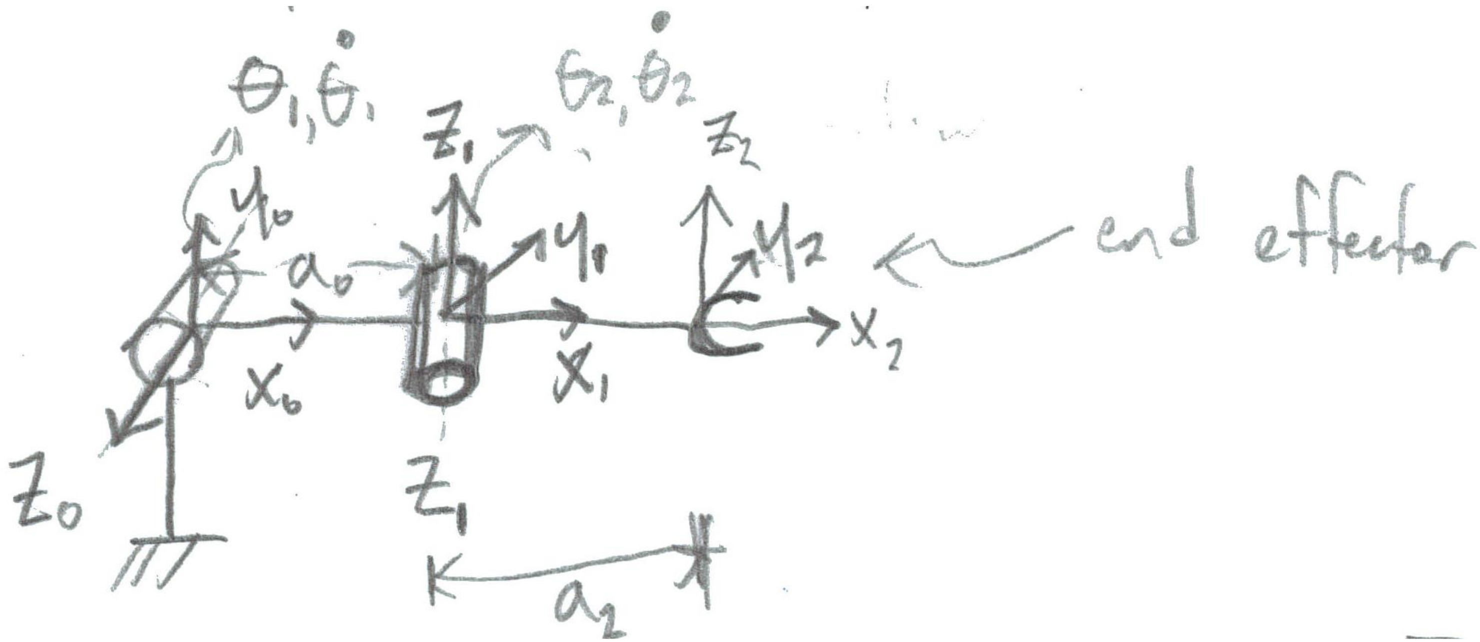
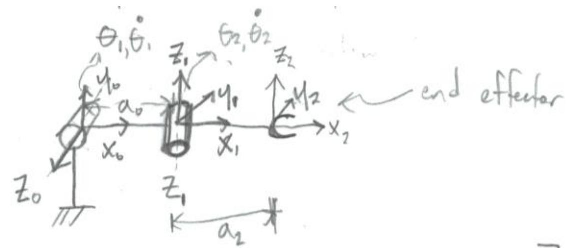
Jacobian - 2 Link Example



Jacobian Example (two link)



θ_i	d_i	a_i	α
θ_1	0	a_1	$-\pi/2$
θ_2	0	a_2	0

$$J(q) = \begin{bmatrix} z_0^0 \times (o_2^0 - o_0^0) & z_1^0 \times (o_2 - o_1) \\ z_0^0 & z_1^0 \end{bmatrix}$$

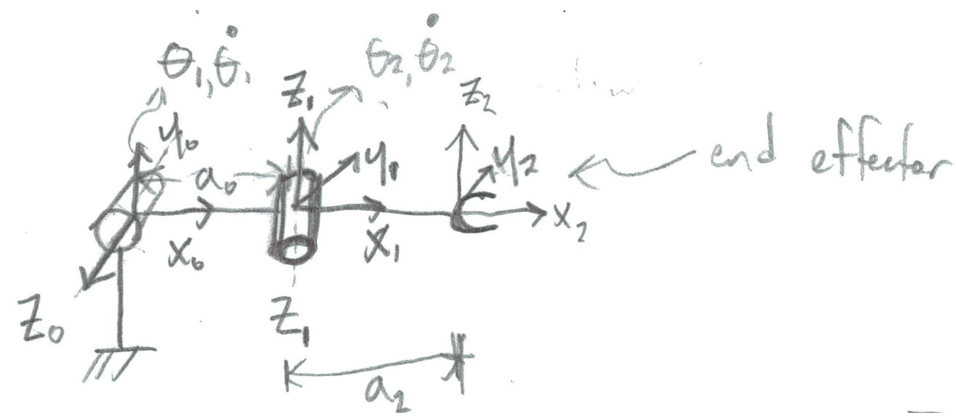
from eqn 3.10

$$A_1 = \begin{bmatrix} C_{\theta_1} & -S_{\theta_1} C_{\alpha_1} & S_{\theta_1} S_{\alpha_1} & a_1 C_{\theta_1} \\ S_{\theta_1} & C_{\theta_1} C_{\alpha_1} & -C_{\theta_1} S_{\alpha_1} & a_1 S_{\theta_1} \\ 0 & S_{\alpha_1} & C_{\alpha_1} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{\theta_1} & 0 & -S_{\theta_1} & a_1 C_{\theta_1} \\ S_{\theta_1} & 0 & C_{\theta_1} & a_1 S_{\theta_1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} C_{\alpha_2} & S_{\theta_2} S_{\alpha_2} & a_2 C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} C_{\alpha_2} & -C_{\theta_2} S_{\alpha_2} & a_2 S_{\theta_2} \\ 0 & S_{\alpha_2} & C_{\alpha_2} & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & a_2 C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & a_2 S_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

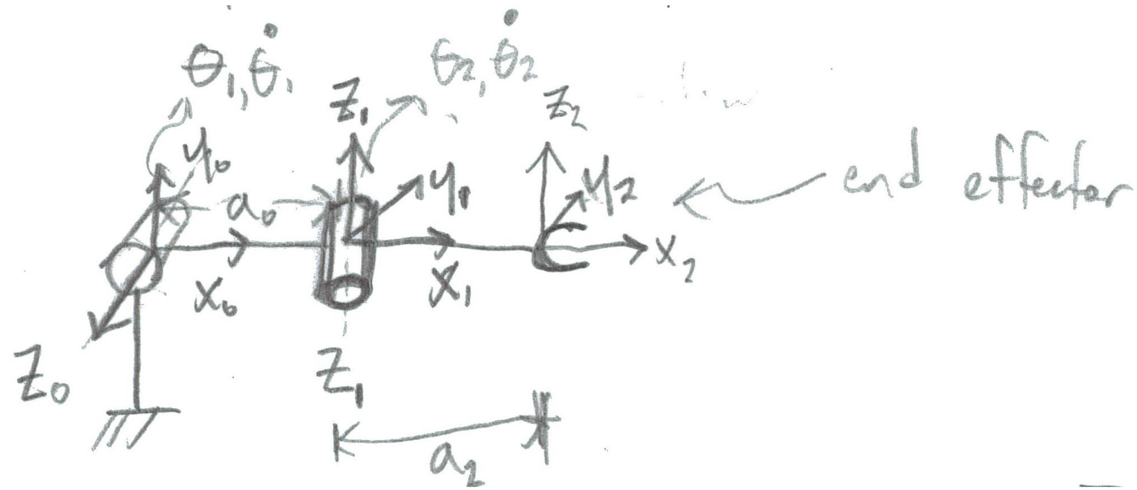
$$T_1^0 = A_1$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} C_{\theta_1} C_{\theta_2} & -S_{\theta_2} C_{\theta_1} & -S_{\theta_1} & a_2 C_{\theta_1} C_{\theta_2} + a_1 C_{\theta_1} \\ S_{\theta_1} C_{\theta_2} & -S_{\theta_1} S_{\theta_2} & C_{\theta_1} & a_2 S_{\theta_1} C_{\theta_2} + a_1 S_{\theta_1} \\ -S_{\theta} & -C_{\theta_2} & 0 & -a_2 S_{\theta_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$J(q) = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} (a_2 c_{\theta_1} c_{\theta_2} + a_1 c_{\theta_1} - 0) \\ (a_2 s_{\theta_1} c_{\theta_2} + a_1 s_{\theta_1} - 0) \\ (-a_2 s_{\theta_2} - 0) \end{bmatrix} & \begin{bmatrix} -s_{\theta_1} \\ c_{\theta_1} \\ 0 \end{bmatrix} \times \begin{bmatrix} (a_2 c_{\theta_1} c_{\theta_2} + a_1 c_{\theta_1} - a_1 c_{\theta_1}) \\ (a_2 s_{\theta_1} c_{\theta_2} + a_1 s_{\theta_1} - a_1 s_{\theta_1}) \\ (-a_2 s_{\theta_2} - 0) \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} -s_{\theta_1} \\ c_{\theta_1} \\ 0 \end{bmatrix} \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \begin{bmatrix} -a_2 s_{\theta_1} c_{\theta_2} - a_1 s_{\theta_1} \\ a_2 c_{\theta_1} c_{\theta_2} + a_1 c_{\theta_1} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} -a_2 s_{\theta_2} c_{\theta_1} \\ -a_2 s_{\theta_2} s_{\theta_1} \\ -a_2 s_{\theta_1} s_{\theta_1} c_{\theta_2} - a_2 c_{\theta_1} c_{\theta_1} c_{\theta_2} \\ -s_{\theta_1} \\ c_{\theta_1} \\ 0 \end{bmatrix} \end{bmatrix}$$



$$
\begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix} = \left[\begin{bmatrix} -a_2 s_{\theta_1} c_{\theta_2} - a_1 s_{\theta_1} \\ a_2 c_{\theta_1} c_{\theta_2} + a_1 c_{\theta_1} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -a_2 s_{\theta_2} c_{\theta_1} \\ -a_2 s_{\theta_2} s_{\theta_1} \\ -a_2 s_{\theta_1} s_{\theta_1} c_{\theta_2} - a_2 c_{\theta_1} c_{\theta_1} c_{\theta_2} \\ -s_{\theta_1} \\ c_{\theta_1} \\ 0 \end{bmatrix} \right] \dot{q}
$$