774 Homework Differential Flatness

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Suppose that the equations of motion for a two wheeled mobile robot are given by

$$\dot{r}_x = v\cos\psi \tag{1}$$
$$\dot{r}_y = v\sin\psi \tag{2}$$
$$\dot{\psi} = -\psi + r \tag{3}$$
$$\dot{v} = -0.1v + a, \tag{4}$$
$$\dot{r} = \frac{1}{J_z}\tau \tag{5}$$
$$\dot{a} = \frac{1}{m}F \tag{6}$$

where τ and F are the torque and force on the robot respectively, and where $m = 1$ kg, and $J_z = 0.01\ N\ m^2$. Show that the system is differentially flat with flat output

$$y = \begin{pmatrix} r_x \\ r_y \end{pmatrix}.$$

Path planning for the flat output can be accomplished by specifying $y^d(t)$

for $t \in [0, T]$. Let

$$\begin{aligned} y^d(t) &= \begin{pmatrix} c_0 + c_1 t + c_2 t^2 + c_3 t^3 \\ d_0 + d_1 t + d_2 t^3 + c_3 t^3 \end{pmatrix} \\ &= \begin{pmatrix} 1 & t & t^2 & t^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & t & t^2 & t^3 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ d_0 \\ d_1 \\ d_2 \\ d_3 \end{pmatrix} \\ &\triangleq \Phi(t) C, \end{aligned}$$

where $\Phi(t)$ and C are suitably defined. Note that $\dot{y}^d(t) = \dot{\Phi}(t) C$, where

$$\dot{\Phi}(t) = \begin{pmatrix} 0 & 1 & 2t & 3t^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2t & 3t^2 \end{pmatrix}.$$

Higher order derivatives of the reference trajectory can be similarly defined.

Since there are 8 coefficients in C, a trajectory can be specified by 8 constraints. Accordingly, suppose that $y(0)$, $\dot{y}(0)$, $y(T)$, and $\dot{y}(T)$ are all specified, there T is a free parameter. Write a simple function that returns C where the final time T is selected so that the maximum acceleration along the trajectory is limited by

$$\|\ddot{y}(T)\| \leq A = 1 \ m/s^2.$$

Note that you will need to solve an optimization problem to find T and C. In Matlab, you could use `fmincon`.

Implement the differential flatness based controller in Simulink where the initial position and orientation, and the final position and orientation are randomly generated at the beginning of each simulation. Use a linear controller so that the robot follows the desired trajectory.