Problem Set #2

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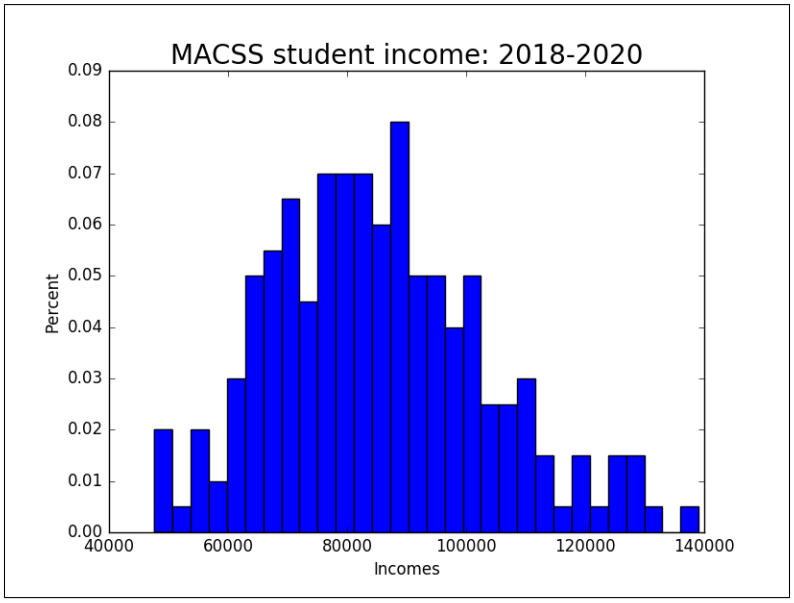
Problem 1. Some income data, lognormal distribution, and hypothesis testing.

Part (a).



Part (b).

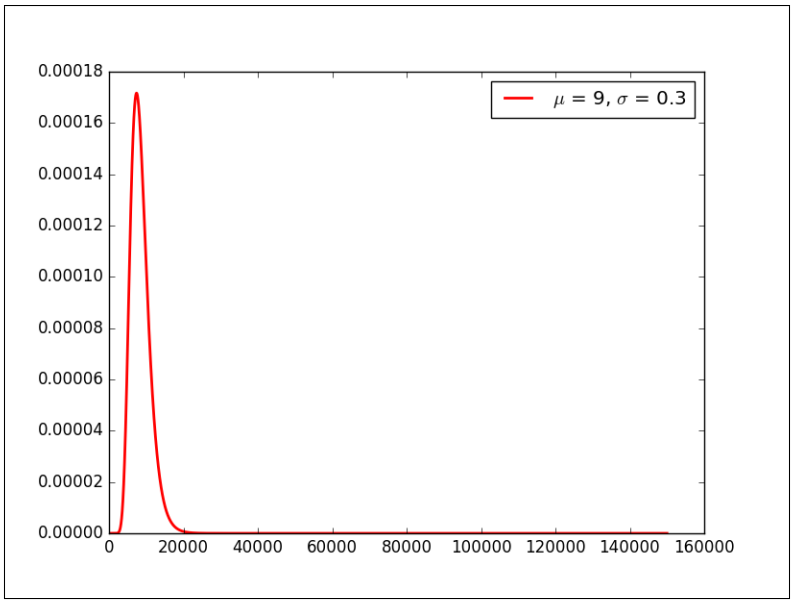
The value of the log likelihood value for this parameterization of the distribution and given this data is -8298.64.

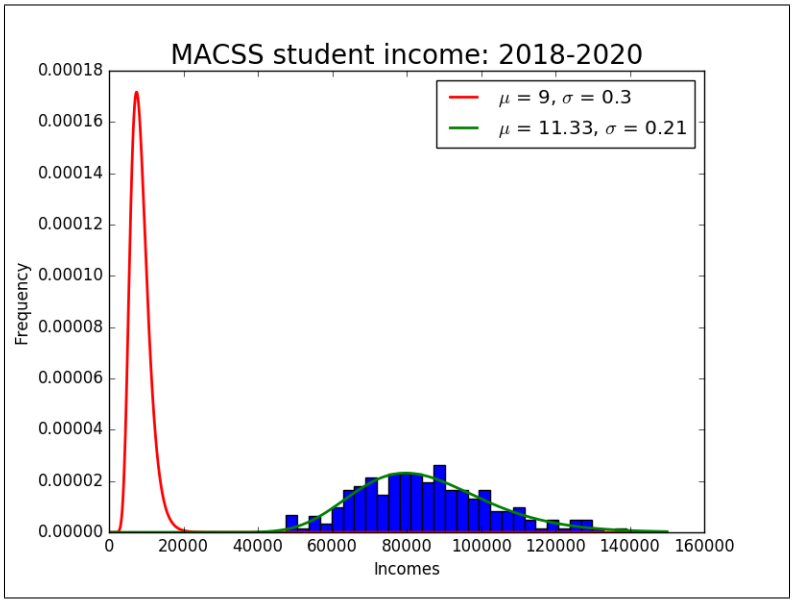
Part (c).

Firstly, $\mu_{mle} = 11.331, \sigma_{mle} = 0.212$.

The log likelihood value of the data given these parameters is -2239.53.

The variance-covariance matrix is: $\begin{bmatrix} 0.00023979 & 0.00000351 \\ 0.00000351 & 0.00011245 \end{bmatrix}$





Part (d). The Chi square of H0 with 2 degrees of freedom p-value = 0.00000000. Since this probability is small (< 0.05), the data is unlikely coming from the distribution in part(b).

Part (e). The probability that a student would earn more than \$100,000 is: 19.56%. The probability of a student earn less than \$75,000 is: 30.79%.

Problem 2. Linear regression and MLE.

Part (a).

$\beta_0^{mle} = .252 \quad \beta_1^{mle} = 0.013 \quad \beta_2^{mle} = 0.401 \quad \beta_3^{mle} = -0.009992 \quad \sigma^2_{mle} = 0.00000911$
The value of the log likelihood function is: 876.87.

The variance-covariance matrix is: $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Part (b).
Likelihood Ratio Test p-value is: 0.00000000.

This number is really low ($< .05$), so it is unlikely that age, number of children, and average winter temperature have no effect onthe number of sick days.