

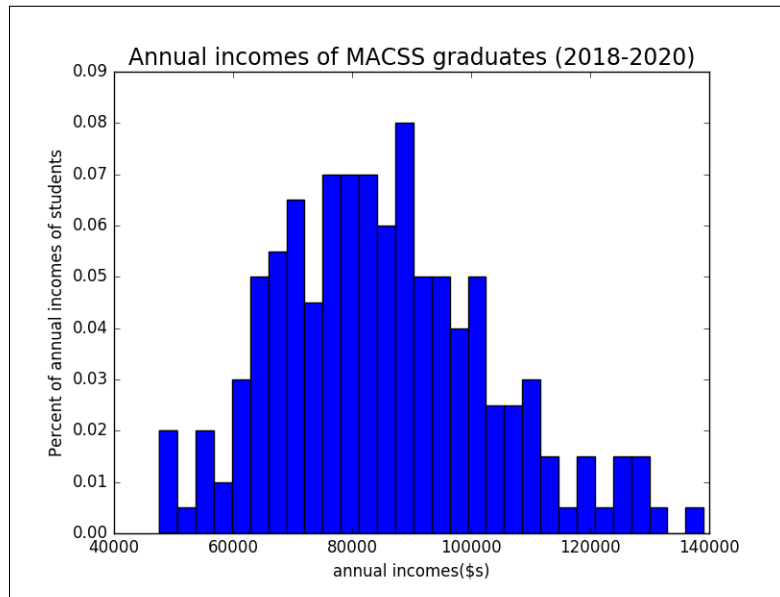
# Problem Set #[2]

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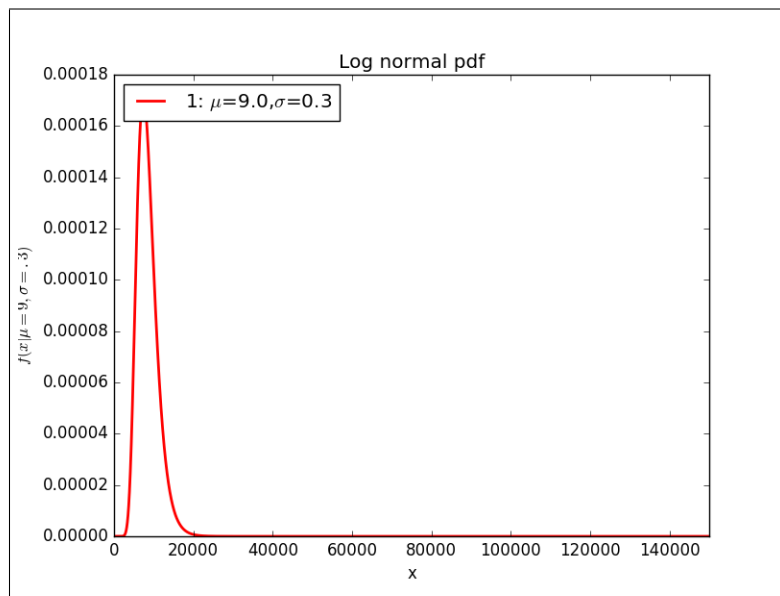
## Problem 1

### Part (a).Plot a histogram

The histogram of annual incomes of students who graduated in 2018, 2019, and 2020 from the University of Chicago MACSS program is as below:

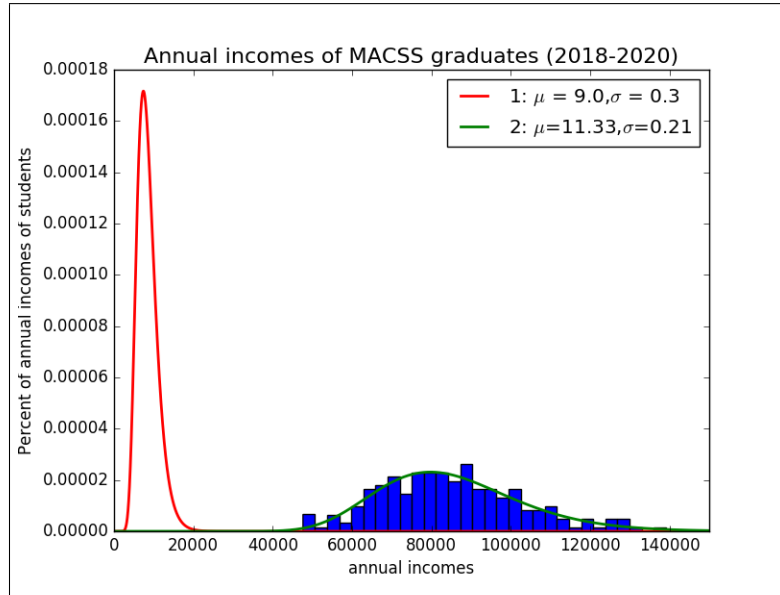


### Part (b).Log PDF and log likelihood



Value of the log likelihood is: -8298.63695601

### Part (c).MLE



The value of Log-likelihood function is: -2239.534744

The MLE estimate parameters are as below:

$$MLE_{estimate} \mu = 11.3314403262$$

$$MLE_{estimate} \sigma = 0.211674580397$$

$$VCV_{(MLE)} = \begin{bmatrix} 2.22214155e-04 & -5.61369430e-07 \\ -5.61369430e-07 & 1.12310760e-04 \end{bmatrix}$$

### Part (d).likelihood ratio test

From chi squared of  $H_0$  with 2 degrees of freedom, we can know the p-value is 0.0.

### Part (e).

Probability that I will earn more than \$100,000 is 0.196  
Probability that I will earn less than \$75,000 is 0.308

## Problem 2

### Part (a).MLE

The value of Log-likelihood function is: 876.865046414609

The MLE estimate parameters are as below:

$$MLEestimate\beta_0 = 0.251646265218$$

$$MLEestimate\beta_1 = 0.0129333458728$$

$$MLEestimate\beta_2 = 0.40050209353$$

$$MLEestimate\beta_3 = -0.0099916690296$$

$$MLEestimate\sigma = 0.00301768768568$$

$$VCV_{(MLE)} = \begin{bmatrix} 8.04161587e-07 & 1.22767134e-09 & -9.08556283e-08 & -1.56112623e-08 & -1.25989887e-08 \\ 1.22767134e-09 & 2.00398389e-09 & -1.63752155e-08 & -1.23747827e-09 & -5.92335814e-10 \\ -9.08556283e-08 & -1.63752155e-08 & 1.72498499e-07 & 1.04474878e-08 & 6.52192033e-09 \\ -1.56112623e-08 & -1.23747827e-09 & 1.04474878e-08 & 1.09878616e-09 & 4.84193753e-10 \\ -1.25989887e-08 & -5.92335814e-10 & 6.52192033e-09 & 4.84193753e-10 & 2.07961106e-08 \end{bmatrix}$$

#### **Part (b).likelihood ratio test**

From chi squared of H0 with 5 degrees of freedom, we can know the p-value is 0.0. This result implies that the likelihood that age, number of children, and average winter temperature have no effect on the number of sick days is very low