass of the athlete plus equipment) through which an advantageous take-off angle has to be attained. The take-off velocity vector $\vec{v_{00}}$ is given by $\vec{v}_{00} = \vec{v}_0 + \vec{v}_{\mathrm{p0}}$ with $\vec{v}_{\mathrm{p0}}$ being the velocity perpendicular to the ramp due to the athlete's jump. Simultaneously, the athlete must produce an angular momentum forwards in order to obtain an advantageous angle of attack as soon as possible after leaving the ramp. During the jump phase the athlete must anticipate the magnitude of the backward torque due to the air-stream so that the forward rotation will be stopped at the right moment. If the forward angular momentum is too low, a disadvantageous flight position reduces velocity and, therefore, results in bad competitive performance. Worse is the production of too much forward angular momentum because this substantially increases the tumbling risk. During the flight the gravitational force F_{g}, the lift force F_{l}, and the drag force F_{d} act upon the athlete:

$$F_{\mathrm{g}} = mg\,; \quad F_{\mathrm{l}} = \frac{\rho}{2}v^2 c_{\mathrm{l}} A = \frac{\rho}{2}v^2 L\,; \quad F_{\mathrm{d}} = \frac{\rho}{2}v^2 c_{\mathrm{d}} A = \frac{\rho}{2}v^2 D\,.$$

The velocity of motion along the flight path v has the components $\dot{x}$ and $\dot{y}$

$$v^2 = \dot{x}^2 + \dot{y}^2\ .$$

The athlete can strongly influence the aerodynamic forces by changing his posture. He can affect the drag force, the lift force and the torque, and thereby significantly influence changes in his flight position relative to the air stream. The flight path is described by the following non-linear differential equations which can be solved numerically by using proper iterative procedures:

$$\dot{v}_x = \frac{(-F_{\mathrm{d}}\cos\varphi - f_{\mathrm{l}}\sin\varphi)}{m} \qquad \dot{v}_y = \frac{(-F_{\mathrm{d}}\sin\varphi - f_{\mathrm{l}}\cos\varphi)}{m} - g\ .$$

$$\dot{x} = v_x \qquad \dot{y} = v_y$$

In order to achieve highly realistic computer simulations, it is necessary to be able to simulate changes in posture and position during the simulated flight, i.e., changes in the resulting aerodynamic forces. We developed such a computer model of ski jumping in 1995 [3, 4].

2.1 Basic aerodynamic problems

Aerodynamic questions related to sports are complex and manifold. For this reason the influencing phenomena should be investigated by both theoretical and experimental approaches.

2.1.1 Theoretical approach

The Navier–Stokes equations which describe the dynamics of Newtonian fluids have inherent major mathematical difficulties. Exact solutions are only possible in special cases involving objects with simple geometries that do not exist in sports. The numerical solution becomes increasingly difficult as Reynolds numbers increase, even when supercomputers are used for the numerical solution of these non-linear partial differential equations. Owing to the non-linearity of the equations, a variation of the geometrical and fluid mechanical parameters can result in bifurcations and the non-linear fluid system can display deterministic chaos. Computational fluid dynamics is proving to be invaluable at the early stage of trend analysis prior to prototype testing in several kinds of sports (e.g., Formula 1, yachting). Initial studies of ski jumping have also been made. However, the thin boundary layers around moving bodies have to be resolved accurately and the associated physical effects are notoriously difficult to predict accurately in CFD. A combination of measurement and CFD is the state-of-the-art approach necessity when appropriate predictions for sports aerodynamics are desired. The best configurations found in wind tunnel tests still have to be tested in the field by the athletes before their competition debut. In ski jumping (and many other sports as well) the characteristic dimensions of the body and/or equipment and the typical velocities result in Reynolds numbers between Re = 10^4 and Re = 10^6 where pronounced changes in the drag coefficient may occur: $c_{\mathrm{d}} = c_{\mathrm{d}}(\mathrm{Re})$. This was already shown for the sphere in 1912 by G. Eiffel [8] and in 1914 by C. Wieselsberger [9]. The sensitivity of the transition from laminar to turbulent flow on the roughness of the surface (or on small surface obstacles) was also shown. The theoretical approach describing the lift forces is at least as difficult as the discussion of the drag, and science still does not have a complete understanding when turbulence phenomena occur. Micro effects can be the starting point for major flow changes in a system and to discount them in modern turbulence models or even to ignore them leads to completely inaccurate predictions of the whole process.

2.1.2 Experimental approach

Small changes in the lift and/or drag coefficients can have pronounced consequences for the sport concerned. Therefore, accurate measurements in the wind tunnel are necessary when these data are to be used for subsequent mapping of 'real world' problems. A wind tunnel with a sufficiently large cross-section is necessary to minimize blockage effects. Because the athletes must be studied in various postures in full gear, adequate positioning devices are required. However, these devices can cause secondary errors even when the forces acting solely on the devices are considered. Positioning devices that lead to small disruptions in the air flow around the athletes need to be designed. The experimental problems are smaller for those questions where only relative changes in the aerodynamical parameters are to be considered. However, the design of experiments reflecting the special circumstances of different sports is not trivial at all. A reliable interpretation of the effects associated with different aerodynamical characteristics usually necessitates a computer-based analysis of the experimental data. So, for instance, a given increase of the drag area $D = c_{\mathrm{d}}A$ during the initial flight phase reduces the jump length much more than would be the case during the final phase of the flight. Analogously, a change in the drag coefficient of the athlete in the crouched position may occur during the in-run phase due to a Reynolds-number increase beyond the critical value (e.g., when the velocity increases). These phenomena may strongly depend on individual body dimensions of the athletes. Therefore, a computer-assisted analysis of the resulting effects based on experimental aerodynamic data is the only way of appropriate treatment (based on the equations of motion for the in-run phase). For all kinds of sports where a minimal aerodynamic drag is important, the factors influencing the shift of the drop in the drag coefficient within the critical Reynolds-number range are most important. These factors have not been sufficiently understood in the context of sports involving complex objects like human bodies. The surface structure (dress, jumping suit, downhill suit, etc.), temperature effects, vibration of surfaces like human skin, tension of the surface material etc. are all influential factors. Considering the significant influence of, for example, the geometry of a sphere versus an aerodynamically well-shaped object on the lift and drag coefficients (the difference can be a factor of 100; see, for example, H. Schlichting and K. Gersten [10]) the predominant importance of improvements in this direction is evident.

Very little is known about the drag and lift forces acting on the complex structure of a human body in an air-stream. From the aerodynamic data according to flight positions it is evident that the lift forces in ski jumping are of the same magnitude as the drag forces and that the flight length is very sensitive to changes in both. It is well known that pronounced changes of the lift coefficients of wings occur in the critical range of the Reynolds number, and it has been found from measurements in low-turbulence wind tunnels that the degree of turbulence of the outer flow has an important influence on the lift coefficient of a wing. Yet, no systematic study of these characteristics in relation to human bodies in the air-stream has been made. Our knowledge of wings cannot be adequately transferred to the flow around human bodies because the geometric form is not at all similar and wings usually do not work at angles of attack of up to 50 degrees.

Acknowledgements

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References

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Figures

Examples of experimental results

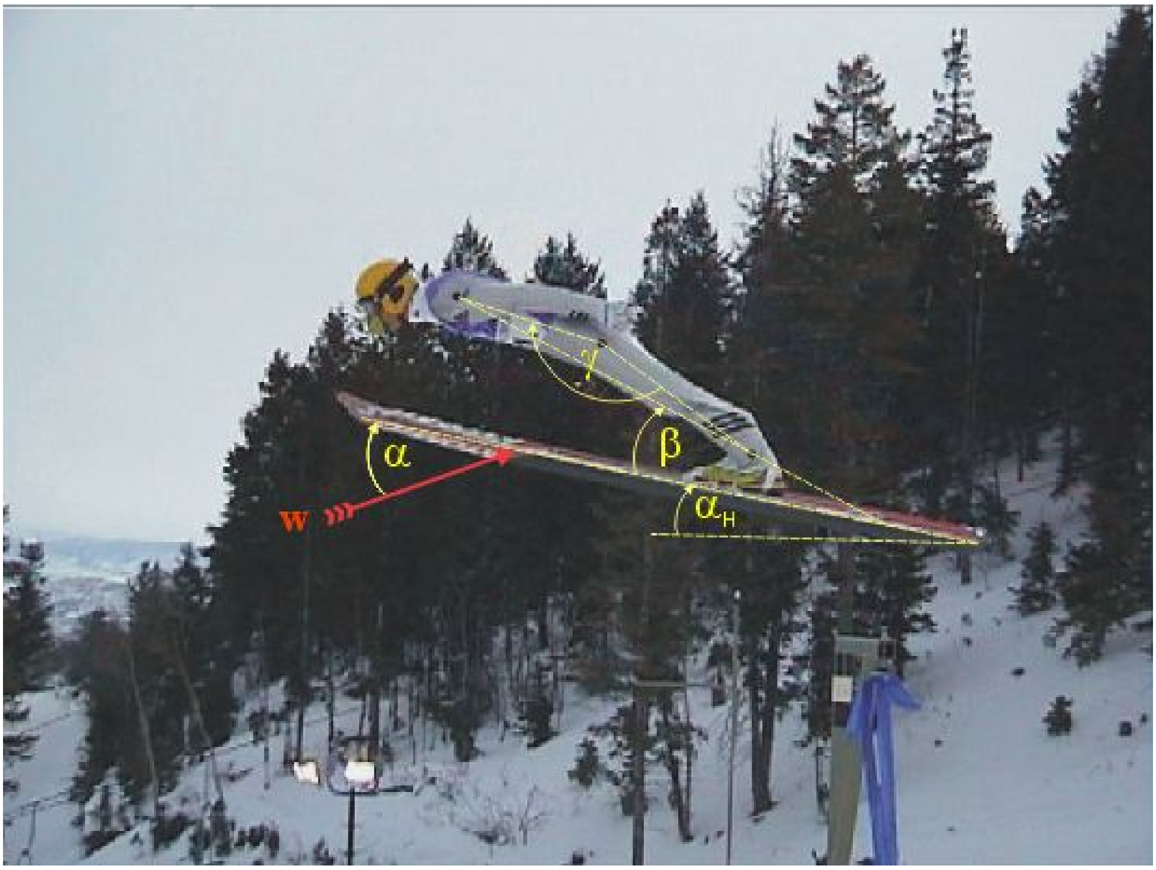


Fig. 1: A ski jumper at the $K = 120$ m jumping hill in Park City. The nomenclature used for the position angles is indicated, w is the resulting air stream vector.

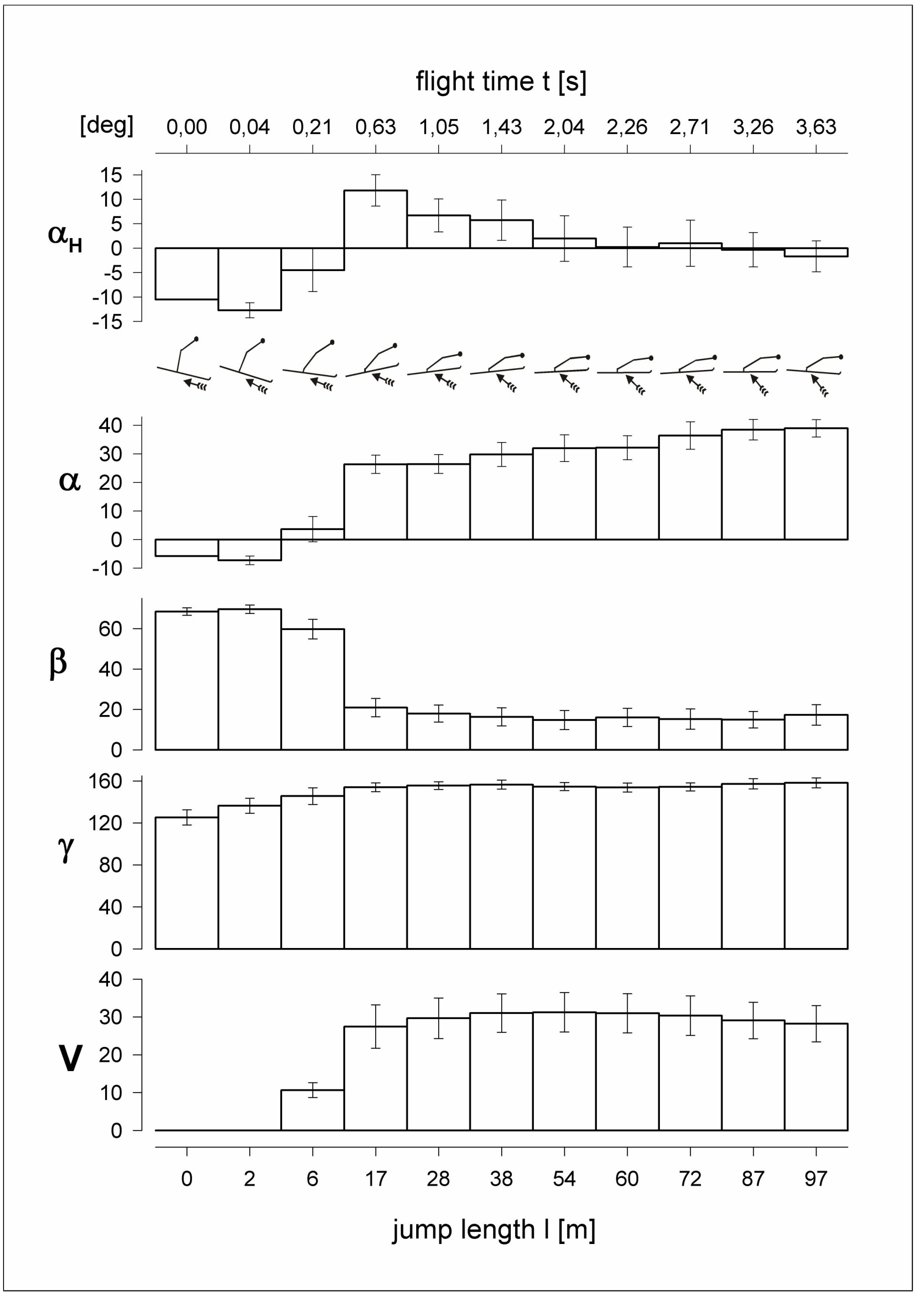


Fig. 2: Field research results obtained during the 19th Olympic Winter Games (Salt Lake City, 2002; venue in Park City). The histograms show the average values and standard deviations of position angles from the best ten athletes in each of the five runs at the K = 120 m jumping hill. The number of angle measurements ranged from 18 to 50 at each position. The angle V of the skis to each other was determined from digitized images taken from the end of the run out.

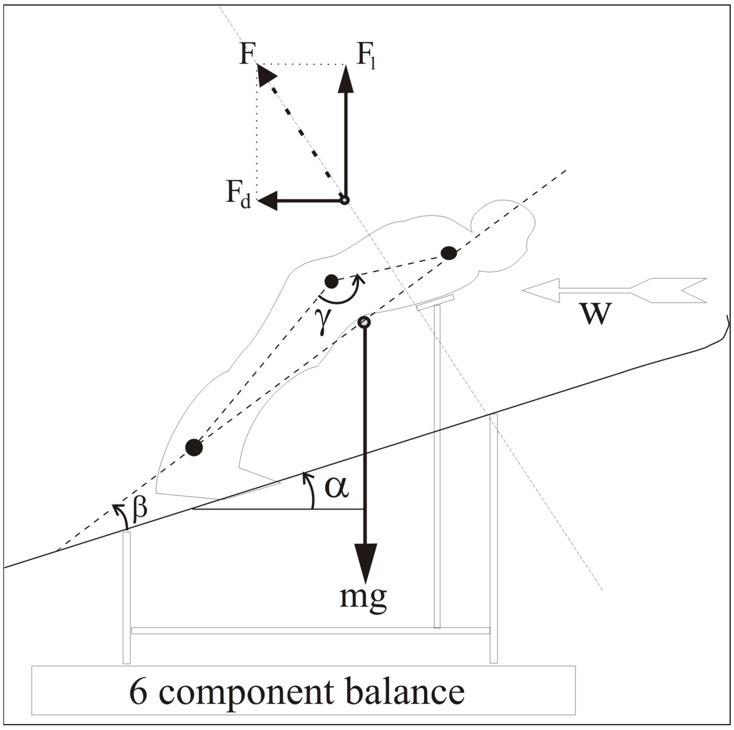
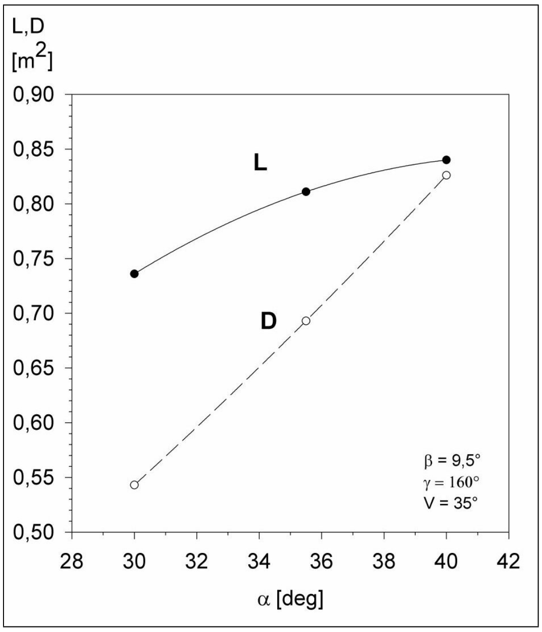
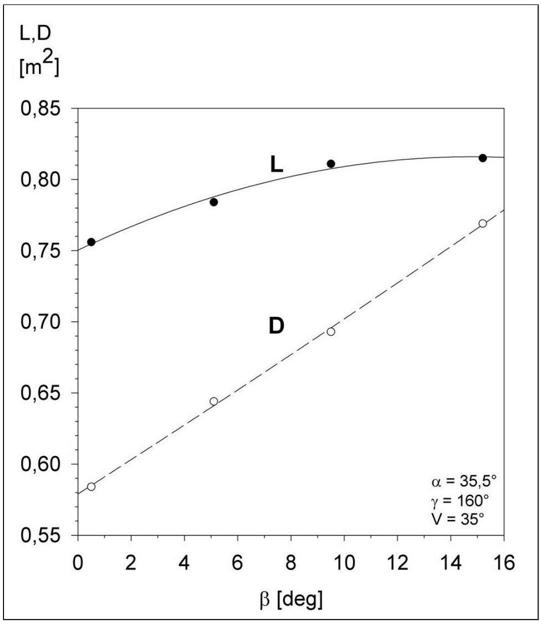


Fig. 3: Schematics of the wind tunnel measurements. The figure shows the apparatus, which enabled almost all postures of athletes and skis imaginable, and demonstrates the nomenclature used for the position angles. This study used the large wind tunnel at Arsenal Research in Vienna. The tunnel has a cross-section of 5×5 m^2. A 1.8 MW motor produces a maximum wind speed of 32 m s^{-1}.

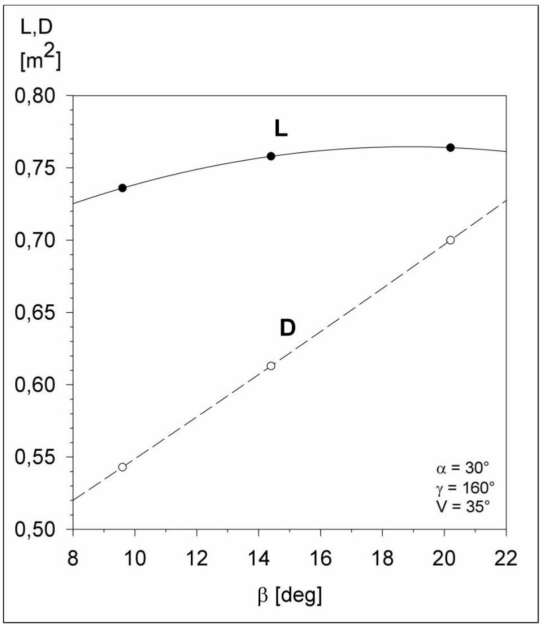
Fig. 4: Wind tunnel measurements: The aerodynamic forces largely depend on the flight style. Large-scale wind tunnel in Vienna; Andreas Goldberger, wr 225 m (2000).



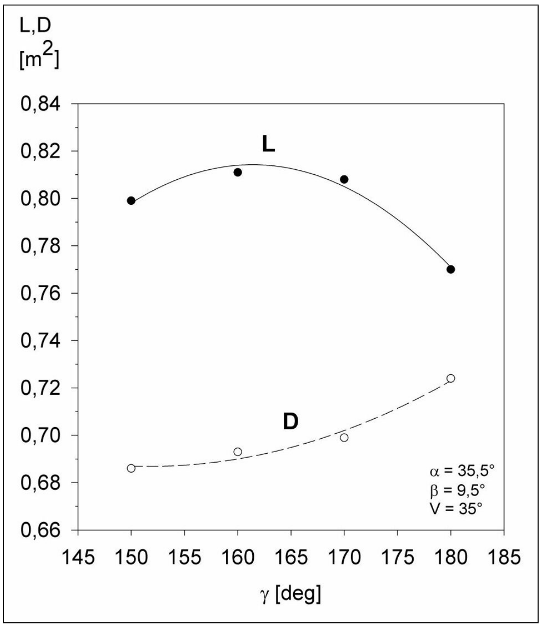
(a) Different angles of attack α. The body position was held constant ($\beta = 9.5°$ and $\gamma = 160°$) and the angles of attack were $30°$, $35.5°$ and $40°$. The opening of the skis was held constant at $V = 35°$. The interpolating functions are: $L = -0.43903 + 0.060743\alpha - 7.192 \times 10^{-4}\alpha^2$; $D = -0.032061 + 0.01232\alpha + 2.283x10^{-4}\alpha^2$.



(b) L and D values depending on the body-to-ski angle β. The values shown have been taken at $\alpha = 35.5°$, $\gamma = 160°$ and $V = 35°$. The interpolating functions are: $L = -0.645718 + 0.0126185\beta - 3.348 \times 10^{-4}\beta^2$; $D = 0.408434 + 0.01364\beta + 3.9308 \times 10^{-5}\beta^2$.

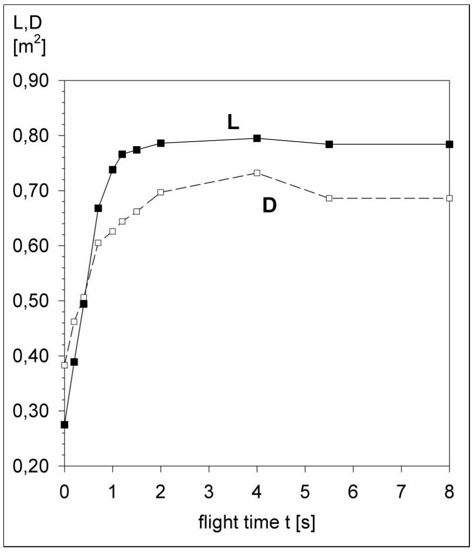


(c) L and D values depending on the body-to-ski angle β. The values shown here have been taken at $\alpha = 30°$, $\gamma = 160°$ and $V = 35°$. The interpolating functions are: $L = 0.75037 + 8.86746x10^{-3}\beta - 2.99665 \times 10^{-4}\beta^2$; $D = 0.578995 + 0.01201\beta + 2.91724 \times 10^{-5}\beta^2$.

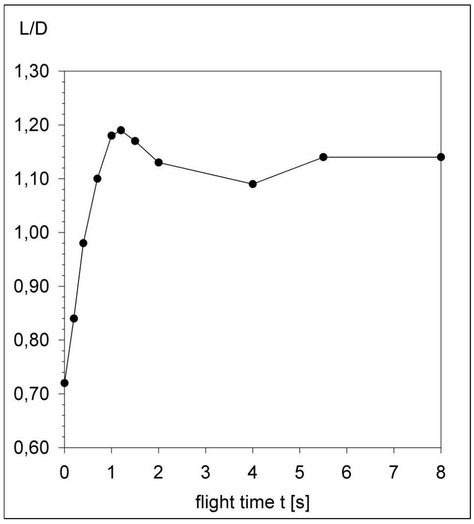


(d) L and D values depending on the hip angle γ. The body-to-ski angle β and the angle of attack α were held constant ($\beta = 9.5°$ and $\alpha = 35.5°$). The opening of the skis was held constant at $V = 35°$. The interpolating functions are: $L = -2.442 + 0.04035\gamma - 1.25 \times 10^{-4}\gamma^2$; $D = 1.722 - 0.01365\gamma + 4.5 \times 10^{-5}\gamma^2$.

Fig. 5: L and D values for model A (height $h = 1.78$ m)



(a) Values are functions of time reflecting the athlete's position changes during the flight



(b) L/D ratio for the reference jump with model A

Fig. 6: L and D values of reference jump of model A

Examples of computer simulations of ski jumping

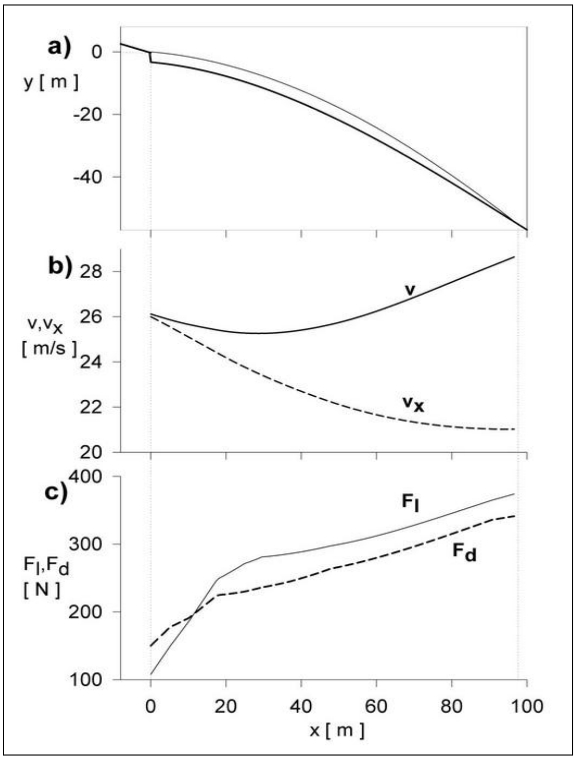
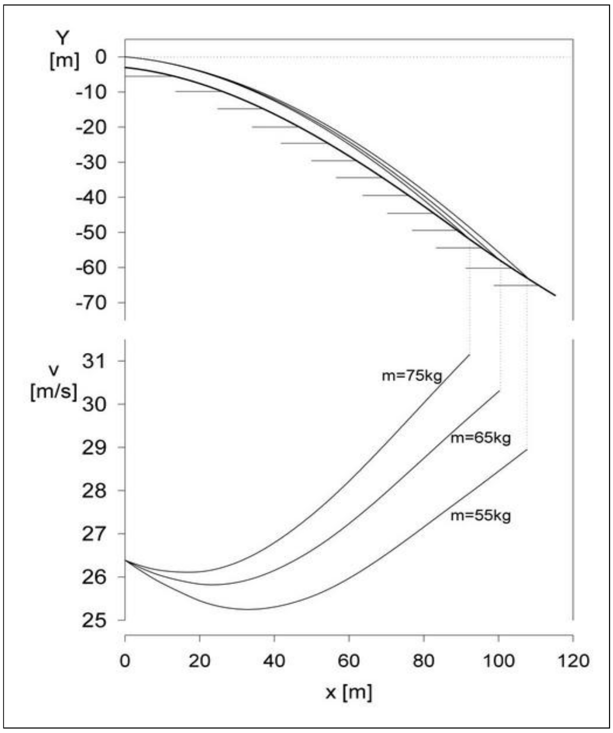
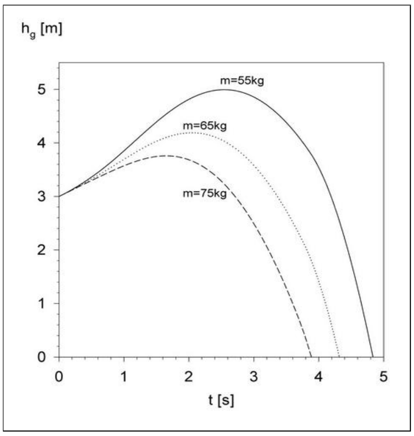


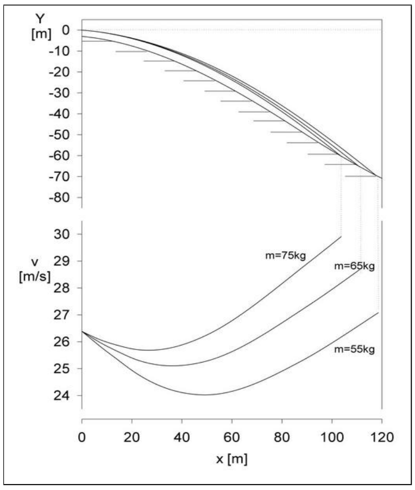
Fig. 7: Results with the reference jump for model A. **(a)** shows the profile of the jumping hill in Sapporo and the trajectory $y = y(x)$. Jumping hill parameters for Sapporo (K = 120 m): $a = 11°$, $b = -37°$, $c = 35°$, $H(K) = 59.449$ m, $N(K) = 103.391$ m, $r_1 = 105$ m, $R_2 = 120$ m, $M = 20$ m, $T = 7$ m, $S = 3.3$ m. **(b)** is the velocity of motion v (solid line) and the horizontal component of this velocity v_x (broken line). **(c)** shows the lift force F_l and drag force F_d acting on the athlete and his equipment. The air density was set to 1.15 kg m^{-3}; the mass of the athlete with equipment was 65 kg.



(a) Jumping hill parameters for Park City (K = 120 m): $\alpha = -10.5°$, $\beta = 35°$, $\beta_P = 38°$, $\beta_L = 37.77°$, $\gamma = 35°$, $h = 59.52$ m, $n = 103.51$ m, $r_1 = 93$ m, $r_2 = 105$ m, $r_L = 356.5$ m, $l_1 = 18.67$ m, $l_2 = 13.90$ m, $t = 6.7$ m, $s = 3$ m. For all jumping hills approved by the FIS the parameters can be found in the FIS Certificates of Jumping Hills. The trajectories and velocities for three different masses (55, 65 and 75 kg) are shown. The approach velocity v_0 was 26.27 m s^{-1}, the air density $\rho = 1.0$ kg m^{-3} and v_{p0} was 2.5 m s^{-1}. The gust velocity v_g was set to 0.



(b) Height above ground h_g for three different masses as a function of the flight time t. The solid line shows a jump simulation using a mass of m = 55 kg, the dotted line for m = 65 kg, and the broken line for m = 75 kg.



(c) Analogous to Fig. (a), however, in this case a gust blowing constantly with $v_g = 3$ m s^{-1}($\zeta = 135°$) during the whole flight was used.

Fig. 8: Simulated jumps using the L and D tables from the reference jump for model A and the profile of the jumping hill in Park City