Jan Suchánek

$a = 3$
$b = -1$
$c = -4$
$d = 2$

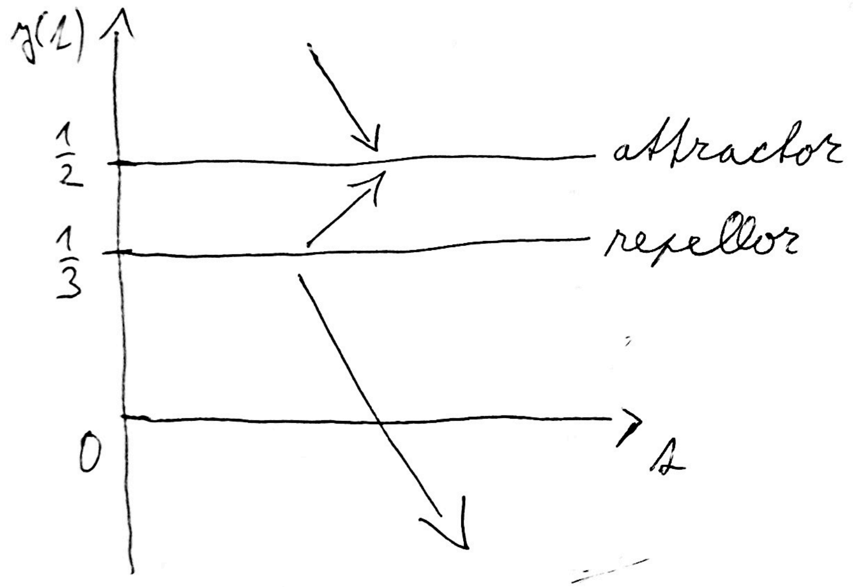
1) $y'(t) = (3y(t) + (-1))(-4y(t) + 2)$

$y'(t) = (3y(t) - 1)(-4y(t) + 2)$

Find equilibria:

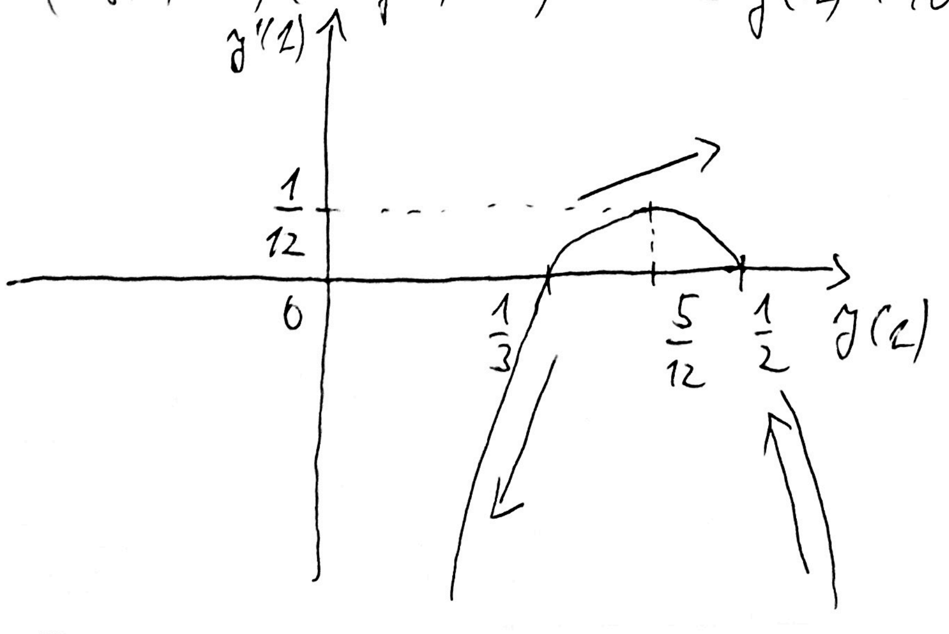
$y'(t) = 0$ when $y(t) = \frac{1}{3}$ or $y(t) = \frac{1}{2}$

Direction field:



Phase diagram:

$y'(t) = (3y(t) - 1)(-4y(t) + 2) = -12y^2(t) + 10y(t) - 2$



2) a) $y'' - 3y' + 2y = 0$

Jan Sochánek

Characteristic polynomial:

$\lambda^2 - 3\lambda + 2 = 0$ $\qquad FS = \{e^{2t}, e^t\}$

$(\lambda - 2)(\lambda - 1) = 0$ $\qquad y_H(t) = ae^{2t} + be^t, \quad a, b \in \mathbb{R}$

$y(t) = y_H(t)$

Br. $y(0) = 1, \; y(1) = e$

$y(1) = ae^2 + be = e$

$y(0) = a e^{2.0} + be^0 = 1$

$e(a.e + b) = e \quad / \cdot \frac{1}{e}$

$a + b = 1$

$a.e + b = 1$

$a + b = 1$

$a.e + b - a - b = 1 - 1$

$ae - a = 0$

$a(e-1) = 0 \quad / \cdot \frac{1}{e-1}$

$a = 0 \Rightarrow b = 1$

Specific solution:

$y(t) = e^t$

2) b) Jan Sadlonek

$y'' - 2y' - 15y = 0$ $FS = \{e^{5t}, e^{-3t}\}$

Ch. e.

$\lambda^2 - 2\lambda - 15 = 0$ $y(t) = y_H(t) = a e^{5t} + b e^{-3t}, \quad a, b \in \mathbb{R}$

$(\lambda - 5)(\lambda + 3) = 0$

$y(0) = 4, \quad y'(0) = 4$ $y'(t) = 5a e^{5t} - 3b e^{-3t}$

$y(0) = a \cdot e^{5 \cdot 0} + b e^{-3 \cdot 0} = 4$ $3a + 3b + 5a - 3b = 16$

$y'(0) = 5a e^{5 \cdot 0} - 3b e^{-3 \cdot 0} = 4$ $8a = 16$

$a = 2$

$a + b = 4$

$5a - 3b = 4$ $2 + b = 4$

$b = 2$

$3a + 3b = 12$

$5a - 3b = 4$ $y(t) = \cancel{2a}\, 2e^{5t} + 2e^{-3t}$

Bonus: Find initial condition, for which the specific solution is converging, but not constant. For example $a = 0$ and $b = 1$ could work, as the solution would be $y(t) = e^{-3t}$, which is converging but not constant. Initial condition in this case could be $y(0) = 1$ and $y'(0) = -3$.

3) $\dot{k}(t) = s \cdot a \cdot k^{\alpha} - (n+\delta)k$

Find equilibria:

$$\dot{k}(t) = 0$$

$$0 = s \cdot a \cdot k^{\alpha} - (n+\delta)k \quad \Big/ \cdot \frac{1}{k}$$

assuming that $k > 0$, which is reasonable

$$0 = s \cdot a \cdot k^{\alpha-1} - (n+\delta)$$

$$n+\delta = s \cdot a\, k^{\alpha-1}$$

$$k^{\alpha-1} = \frac{n+\delta}{s \cdot a} \Rightarrow k^{*} = \left(\frac{n+\delta}{s \cdot a}\right)^{\frac{1}{\alpha-1}}$$