Homework Problem Set #4

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Let R be a ring with identity $1 \neq 0$, and let F be a field

9.4.1 Determine whether the following polynomials are irreducible in the rings indicated. For those that are reducible, determine their factorization into irreducibles. The notation F_p denotes the finite field $\mathbb{Z}/p\mathbb{Z}$, p a prime.

b) $x^3 + x + 1$ in $F_3[X]$

c) $x^4 + 1$ in $F_5[X]$

d) $x^4 + 10x^2 + 1$ in $\mathbb{Z}[X]$

9.4.2 Prove that the following polynomials are irreducible in $\mathbb{Z}[X]$:

$$\text{b) } x^6 + 30x^5 - 15x^3 + 6x - 120$$

$$\text{c) } x^4 + 4x^3 + 6x^2 + 2x + 1 \text{ [Substitute } x - 1 \text{ for } x.]$$

9.4.6 Construct fields of each of the following orders:

(a) 9 **(b)** 49 **(c)** 8 **(d)** 81

(you may exhibit these as $F[x]/(f(x))$ for some F and f). [Use Exercises 2 and 3 in Section 2.]

9.4.11 Prove that $x^2 + y^2 - 1$ is irreducible in $\mathbb{Q}[x, y]$.

13.1.1 Show that $p(x) = x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$. Let θ be a root of $p(x)$. Find the inverse of $1 + \theta$ in $\mathbb{Q}[\theta]$.