Seminar 3

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Overview

Time Series

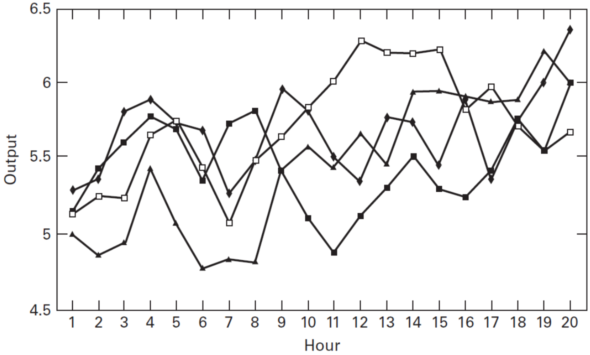


FIGURE 2.1 Hourly Output of Four Machines

A time series $y_t \in R^m$ is a process observed in sequence over time: $t = 1, ..., n$. The time series is **univariate** if $m = 1$ and **multivariate** if $m > 1$.

Time Series - Stationarity

- **Recall** that cross-sectional observations are conventionally treated as random draws from an underlying population.
- **Instead**, we treat the observed sample $\{y_1, ...y_n\}$ as a realization of a dependent stochastic process and $\{y_1, ...y_n\} \in \{..., y_{t-1}, y_t, y_{t+1}, ...\}$.
- Since there is only one observed time series sample, in order to learn about these distributions there needs to be some sort of **constancy**.
- The most commonly assumed form of constancy is **stationarity**.

Time Series - Weak Stationarity

Def: $\{y_t\}$ is **covariance** or **weakly stationary** if the mean

$$\mu = E(y_t)$$

and variance matrix

$$\Sigma = var(y_t) = E\big((y_t - \mu)(y_t - \mu)'\big)$$

are independent of t, and the **autocovariances**

$$\Gamma(k) = cov(y_t, y_{t-k}) = E\big((y_t - \mu)(y_{t-k} - \mu)'\big)$$

are independent of t for all k.

Univariate case: the variance is written as σ^2 ; autocovariances as $\gamma(k)$.

The **autocovariances** summarize the linear dependence between y_t and its lags. A scale-free measure of linear dependence in the univariate case are the **autocorrelations**:

$$\rho(k) = corr(y_t, y_{t-k}) = \frac{cov(y_t, y_{t-k})}{\sqrt{var(y_t)var(y_{t-k})}} = \frac{\gamma(k)}{\sigma^2} = \frac{\gamma(k)}{\gamma(0)}$$

Notice by symmetry that $\rho(-k) = \rho(k)$

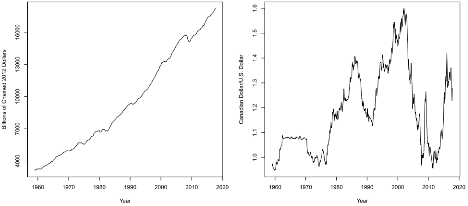
Time Series - Strict Stationarity

The definition of strictly stationary concerns the entire joint distribution.

> **Def**: $\{y_t\}$ is **strictly stationary** if the joint distribution of $(y_t, ..., y_{t+l})$ is independent of t for all l.

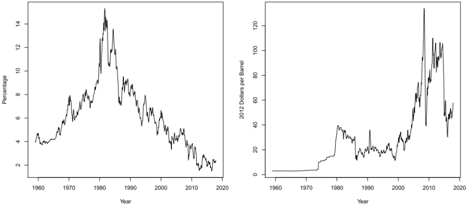
- Strict stationarity implies that the (marginal) distribution of y_t does not vary over time.
- The core meaning of both **weak and strict stationarity** is the same – that the distribution of y_t is stable over time.

Time Series - Stationarity in Figures



(a) U.S. Real GDP (b) U.S.-Canada Exchange Rate

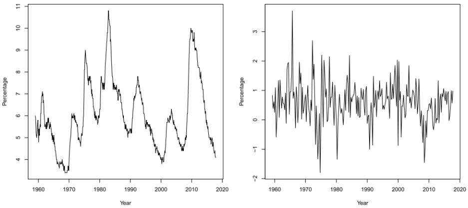
Figure 14.1: U.S. GDP and Exchange Rate



(a) Interest Rate on 10-Year Treasury (b) Real Crude Oil Price

Figure 14.2: Interest Rate and Crude Oil Price

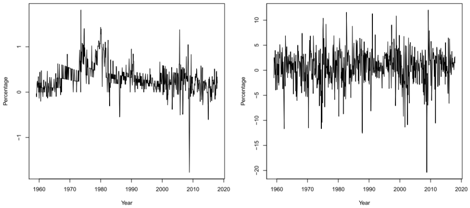
Time Series - Stationarity in Figures



(a) U.S. Unemployment Rate

(b) Consumption Growth Rate

Figure 14.3: Unemployment Rate and Cconsumption Growth Rate



(a) U.S. Inflation Rate

(b) S&P 500 Return

Figure 14.4: U.S. Inflation Rate and S&P 500 Return

Modeling Structural Breaks

- See Jupyter Notebook

Trend versus difference stationarity

A series generated by a linear trend buried in stationary noise, i.e.,

$$y_t = \alpha + \beta t + \epsilon_t \tag{1}$$

is often referred to as **trend stationary** (TS), in contrast to an I(1) process

$$\Delta y_t = \beta + \epsilon_t \tag{2}$$

which is known as **difference stationary** (DS). Accumulating the changes Δy_t from an initial value y_0 yields

$$y_t = y_0 + \beta t + \sum_{t=1}^{t} \epsilon_t \tag{3}$$

which looks superficially like (1), but has two fundamental differences. The intercept is no longer a fixed parameter but now depends upon the initial value y_0, and the error is no longer stationary, for its variances and covariances depend on time.

Trend versus difference stationarity

The distinction between TS and DS processes has important implications for the analysis of both economic growth (trends) and business cycles.

- If y_t is TS then all variation of the series is attributable to fluctuations in the cyclical component, ϵ_t, and any shock must have only a temporary effect as the series always returns to a linear growth path.
- If y_t is DS, however, its trend component must be a non-stationary stochastic process rather than a deterministic function of time, so that a shock to it will have an enduring effect on the future path of the series.

Hence treating y_t as a TS rather than a DS process is likely to lead to an overstatement of the magnitude and duration of the cyclical component and an understatement of the importance and persistence of the trend component.

Trend versus difference stationarity

We can distinguish whether an observed time series is TS or DS by computing a unit root test. The simplest form of the test takes the DS model as the null hypothesis, embodied in the regression as $\rho = 1$ and $\beta = 0$

$$y_t = \alpha + \rho y_{t-1} + \beta t + \epsilon_t$$

An equivalent way of computing the test statistic is to rewrite

$$\Delta y_t = \alpha + \phi y_{t-1} + \beta t + \epsilon_t$$

where $\phi = \rho - 1$ and then apply the usual t-ratio testing the null $\phi = 0$ against the alternative $\phi < 0$.

Unit Root and Explosive AR(1) Processes

The AR(1) process is stationary if $|\alpha| < 1$. What happens otherwise?

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \epsilon_t$$

If $\alpha_0 = 1$ and $\alpha_1 = 0$ the model is known as a random walk.

$$y_t = y_{t-1} + \epsilon_t$$

This is also called a **unit root process**, a martingale, or an integrated process. By back-substitution we find:

$$y_t = y_0 + \sum_{j=1}^{t} \epsilon_j$$

The initial condition does not disappear for large t. Consequently the series is non-stationary – meaning that y_t cannot be written as a convergent function of the infinite past history of e_t .

Dickey-Fuller Test

- See Jupyter Notebook