Problem Set #2

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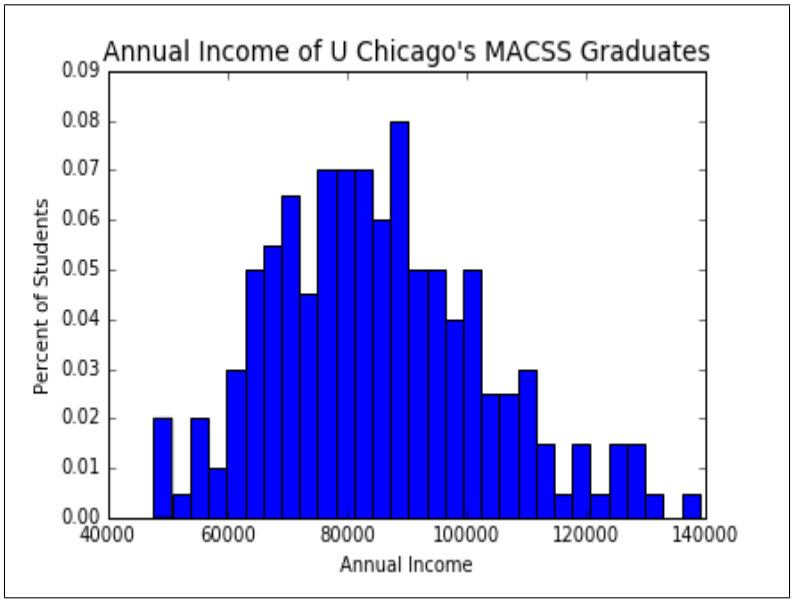
Problem 1: Some income data, lognormal distribution, and hypothesis testing

Part (a). The histogram is shown in Figure 1.

Figure 1: Histogram of Percentages of the MACSS Graduates



Part (b). The log likelihood value is approximately: -8298.64. The plot of lognormal PDF is shown in Figure 2.

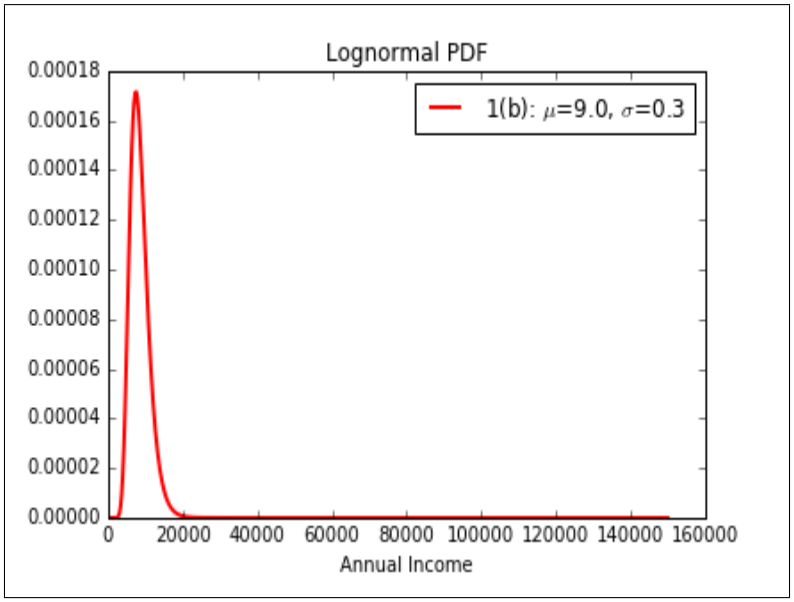
Part (c). The PDF of lognormal distribution by maximum likelihood against the PDF from part(b) and the histogram from part(a) is in Figure 3.
ML estimate for mu is approximately: 11.33
ML estimate for sigma is approximately: 0.21
The value of the likelihood function is approximately: -2239.55

Part (d). Chi squared of null-hypothesis (H0) with 2 degrees of freedom = 0.0000. This means we could reject the hypothesis that the model in part(b) fits the data. The variance-covariance matrix is: $\begin{bmatrix} 0.00030507 & 0.00018687 \\ 0.00018687 & 0.00054403 \end{bmatrix}$

Part (e). The probability of earning more than $100,000 is approximately: 0.1919. The probability of earning less than $75,000 is approximately: 0.3089

Problem 2: Linear regression and MLE

Figure 2: Lognormal PDF



Part (a). The parameters of the model by maximum likelihood are approximately:

$$\sigma_{MLE} = 0.00302, \beta_0 = 0.25160, \beta_1 = 0.01293, \beta_2 = 0.40053, \beta_3 = -0.00999$$

The value of the likelihood function is approximately: 872.18 The estimated variance covariance matrix is: $\begin{bmatrix} 1. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 0. & 1. \end{bmatrix}$

Part (b). The parameters of null-hypothesis are:

$$H_0 : \sigma^2 = 0.01, \beta_0 = 1, \beta_1 = 0, \beta_2 = 0, \beta_3 = 0$$

Chi squared of H0 with 2 degrees of freedom = 0.0000, which means we could reject the null-hypothesis that the variables of age, number of children, and average temperature in winter have no impact on sickness.

Please see the next page for Figure 3, Problem 1(c).

Figure 3: Lognormal PDF