Homework Problem Set #2

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Let R be a ring with identity $1 \neq 0$.

7.4.19 Let R be a finite commutative ring with identity. Prove that every prime ideal of R is a maximal ideal.

7.5.3 Let F be a field. Prove that F contains a unique smallest subfield F_0 and that F_0 is isomorphic to either $\mathbb{Q}$ or $\mathbb{Z}/p\mathbb{Z}$ for some prime p (F_0 is called the *prime subfield* of F). [See Exercise 26, Section 3.]

8.2.2 Prove that any two nonzero elements of a P.I.D. have a least common multiple (cf. Exercise 11, Section 1).

8.2.3 Prove that a quotient of a P.I.D. by a prime ideal is again a P.I.D.

9.1.4 Prove that the ideals (x) and (x, y) are prime ideals in $\mathbb{Q}[x, y]$, but only the latter ideal is a maximal ideal.