**Mathematical methods in Macroeconomics - Problem Set 1**

IES FSV UK

Notes:

The homework is due on **21st October, 13:00.** Upload ONE word file (or pdf or similar) and ONE Python notebook via Moodle. If you face any problem with uploading files, send the files via e-mail to [josef.strasky@gmail.com](mailto:josef.strasky@gmail.com) and [kubistmi@gmail.com](mailto:kubistmi@gmail.com). You can also bring the hand-written part of the solution to the lecture on 21st October, 2 PM (but submit the rest of the solution beforehand).

1. Consider equation , where *y(t)* is a function and *a,b,c,d* are coefficients



* Find out the first letter of your first name and the first letter of your surname (English alphabet)
* Find appropriate values of coefficients *a,b,c,d*
* *Example: Josef Stráský - first name:* ***J****; surname:* ***S*** --> *a* = 3; *b* = -1; *c* = -4; *d* = 2;

| first name | surname | A | B | C | D | E | F | G | H | I | J | K | L | M |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **c** | -2 | -1 | 1 | 2 | 3 | -2 | -1 | 1 | 2 | 3 | -2 | -1 | 1 |
| **B** | **d** | -2 | -2 | -2 | -2 | -2 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 4 |
| first name | surname | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| **A** | **c** | 2 | 3 | -2 | -1 | 1 | -4 | 3 | -2 | -1 | 1 | 2 | 3 | 4 |
| **B** | **d** | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |

* Find equilibria of your personalized differential equation.
* Draw direction field (by hand is fine) and discuss the stability of equilibria.
* Draw a phase diagram (either by hand or using software) and discuss the stability of equilibria using a phase diagram

*Note: if your coefficients give especially strange equations and result then choose another combination.*

1. Find the general solution of the following equations (by manual calculation – no software needed), then use initial conditions and find a specific solution for given initial conditions.

a) *y(0) = 1; y(1) = e;;*



b) *y(0) = 4; y´(0) = 4;*



- BONUS: find some **initial condition** for that the specific solution is converging, but not constant

1. Let us have the following differential equation (actually, it is the central equation of Solow model with Cobb-Douglas function):



1. find fixed points (equilibria) *k\**, by hand-writing

4) Python functions

Prepare a function **optimize()** with the following parameters:

* **start** - single float, the start of the interval
* **end** - single float, end of the interval
* **step** - single float, the incremental step of the sequence
* **fun** - function to be optimized

Your function should construct a sequence of numbers from **start** to **end** with the increment of size **step**, then find the function’s maximum value and where it was found, in other words:

**x** and **fun(x)** s. t. **fun(x)** = max **fun(x)** for **x ∊** **{**start, start+step, start+2\*step, … , stop**}**

E.g.

***parameters:*** start = -1, stop = 1, step = 0.1, fun = cos

***output:*** (0, 1) - maximum is at position 0 with value of 1

*Note: don’t use the build-in / package functions* ***max()*** *and* ***argmax()***

5) Python data wrangling

Prepare a single python notebook with the following steps

* Read the data from covid.zip files
* Choose data for one country
* Prepare your data
  + Summarise the variable values, prepare nice plots
  + Comment your results extensively– what can you observe in your data, what are the consequences – show that you understand the dataset.
  + Utilize meaningful data transformations
    - Think about what are the model expectation?
* The Malthusian population growth model
  + Describe the theoretical framework (how it should work)
  + Fit the model to the data
  + Comment on the model fit – is it meeting assumptions, what do the residuals look like, what are the problems (and where)?
* Prepare the interactive plot
  + For a selected data range:
  + Fit the model to the selection and print against the actual values
  + Present the formula (with parameters), R0, and R^2 values

*Note 1: Please comment on all your steps extensively, using both code comments (# comment) and markdown cells, it helps us understand that you understand what you are doing.*

*Note 2: Do not blindly copy code from the seminar. This exercise serves to make you get used to working in python, not in the art of copy-pasting. Chunks of code directly from the seminar will yield negative points.*

*Note 3: The model from the seminar had a significant shortcoming that was discussed in the lecture and you are asked to overcome this limitation.*