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MACS 30100
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2017/01/23

1 Problem Set 2

1.1 Problem 1

1.a

For a histogram of the actual income distribution, please see figure 1.

1.b

For a histogram of the actual income distribution with the given PDF curve, please see figure 2.

The value of the log-likelihood for the given parametrization of this distribution and given this data is -8298.637.

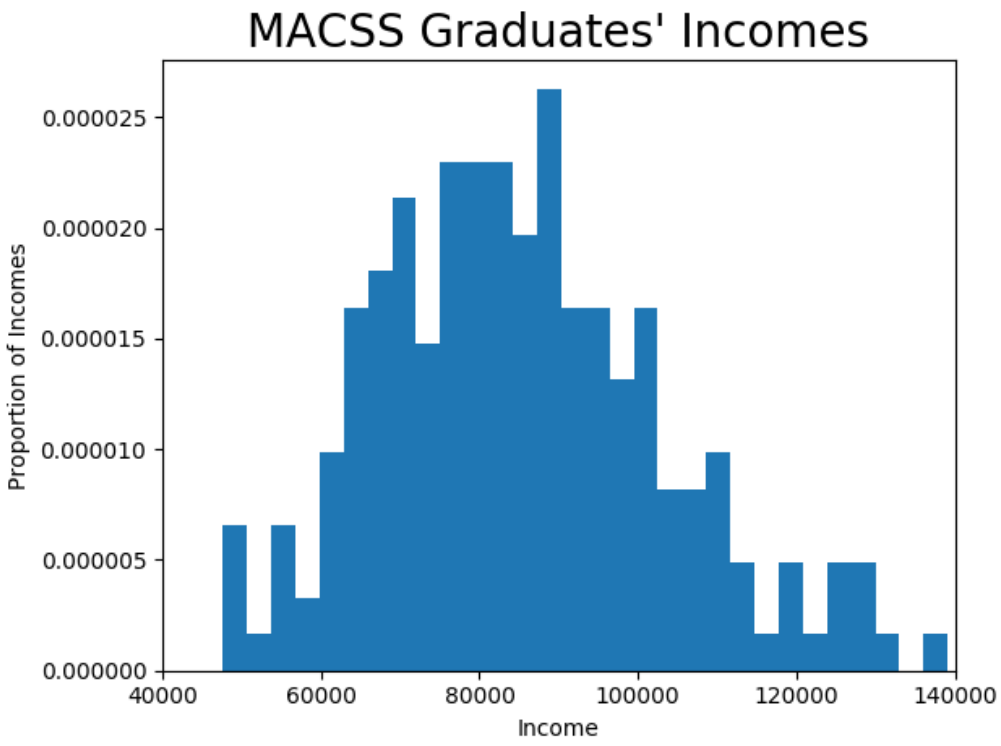
1.c

For a histogram of the actual income distribution with the given PDF curve as well as the maximum likelihood estimation, please see figure 3.

For the resulting maximum likelihood estimation function, $\mu = 11.331$ and $\sigma = 0.212$. The maximum log-likelihood is -2239.535.

For the variance–covariance matrix, please see figure 4.

Figure 1: Histogram of Incomes of MACSS Graduates (2018–2020)



1.d

We perform a likelihood ratio test to determine the probability that the given data came from the distribution given in 1b rather than the maximum likelihood estimation made in 1c. χ^2 of H_0 with two degrees of freedom returns a p-value of 0.000; we find that there is no evidence to support the claim that the given data came from the distribution given in 1b rather than from the maximum likelihood estimation made in 1c. (That is, the probability that the actual income data come from the distribution given in 1b is zero.)

1.e

The probability of a given MACSS graduate earning more than \$100,000 is 0.196. The probability of a given MACSS graduate earning less than \$75,000 is 0.308.

Figure 2: Histogram of Incomes of MACSS Graduates (2018–2020) with log-normal PDF $(\mu = 9.0, \sigma = 0.3)$

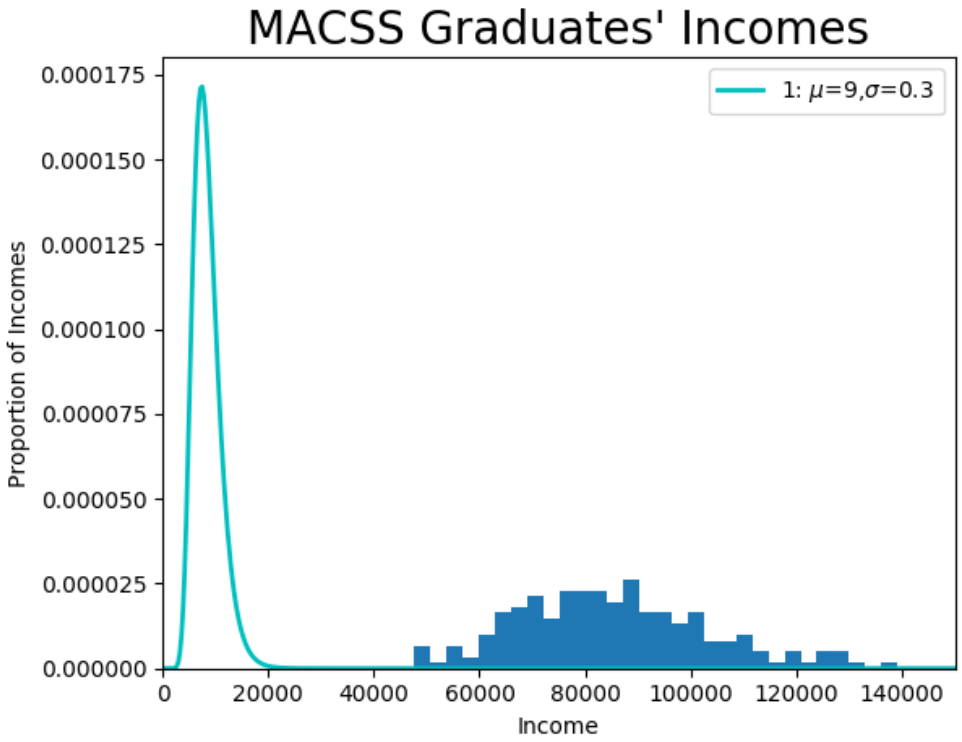


Figure 3: Histogram of Incomes of MACSS Graduates (2018–2020) with log-normal PDF ($\mu = 9.0, \sigma = 0.3$) and Maximum Likelihood Estimation

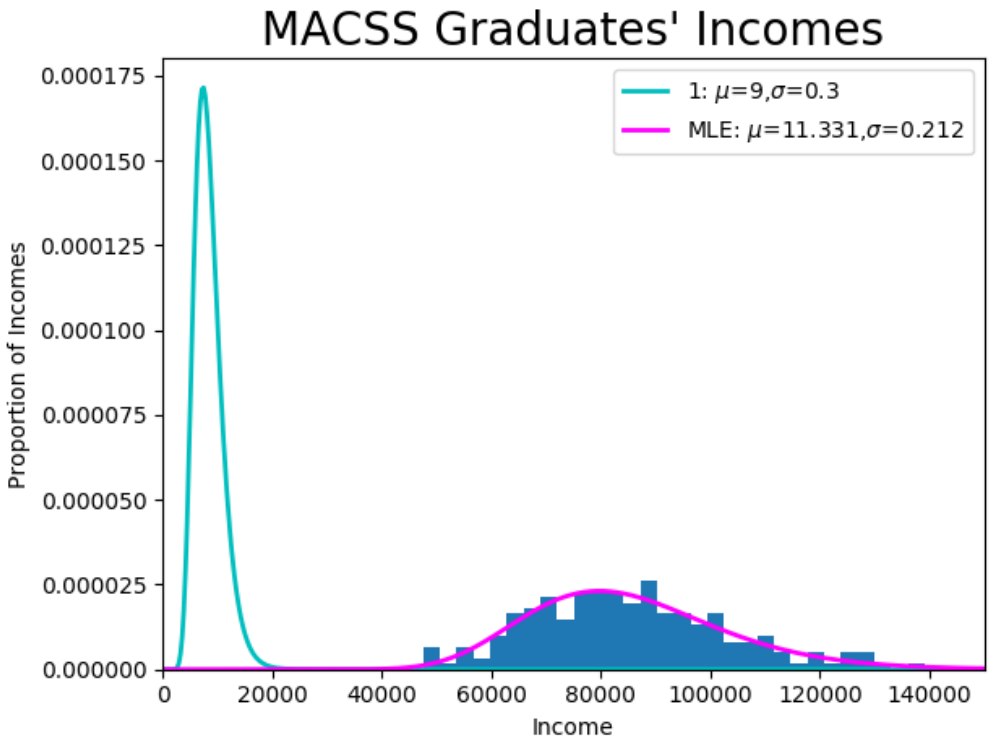


Figure 4: Variance–Covariance Matrix for Maximum Likelihood Estimation of Income Distribution

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[  2.24988903e-04   2.55269928e-07]
[  2.55269928e-07   1.11129364e-04]
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Figure 5: Variance–Covariance Matrix for Maximum Likelihood Estimation for Sick-Weeks Model

1.2 Problem 2

1.a

Using the given model, we estimate the parameters by maximum likelihood:

$\beta_0 = 0.252$

$\beta_1 = 0.013$

$\beta_2 = 0.401$

$\beta_3 = -0.010$

$\sigma^2 = 0.003$

The log-likelihood for this estimation is -876.865.

For the variance–covariance matrix, please see figure 5.

1.b

We use a likelihood ratio test to determine the probability that $\beta_0 = 1.0$, $\sigma^2 = 0.01$, and $\beta_1, \beta_2, \beta_3 = 0$. With a p-value of 0.000, we find that the parameters do not have such values. This means that we reject the null hypothesis and find that employee age, number of children, and average winter temperatures do have an effect on the number of sick weeks taken.