MeEn 537 Homework #2 Solution

2-38

By inspection, we can write the following by picking the axes in the frame we are transforming from and writing them as a column in the components of the axes we are transforming to. So for the first one, this looks like this:

$$H_1^0 = \begin{bmatrix} x_1^0 & y_1^0 & z_1^0 & o_1^0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Accordingly, we can get the others by inspection as well:

$$H_2^0 = \begin{bmatrix} x_2^0 & y_2^0 & z_2^0 & o_2^0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} x_2^1 & y_2^1 & z_2^1 & o_2^1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiplying H_0^1 by H_2^1 results in H_2^0.

2-39 – find $H_1^0, H_2^0, H_3^0, H_3^2$

by inspection – we can assume $H = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$, then we can write "$t$" as the distance to the frame, but in the frame of interest. the same goes for the columns of R =>

$$R = \left[\, x_{old}^{new} \;\vdots\; y_{old}^{new} \;\vdots\; z_{old}^{new} \,\right]$$

$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1^0 & y_1^0 & z_1^0 & t_{01}^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{array}{c} \begin{array}{cccc} x_3^0 & y_3^0 & z_3^0 & \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$H_3^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

could also have been H_2^3 since the problem isn't clear.

2-40 find: H_3^0 & H_3^2 after rotating camera $90°$ about z_3 axis

- can find in two ways, either multiply by a $Rot_{z,90°}$ or just rotate the frame & follow the same procedure as in 2-39

$$\Downarrow$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

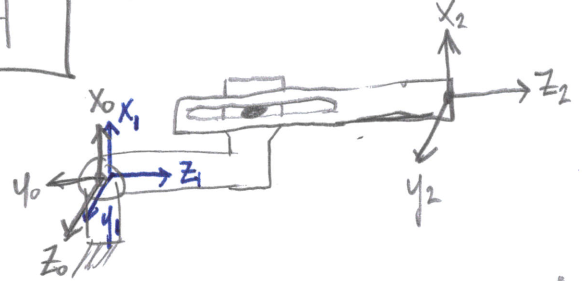
$$H_3^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & -1 & 0 & 1.5 \\ 0 & 0 & -1 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2-43

The first and second matrices commute because the translation is happening along the same axis as the rotation. This is the same for the third and fourth matrices which also commute. The second and third will commute because although they translate about different axes, there is no rotation between them which means the ordering does not matter.

3-4



DH params:

θ	d	a	α
θ_1^*	0	0	$\frac{\pi}{2}$
0	$d_2^* + d_{off}$	a_2	0

$*$ – joint variables

d_{off} = constant offset

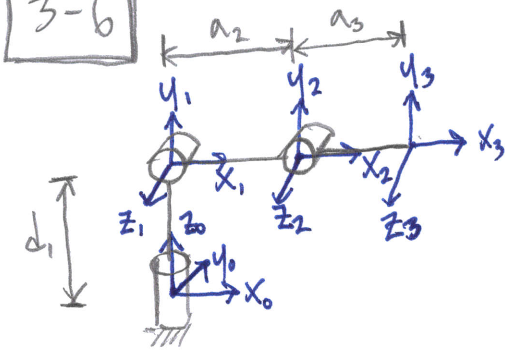
$T_2^0 = A_1 A_2$, from eqn 3.10 $\Rightarrow$

$$A_1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 + d_{off} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_2^0 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & a_2 C_{\theta_1} + (d_2 + d_{off}) S_{\theta_1} \\ S_{\theta_1} & 0 & -C_{\theta_1} & a_2 S_{\theta_1} - (d_2 + d_{off}) C_{\theta_1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

3-6



DH params:

θ	d	a	α
θ_1	d_1	0	$\frac{\pi}{2}$
θ_2	0	a_2	0
θ_3	0	a_3	0

$T_3^0 = A_1 A_2 A_3$, from eqn 3.10 $\Rightarrow$

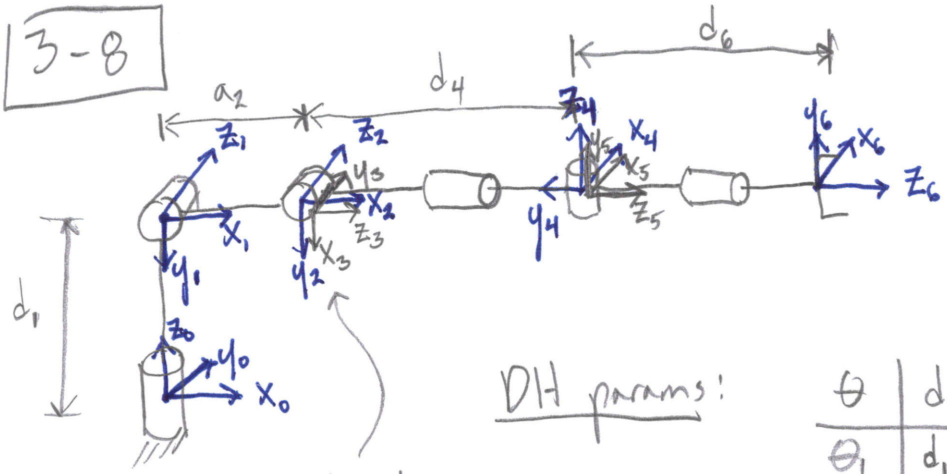
$$A_1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & a_2 C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & a_2 S_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & a_3 C_{\theta_3} \\ S_{\theta_3} & C_{\theta_3} & 0 & a_3 S_{\theta_3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

I don't care if you multiply these together or not.
You should at least be able to do 2 like in problem 3-4.

3-8



DH params:

θ	d	a	α
θ_1	d_1	0	$-\frac{\pi}{2}$
θ_2	0	a_2	0
$\theta_3+\frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
$\theta_4+\frac{\pi}{2}$	d_4	0	$-\frac{\pi}{2}$
θ_5	0	0	$\frac{\pi}{2}$
θ_6	d_6	0	0

$T_6^0 = A_1 A_2 A_3 A_4 A_5 A_6$ — I will assume that you can now evaluate eqn 3.10 with the above parameters.

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for software part, let each link length be 20 cm $\Rightarrow$

$d_1 = 0.2\,m$

$a_2 = 0.2\,m$

$d_4 = 0.4\,m$

$d_6 = 0.4\,m$

now see attached code and plot for approximation of workspace.

MATLAB code is included on Learning Suite. Here is a figure representing a quarter of the workspace assuming each joint has plus and minus $\pi/2$ for the joint limits.