1.

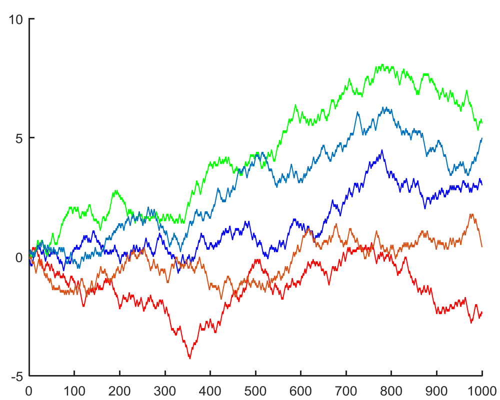
```
% problem 1
clear;
close all;
clc;

p = 0.5;
q = 1-p;
n = 1000;
trials = 10000;
s = 0.1;

x = cumsum((rand(n,trials)<p).*2*s-s);  % generate 10000 random walks. each
column represent one random walk.

figure(1);
hold on;
plot(x((1:n),1),'r');
plot(x((1:n),2),'b');
plot(x((1:n),3),'g');
plot(x((1:n),4));
plot(x((1:n),5));
hold off;

mean_sim = mean(x(n,(1:trials)))
mean_est = n*(p-q)*s
var_sim = var(x(n,(1:trials)))
var_est = 4*n*p*q*s^2
```




```
mean_sim = -0.0465
mean_est = 0
var_sim = 10.1783
var_est = 10.0000
```

2.

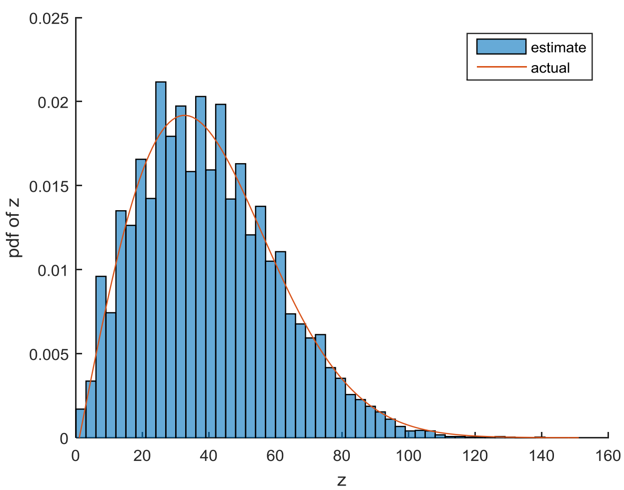
```
% problem 2
clear;
close all;
clc;

p = 0.5;
q = 1-p;
n = 1000;    % the number of steps in a random walk
T = 0.5;       % each time step length
t = n*T;
trials = 10000;
s = 1;
alpha = s^2/T;

x = cumsum((rand(n,trials)<p).*2*s-s);   % generate 10000 random walks. each
column represent one random walk
y = cumsum((rand(n,trials)<p).*2*s-s);
x_est = x(n,1:trials);       % end points of 10000 random walk
y_est = y(n,1:trials);
z_est = sqrt(x_est.^2+y_est.^2);

x_actual = 0:1:150;
z_actual = raylpdf(x_actual,sqrt(alpha*t));     % parameters: (x, std)

figure(1); hold on;
histogram(z_est,'normalization','pdf');  % estimated plot for z(rayleigh)
plot(z_actual);
legend('estimate','actual');
xlabel('z');
ylabel('pdf of z');
```



Analytical parts for 1,2

1. b) because each step in a random walk is likely to increase more than to decrease

   c) because the distribution of random walks will spread more

2. By eq(10-52) on p. 446 when $t \gg T$. (wiener process)

$$f_X(x,t) = \frac{1}{\sqrt{2\pi\alpha t}} e^{-x^2/2\alpha t} \qquad f_Y(y,t) = \frac{1}{\sqrt{2\pi\alpha t}} e^{-y^2/2\alpha t}$$

Since $Z(t) = \sqrt{X^2(t) + Y^2(t)}$, by eq(6-70) on p. 190

$$f_Z(z) = \frac{z}{\sigma^2} e^{-z^2/2\sigma^2} U(z)$$

In this case the variances of $f_X(x,t)$ and $f_Y(y,t)$ are αt instead of σ^2

Thus $f_Z(z,t) = \frac{z}{\alpha t} e^{-z^2/2\alpha t} U(z) \qquad s^2 = \alpha T \Rightarrow \alpha = \frac{s^2}{T}$

3.

3. $H_1(s) = \dfrac{1}{LCs^2 + RCs + 1}$ $\quad H_1(w) = \dfrac{1}{-LCw^2 + RCjw + 1}$

$V(t)$ is output, $n_e(t)$ is input

by eq. (9-149), (10-73), (10-74)

$$S_v(w) = S_{n_e}(w)\,|H(w)|^2 = \frac{2kTR}{(1-w^2LC)^2 + w^2R^2C^2}$$

$$|H(w)|^2 = H(w)H^*(w) = \frac{1}{\{(1-w^2LC) + RCjw\}\{(1-w^2LC) - RCjw\}} = \frac{1}{(1-w^2LC)^2 + w^2R^2C^2}$$

$Z(s) = \dfrac{Ls+R}{LCs^2+RCs+1}$ $\quad Z(jw) = \dfrac{(R+Ljw)((1-w^2LC) - RCjw)}{(-LCw^2+RCjw+1)((1-w^2LC)-RCjw)}$

$$Z(jw) = \frac{R - w^2\cancel{LCR} - R^2Cjw + Ljw - w^3L^2Cj + \cancel{RCLw^2}}{(1-w^2LC)^2 + w^2R^2C^2}$$

$$Re\ Z(jw) = \frac{R}{(1-w^2LC)^2 + w^2R^2C^2}$$

Thus, $S_v(w) = 2kT \cdot Re\ Z(jw)$ verified Nyquist Theorem (10-75)

$H_2(s) = \dfrac{1}{R+Ls}$ $\quad H_2(w) = \dfrac{1}{R+Ljw}$ $\quad |H_2(w)|^2 = \dfrac{1}{R^2+L^2w^2}$

$$S_i(w) = S_{ne}(w)\,|H_2(jw)|^2 = \frac{2kTR}{R^2+L^2w^2}$$

short circuit admittance

$Y(s) = \dfrac{1}{R+Ls}$ $\quad Y(jw) = \dfrac{1\,(R-Ljw)}{(R+Ljw)(R-Ljw)} = \dfrac{R-Ljw}{R^2+L^2w^2}$

$S_i(w) = 2kT\ Re\ Y(jw)$ Verified Nyquist (10-78)

4.

4. $P(Z_n=1)=p$, $P(Z_n=-1)=q=1-p$ $\qquad S=1$

$X_n = \sum_{i=1}^{n} Z_i$

a) $X_4 = Z_1+Z_2+Z_3+Z_4 = -2$ $\qquad P(X_4=-2) = pq^3$

b) $E[X_n] = n(p-q)$ by eq (10-55)

c) $\mathrm{Var}\, X(t) = 4npq$ "

d) $R_{XX}(n,m) = E[X_n X_m] = \sum_{i=1}^{n}\sum_{j=1}^{m} E[Z_i Z_j]$

$$= \sum_{i=1}^{n}\sum_{\substack{j=1\\ i=j}}^{m} E[Z_i Z_j] + \sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{m} E[Z_i Z_j] = \sum_{i=1}^{\min(n,m)} E[Z_i^2] + \sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{m} E[Z_i]E[Z_j]$$

$E[Z_i^2] = p(1)^2 + q(-1)^2 = p+q = 1$

$$= \boxed{\underbrace{\min(n,m)\times 1}_{\#\text{ of the occurrence } i=j} + \big(\underbrace{nm - \min(n,m)}_{\#\text{ of the occurrence } i\neq j}\big)(p-q)^2}$$

e) no, because the mean of X_n changes as n changes