Problem Set #[2]
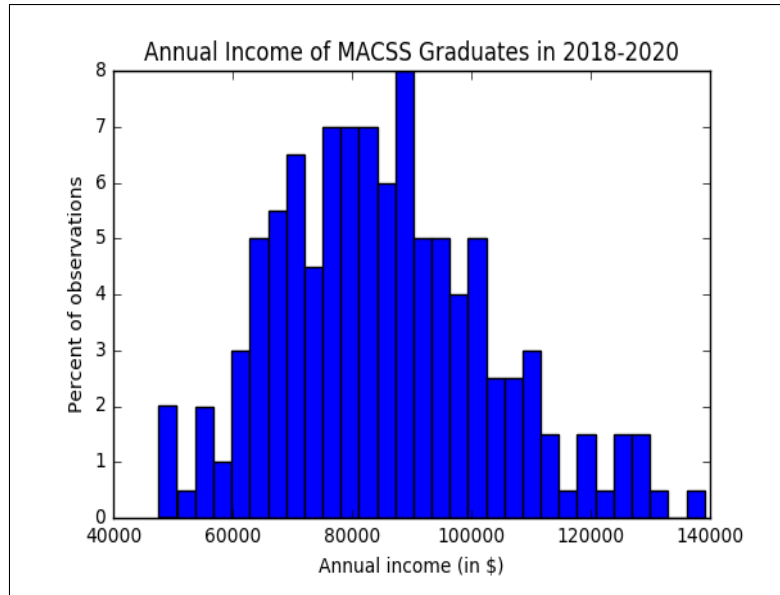
MACS 30100, Dr. Evans

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Problem 1 Some income data, lognormal distribution and Hypothesis testing

Part (a).

Part (b).

The log-likelihood value for given parametrization and given this data is -8298.636956005032.

Part (c).

Firstly, $\mu_{mle} = 11.3314402912, \sigma_{mle} = 0.211674548907$.

The log likelihood value of the data given these parameters is -2239.5347439980173.

The variance-covariance matrix is: $\begin{bmatrix} 0.00030509 & 0.00018702 \\ 0.00018702 & 0.00054462 \end{bmatrix}$

Lognormal PDF

Annual Income of MACSS Graduates in 2018-2020 with the lognormal PDFs
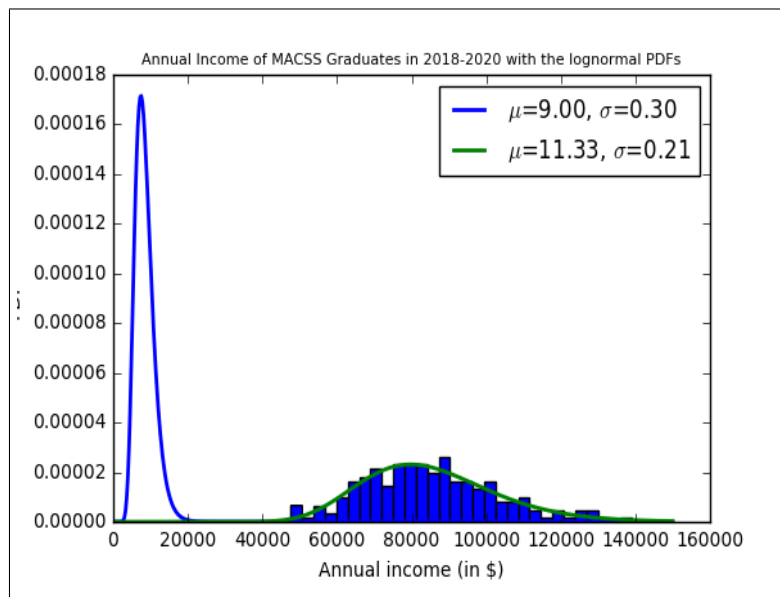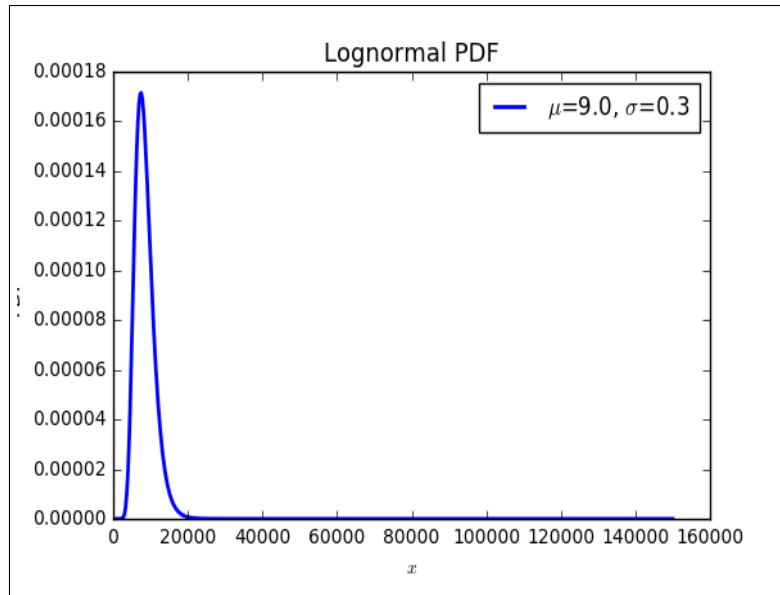
Part (d). The chi squared of H_0 with 2 degrees of freedom p-value is 0.0. So we can reject the null hypothesis. So the data did not come from distribution in part b.

Part (e). The probability that a studentI will earn more than \$100,000 is 0.1958. The probability that a student will earn less than \$75,000 is 0.3077.

Problem 2. Linear regression and MLE.

Part (a).

$\beta_0^{mle} = 0.251647161896$ $\beta_1^{mle} = 0.0129333894972$ $\beta_2^{mle} = 0.400502025415$ $\beta_3^{mle} = -0.00999171308743$ $\sigma_{mle}^2 = 9.10630265262e^{-06}$.
The value of the log likelihood function is: 876.8650389669244.

The variance-covariance matrix is:
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Part (b).
Likelihood Ratio Test p-value is: 0.00000000. So it is unlikely that age, number of children, and average winter temperature have no effect on the sick days.