Homework Problem Set #5

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Let R be a ring with identity $1 \neq 0$, let F be a field, and let x be an indeterminate over F.

13.1.3 Show that $x^3 + x + 1$ is irreducible over F_2 and let θ be a root. Compute the powers of θ in $F_2(\theta)$.

13.1.5 Suppose α is a rational root of a monic polynomial in $\mathbf{Z}[x]$. Prove that α is an integer.

13.2.3 Determine the minimal polynomial over $\mathbf{Q}$ for the element $1 + i$.

13.2.4 Determine the degree over $\mathbf{Q}$ of $2+\sqrt{3}$ and of $1+\sqrt[3]{2}+\sqrt[3]{4}$.

13.2.12 Suppose the degree of the extension K/F is a prime p. Show that any subfield E of K containing F is either K or F.

13.2.14 Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$.