Adaptive Control for Image-Based Pointing

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April 12, 2016

1 Introduction

Image-Based gimbal control is an ongoing research project in MAGICC Lab. For target tracking application, this is an important issue because gimbal has to be controlled elaborately in order to not lose the tracks from the field of view of the camera. Reference [1] showed that a single target is trackable if the pixel location of the target is known. This approach may not be robust because motion of the platform is not accounted for. Z. Hur and M. Rez *examined* the gimbal pointing problem with a control scheme that compensates for the motion of the platform [2]. In order to use their approach, calculating the depth into optical axis of the camera term which is expressed in 'z' is essential to account for the influence of mutual translational motion of the camera and the target on the ground (see Figure 1). The 'z' term can be approximated as the distance from the camera to the ground target and this can be calculated using the geolocation method described in [1]. However, this can be an inaccurate parameter to the controller and in this paper adaptive control scheme is studied to adapt the controller for uncertain 'z' term.

2 Background

The relationship between the target's velocity in the image plane and the translational and rotational motion of the camera can be expressed as

$$\begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -\frac{\lambda}{z} & 0 & \frac{u}{z} & \frac{uw}{\lambda} & -\frac{\lambda^2+u^2}{\lambda} & w \\ 0 & -\frac{\lambda}{z} & \frac{w}{z} & \frac{\lambda^2+w^2}{\lambda} & -\frac{uw}{\lambda} & -u \end{bmatrix} \begin{bmatrix} v_{Cx} \\ v_{Cy} \\ v_{Cz} \\ \omega_{Cx} \\ \omega_{Cy} \\ \omega_{Cz} \end{bmatrix} \tag{1}$$

where $u, w, \dot{u}, \dot{w}$ are image feature location and velocity in the image plane in meters, λ is the focal length, and v_C, ω_C are the translational and rotational motion of the camera. By examning this relationship, it is clear that 'z' term is used only to calculate the influence of the translational motion of the camera. Thus, it is essential to move the camera to prove that the adaptive control presented in this paper is working. For simplicity later in the paper, the expression above can be altered as

$$\dot{s} = L_v(u, w, z)v_C + L_w(u, w)\omega_C \tag{2}$$

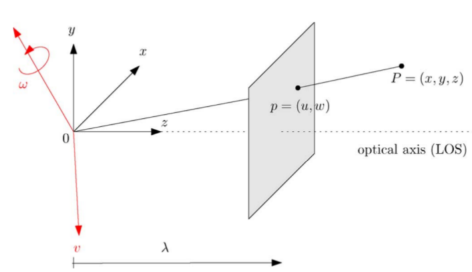

Figure 1: Coordinates of the object on the ground expressed in the camera frame

For a single target tracking case, the purpose of gimbal pointing is to maneuver the gimbal so that the target is pushed to the center of the image plane. Thus, the reference model can be looking like

$$\dot{s}^{ref}(t) = As^{ref}(t) = -\alpha I s^{ref}(t) \tag{3}$$

where $s(t) = [u, w]^T$ and α is some real positive. In this expression, if A is chosen to have negative eigenvalues, exponential stability is guaranteed.

3 Derivation of The Adaptive Controller

From equation (2), ω_C is the control input because it is a common situation to track the target with gimbal movement rather than platform movement. Thus, the equation (2) can be expressed as the following to separate the control input from other terms.

$$\begin{aligned} \dot{s} &= \frac{1}{z}\begin{bmatrix} -\lambda v_{Cx} + u v_{Cz} \\ -\lambda v_{Cy} + w v_{Cz} \end{bmatrix} + \begin{bmatrix} \frac{uw}{\lambda} & -\frac{\lambda^2+u^2}{\lambda} & w \\ \frac{\lambda^2+w^2}{\lambda} & -\frac{uw}{\lambda} & -u \end{bmatrix}\begin{bmatrix} \omega_{Cx} \\ \omega_{Cy} \\ \omega_{Cz} \end{bmatrix} \\ &= \theta\varphi + L_w U \end{aligned} \tag{4}$$

where U is the control input and θ is the uncertain parameter. The expression (4) now is in more convenient form to develop an adaptive controller than the expression (1). In the expression (4), if it is supposed that θ is known, the ideal control input would look like

$$U = L_w^{\#}(-\theta\varphi - K_s s) \tag{5}$$

where $L_w^{\#}$ is the right pseudoinverse of L_w and the subsequent dynamics would become like

$$\dot{s} = -K_s s$$

which in this case if K_s is set to be the same as α in (3), the close-loop response will act exactly same as the reference model. However, since θ is unknown and needs to be adapted, more realistic control input would look like

$$U = L_w^{\#}(-\hat{\theta}\varphi - K_s s) \tag{6}$$

where $\hat{\theta}$ is the estimate of θ. By plugging (6) into (4), the resulting dynamics would be

$$\begin{aligned} \dot{s} &= \theta\varphi - \hat{\theta}\varphi - K_s s \\ &= -\Delta\theta\varphi - K_s s, \ (\Delta\theta = \hat{\theta} - \theta) \\ &= -\Delta\theta\varphi + As \end{aligned}$$

Let the error defined as

$$\begin{aligned} e &= s - s^{ref} \\ \dot{e} &= \dot{s} - \dot{s}^{ref} \\ &= -\Delta\theta\varphi + As - As^{ref} \\ &= -\Delta\theta\varphi + Ae \end{aligned}$$

Using the variables of interest, a Lyapunov function candidate can be constructed as

$$\begin{aligned} V &= \frac{1}{2}e^T e + \frac{1}{2\gamma_\theta}\Delta\theta^2 \\ \dot{V} &= e^T\dot{e} + \frac{1}{\gamma_\theta}\Delta\theta\Delta\dot{\theta} \\ &= e^T(-\Delta\theta\varphi + Ae) + \frac{1}{\gamma_\theta}\Delta\theta\dot{\hat{\theta}}, \ (\dot{\theta} = 0) \\ &= e^T Ae + \Delta\theta\left(\frac{\dot{\hat{\theta}}}{\gamma_\theta} - e^T\varphi\right) \end{aligned}$$

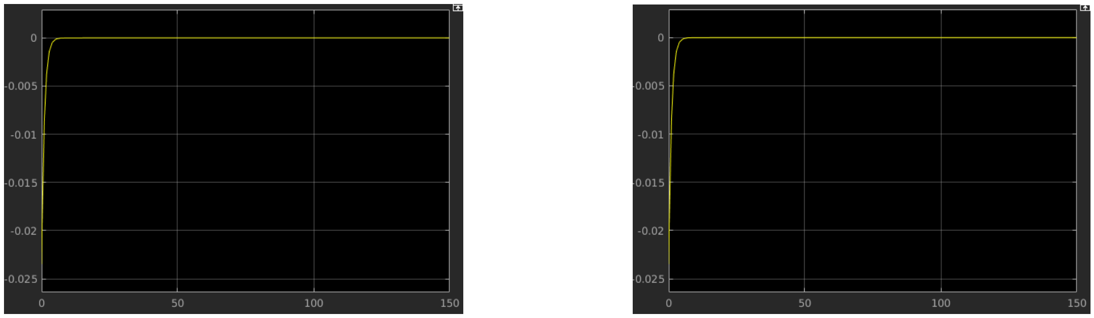

Figure 2: Behavior of the reference model (u^{ref}- left, w^{ref}- right)

Since $\dot{\hat{\theta}}$ is a design parameter, its value can be picked as

$$\dot{\hat{\theta}} = \gamma_\theta e^T \varphi$$

Then,

$$\dot{V} = e^T A e \leq 0 \tag{7}$$

So far, it is still not guaranteed that error would go to zero. By Barbalat's Lemma, asymptotic statility can be proved.

$$\begin{aligned} \ddot{V} &= \dot{e}^T A e + e^T A \dot{e} \\ &= 2e^T A \dot{e} \\ &= 2e^T A(-\Delta\theta\varphi + Ae) \end{aligned} \tag{8}$$

If $\ddot{V}$ is bounded, it is inferred that $\dot{V} = e^T A e \to 0$ as $t \to \infty$. Thus, e, $\Delta\theta$, φ need to be bounded. Since φ is composed of the focal length, target location in the image plane, and the velocity of the platform, it must be bounded. Also, from the equation (7),

$$V(t) \leq V(0)$$

This implies that e and $\Delta\theta$ are bounded because V is bounded. In conclusion, by Barbalat's Lemma $e \to 0$ which means that the target location in the image plane is pushed to the image center.

4 Simulation

First, the behavior of the reference model was confirmed. Suppose that the size of the image plane is square with 30cm for each side and the target is initially located at any random point in this square. As shown in Figure 2, any other trials will show the same result which is the desired behavior of the reference model, converging to the origin of the image plane.

Initial condition of the actual system is also selected randomly to a different location from that of the reference model. The goal is to see the actual system also converges to the origin of the image plane while adapting 'z' term to its actual value. In reality, 'z' is varying term depending on the location of the platform and the target, but in this simulation it is set to any random value between 50 to 150. As mentioned above, in order to see 'z' term being adapted v_C need to hold some values which means the platform is moving. For this reason, v_{Cx}, v_{Cy} are set to random velocities between 0 and 20 meter per second. However, v_{Cz} is kept to be 0 with an assumption that the platform is not flying into or away from the target. Figure 3 shows the block diagram of the whole system.

For the comparison purpose, the ideal controller from the equation (5) was simulated. Figure 4 shows that with the ideal controller the gimbal is actually pointing to the target by locating it to the center in the image plane.

The system response using adaptive controller is shown in Figure 5. From the result, the performance of the adaptive controller is not as good as the ideal controller, but it still seems to be satisfying to fulfill the objective. Also, it is clear that θ or $\frac{1}{z}$ is initially unknown to the controller but adapting to its true value.

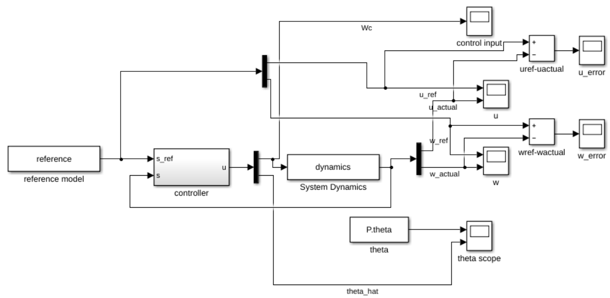

Figure 3: Block diagram of the whole system

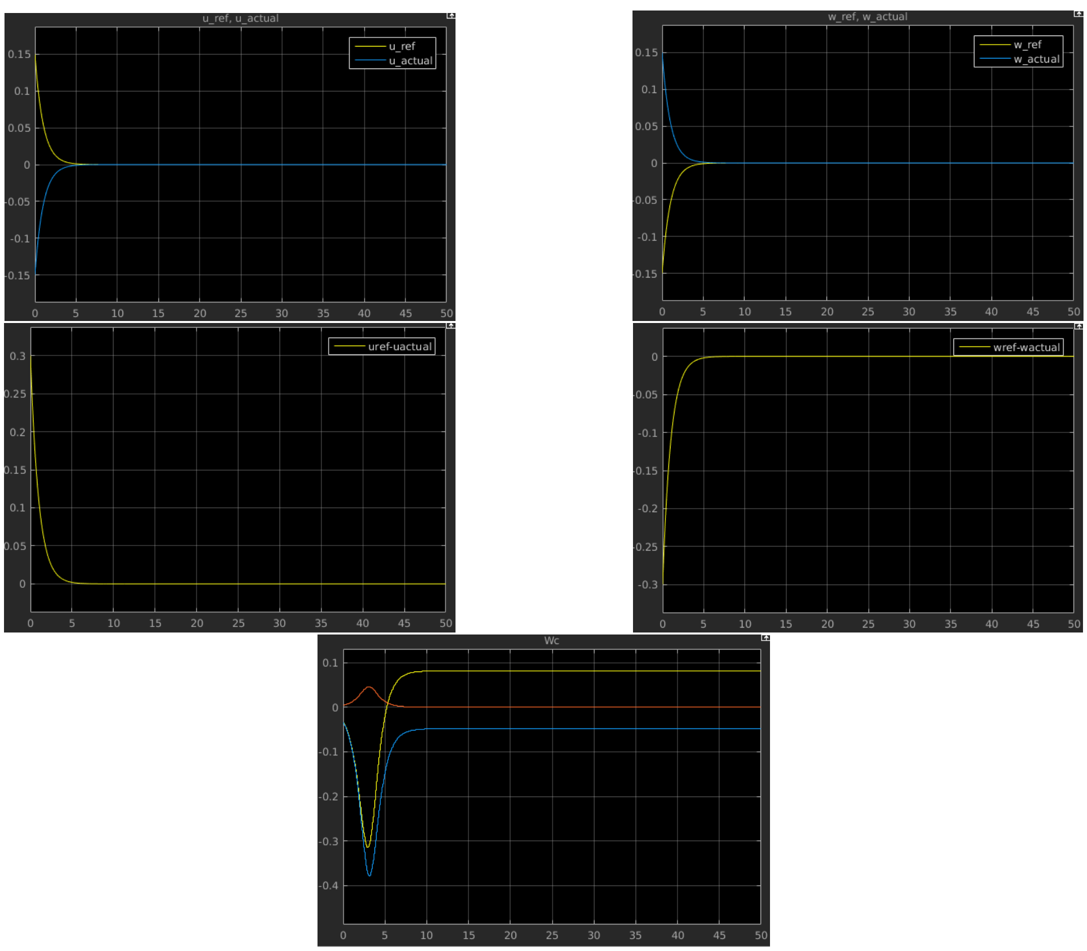

Figure 4: Response of the ideal controller and control effort

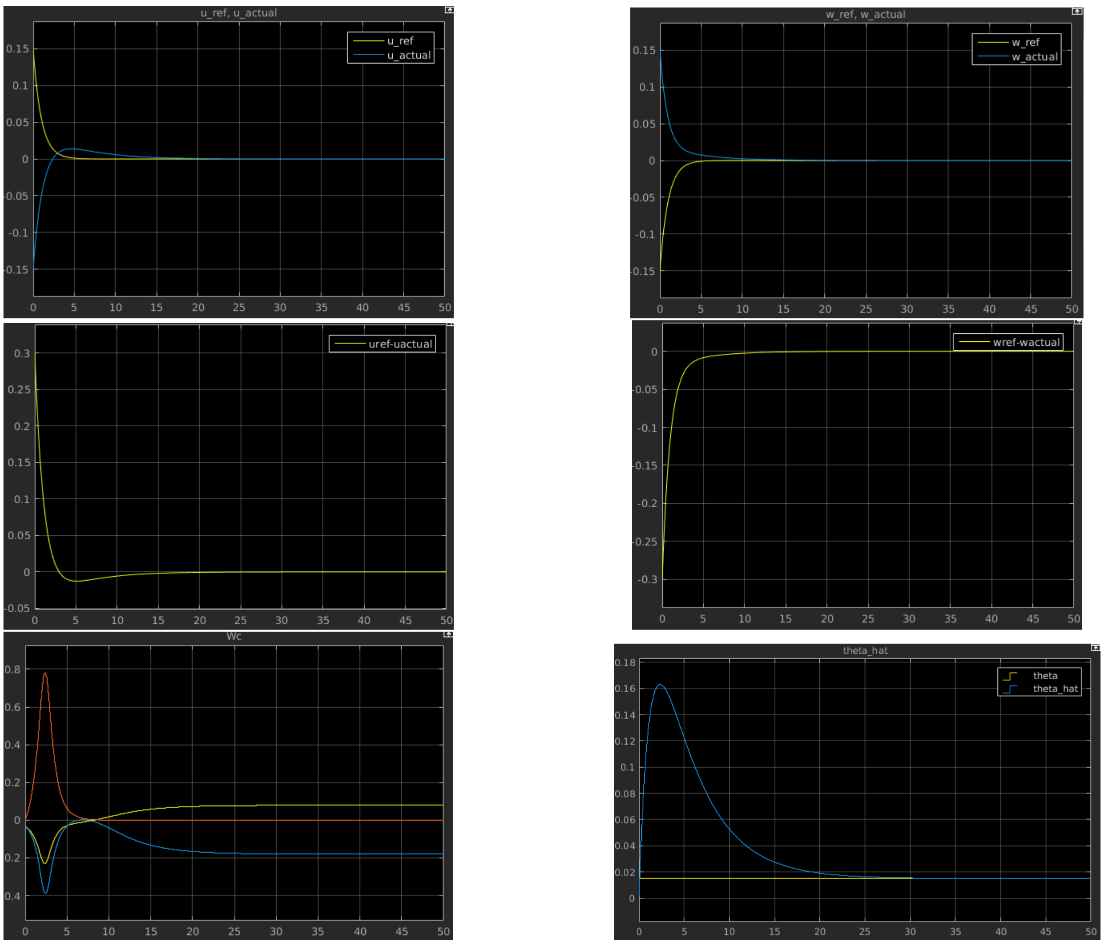

Figure 5: Response of the adaptive controller, control effort, and adapting variable θ

5 Conclusion and Future Work

In conclusion, model reference adaptive control (MRAC) can be used in various applications such as image-based gimbal pointing and can work quite well. The future work of this project is to apply the adpative control derived here into the multi-rotor tracking simulation. The tracking simulation uses a pan-tilt camera which removes ω_{Cx} from control input and this needs to be accounted for in the future work.

References

[1] R. Beard and T. McLain, "Vision-guided Navigation," in *Small Unmanned Aircraft*. Princeton, NJ: Princeton University Press, 2012, ch. 13, pp. 229-234.

[2] Z. Hur and M. Rez, "Image-Based Pointing and Tracking for Inertially Stabilized Airborne Camera Platform," *IEEE Trans. Control Syst. Technol*, vol. 20, no. 5, pp. 1146-1159, Sep. 2012.