Math: Problem Set 4

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1 6.1

The standard form can be expressed in the following way:

$$\begin{aligned}
&\min_{w} - e^{-w^T x} \\
&s.t. w^T A w - w^T A y - w^T x \leq -a \\
&\quad y^T w - w^T x = b
\end{aligned}$$

2 6.5

Now define $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where x_1 denotes the amount of knobs produced; x_2 denotes the amount of milk cartons produced. And $k = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $l = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $p = \begin{bmatrix} 0.05 \\ 0.07 \end{bmatrix}$, $q = \begin{bmatrix} 240 \\ 100 \end{bmatrix}$. Now form $A = \begin{bmatrix} k^T \\ l^T \end{bmatrix}$. Then we can form the following optimization problem:

$$\begin{aligned}
&\min_{x} - p^T . x \\
&s.t. A.x \leq q
\end{aligned}$$

3 6.6

Now we have $\nabla f(x,y) = \begin{bmatrix} 6xy + 4y^2 + y \\ 3x^2 + 8xy + x \end{bmatrix} = \begin{bmatrix} (6x + 4y + 1)y \\ (3x + 8y + 1)x \end{bmatrix} = 0$. Now we should have the following critical point:

$$\begin{cases} x_1 = 0 \\ y_1 = 0 \end{cases} \quad \begin{cases} x_2 = 0 \\ y_2 = -\frac{1}{4} \end{cases} \quad \begin{cases} x_3 = -\frac{1}{3} \\ y_3 = 0 \end{cases} \quad \begin{cases} x_4 = -\frac{1}{9} \\ y_4 = -\frac{1}{12} \end{cases}$$

Now we should have the hessian matrix $H(x,y) = \begin{bmatrix} 6y & 6x + 8y + 1 \\ 6x + 8y + 1 & 8x \end{bmatrix}$

Now we consider $H(x_1, y_1) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Now we should have the eigenvalues $_1 = 1,_2 = -1$. Then this is a indefinite matrix. Then (x_1, y_1) is a saddle point.

$H(x_2, y_2) = \begin{bmatrix} -\dfrac{3}{2} & -1 \\ -1 & 0 \end{bmatrix}$. Again this is an indefinite matrix. Then (x_2, y_2) is a saddle point.

$H(x_3, y_3) = \begin{bmatrix} 0 & -1 \\ -1 & -\dfrac{8}{3} \end{bmatrix}$. Again this is an indefinite matrix. Then (x_3, y_3) is a saddle point.

$H(x_4, y_4) = \begin{bmatrix} -\dfrac{1}{2} & -\frac{1}{3} \\ -\frac{1}{3} & -\dfrac{8}{9} \end{bmatrix}$. Now we should have $\lambda_1\lambda_2 = det(H(x_4, y_4)) = \dfrac{1}{3}$ and $\lambda_1 + \lambda_2 = tr(H(x_4, y_4)) = -\dfrac{25}{18}$. Then we know that both eigenvalues are negative. Then $H(x_4, y_4)$ is a negative definite matrix. Then (x_4, y_4) is a local maximizer.

4 6.11

Now we have $x_1 = x_0 - \dfrac{f'(x_0)}{f'(x_0)'} = x_0 - \dfrac{2ax_0 + b}{2a} = \dfrac{2ax_0 - 2ax_0 - b}{2a} = -\dfrac{b}{2a}$. Then this is the unique local extrema.