# Superposition Wave Function Visualization for free particle

This is a cross platform software that visualizes the **Amplitude**(the square root of **PDF**) and **Phase** of a superposition wave function for a free particle in 1 dimension `x`

This work is done as a bonus project for the Quantum Electronics Course of *KN Toosi University of Technology*, *Tehran, Iran*, presented by Dr. Ebrahim Nadimi (Special Thanks to Dr Nadimi for the excellent lectures that caused a solid understanding on Quantum Physics in me, and the guidance to do this projects)

## General Description

The Schrodinger Equation is:

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\Delta + V(x,t)\right]\Psi(x,t)$$

in a *1D* case, any linear composition complex exponentials can be a solution to the SE:

$$\Psi(x,t) = \sum_{n=1}^{N} A_n \exp i(k_n x - \omega_n t)$$

while we can specify the correct relation between the **wave number** and the **angular frequency** (with the Energy associated with each state)

$$\begin{aligned}\hat{H}\Psi &= E\Psi \\ V(x,t) &= 0 \\ E_n &= \frac{k_n^2\hbar^2}{2m} \\ \omega_n &= \frac{E_n}{\hbar}\end{aligned}$$

The **Born's rule** helps us to find the coefficients; considering that the complex exponentials are *orthonormal*, we can arbitrarily choose the coefficients so that:

$$\sum_n |A_n|^2 = 1$$

as another consequence of **Born's rule** in this solution, the solution is valid in one period, so the wave function will be integrable and integration of **PDF** on one period, gives `1`

we can choose the coefficients so that the state would be equally weighted:

$$A_n = \frac{1}{\sqrt{N}}$$

## Mathematical Calculations

The Amplitude

$$\Psi(x,t) = r(x,t)\exp[i\phi(x,t)]$$

$$|\Psi|^2 = \Psi \times \Psi^* \Rightarrow r(x,t) = \sqrt{\Psi \times \Psi^*}$$

$$\Psi \times \Psi^* = \frac{1}{N}\sum_{n,m} \exp i\left[(k_n - k_m)x - (\omega_n - \omega_m)t\right]$$

And there are `N` terms with `n=m` so:

$$\Psi \times \Psi^* = 1 + \frac{1}{N}\sum_{n \neq m} \exp i\left[(k_n - k_m)x - (\omega_n - \omega_m)t\right]$$

The other `N(N-1)` terms are splittable in two groups in which each term in a group is corresponded to exactly one term in the other group, so that the corresponded terms are complex conjugate of each other (As we know for any `(n,m)` term, there is a `(m,n)` term):

$$\Psi \times \Psi^* = 1 + \frac{2}{N}\sum_{n=1}^{N}\sum_{m=n+1}^{N} \cos\left[(k_n - k_m)x - (\omega_n - \omega_m)t\right]$$

And the **Born's rule** will be satisfied, while the integral of a `cosine` on a period is `0`

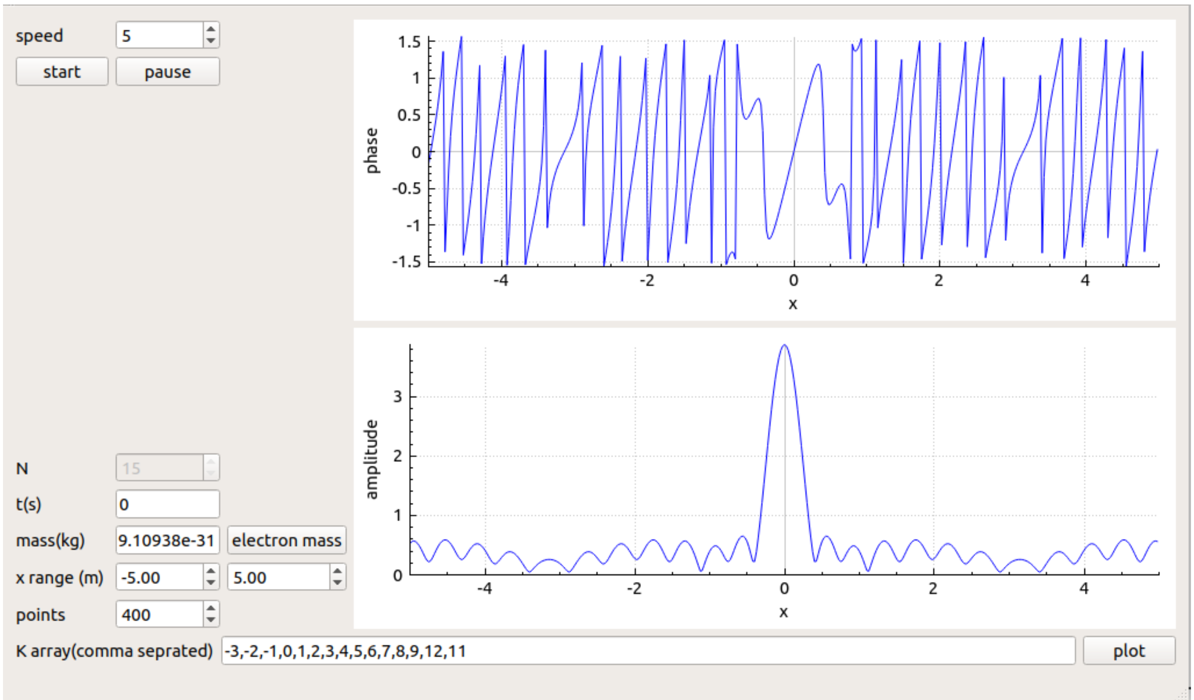
The phase

$$Im(\Psi) = \frac{\Psi - \Psi^*}{2i}, Re(\Psi) = \frac{\Psi + \Psi^*}{2}$$

$$\phi(x,t) = tg^{-1}\left(\frac{\Psi - \Psi^*}{\Psi + \Psi^*}\right)$$

$$\phi(x,t) = tg^{-1}\left(\frac{\sum_{n=1}^{N} \sin\left(k_n x - \omega_n t\right)}{\sum_{n=1}^{N} \cos\left(k_n x - \omega_n t\right)}\right)$$

## Software Description




This numerical calculations are done using pure C++ and the STL (cmath)

The graphical application is wrote using Qt5 and to visualize and plot the results, QCustomPlot is used

## inputs

**K array** comma separated

**x range** the calculation range

**number of points in x range**

**time variation speed**

**The particle mass** and a push button to set the default electron mass