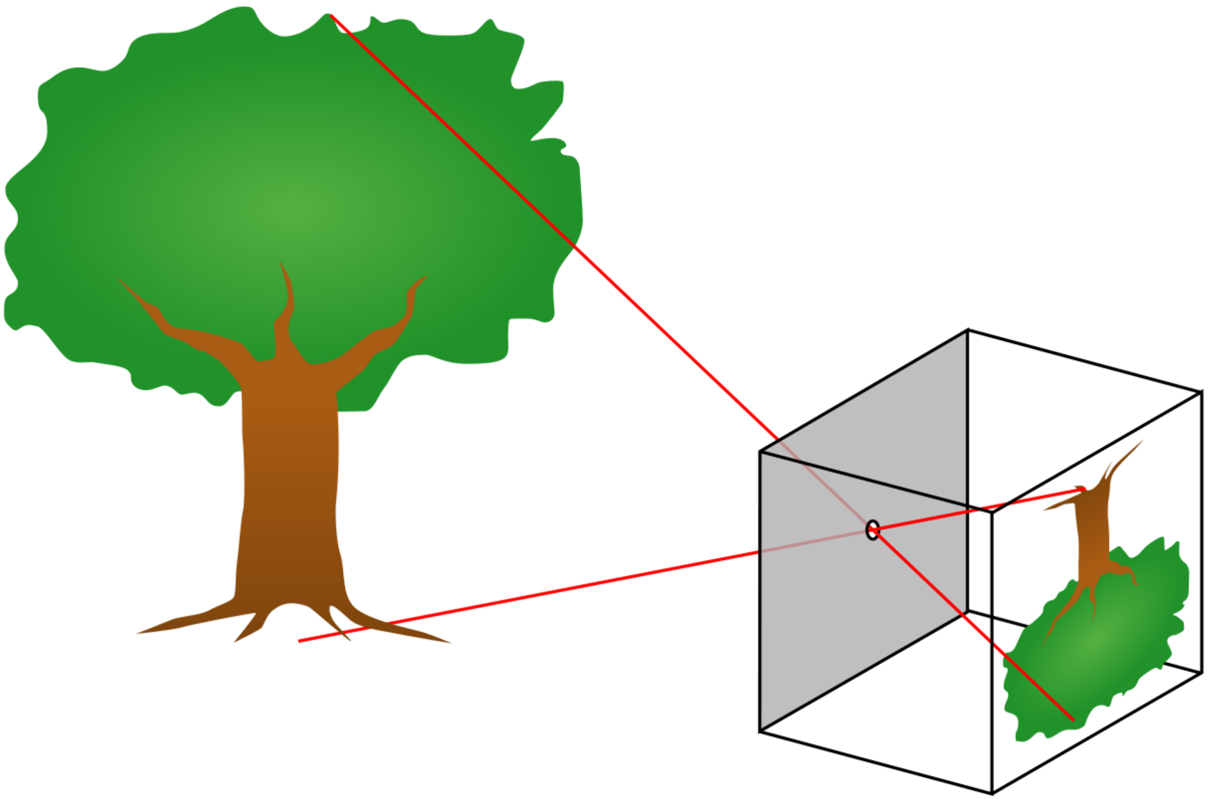
Cameras

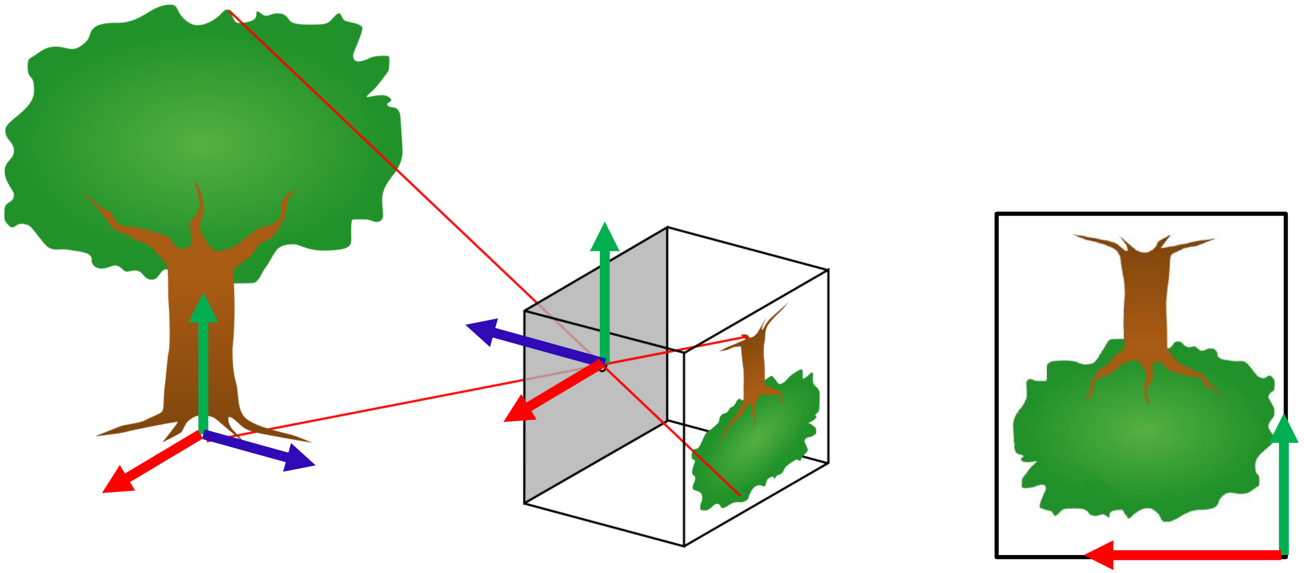
Pinhole camera model



Example of real world pinhole camera

- http://ngm.nationalgeographic.com/2011/05/camera-obscura/oneill-text

Measurement Process for Cameras



$$(X, Y, Z) \Rightarrow (x_p, y_p, z_p) \Rightarrow (u, v)$$

where x,y and z are scaled by the focal length f to get the values in pixel units

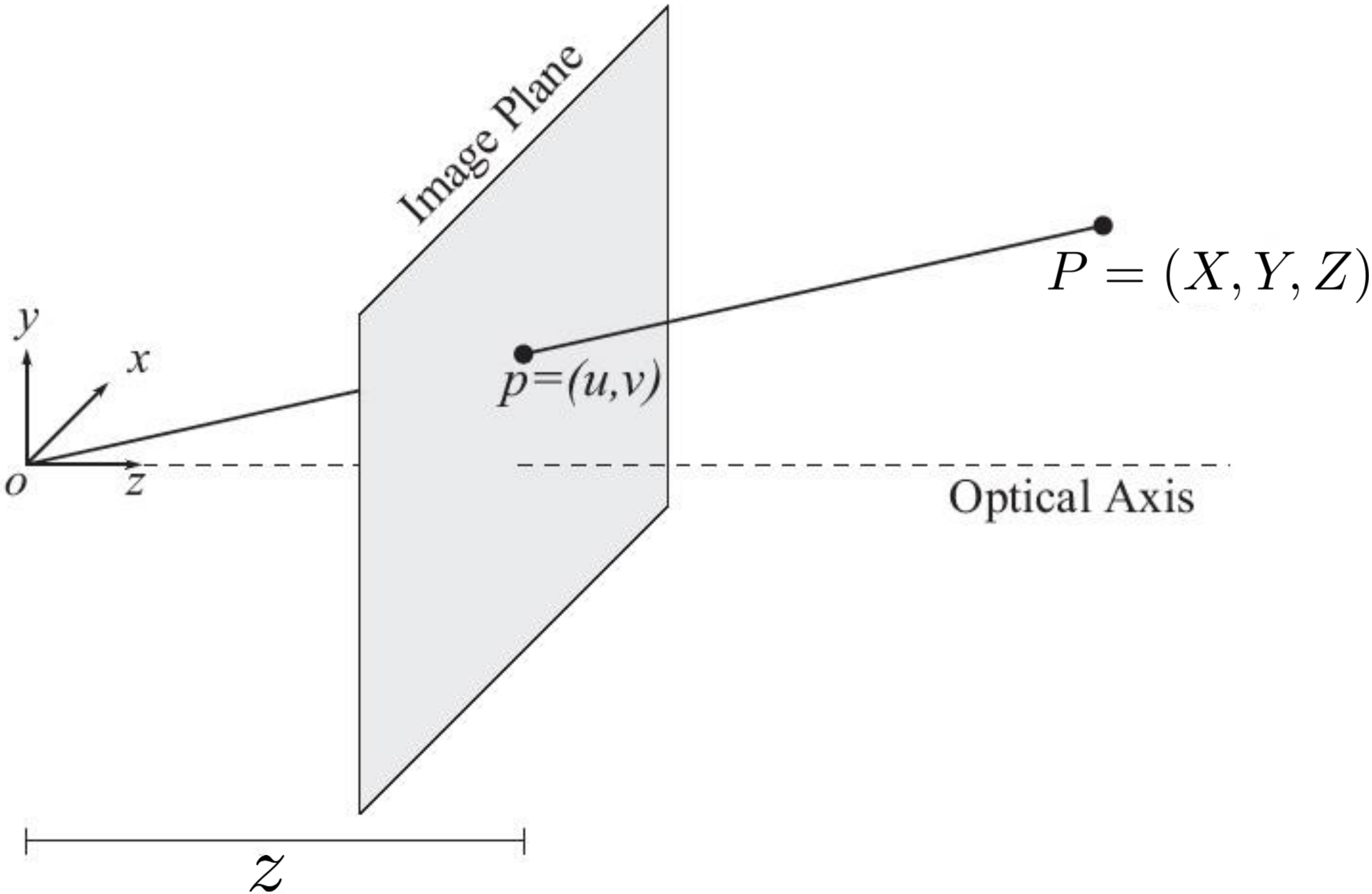


Figure 11.1: Camera coordinate frame.

Calibration

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} [\boldsymbol{R}\ \boldsymbol{t}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} &= \boldsymbol{A}\ [\boldsymbol{R}\ \boldsymbol{t}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \\ u &= \frac{x_p}{z_p} \\ v &= \frac{y_p}{z_p} \end{aligned}$$

Calibration

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \boldsymbol{A}\,[\boldsymbol{R}\ \boldsymbol{t}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Remembering the following =>

$$u = \frac{x_p}{z_p}$$
$$v = \frac{y_p}{z_p}$$

We can say this:

$$u = \frac{M_{11}X + M_{12}Y + M_{13}Z + M_{14}}{M_{31}X + M_{32}Y + M_{33}Z + M_{34}}$$

$$v = \frac{M_{21}X + M_{22}Y + M_{23}Z + M_{24}}{M_{31}X + M_{32}Y + M_{33}Z + M_{34}}$$

Calibration

Manipulation of equations gives us:

$$u(M_{31}X + M_{32}Y + M_{33}Z + M_{34}) = M_{11}X + M_{12}Y + M_{13}Z + M_{14} \quad =>$$

$$uM_{34} = M_{11}X + M_{12}Y + M_{13}Z + M_{14} - uM_{31}X - uM_{32}Y - uM_{33}Z$$

Dividing through by M_{34} **gives the following:**

$$u = M_{11}X + M_{12}Y + M_{13}Z + M_{14} - uM_{31}X - uM_{32}Y - uM_{33}Z$$

Can follow the exact same steps to solve for v to get this:

$$v = M_{21}X + M_{22}Y + M_{23}Z + M_{24} - vM_{31}X - vM_{32}Y - vM_{33}Z$$

Calibration

Which we can write in matrix form like this:

$$\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -uX & -uY & -uZ \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -vX & -vY & -vZ \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \\ M_{21} \\ M_{22} \\ M_{23} \\ M_{24} \\ M_{31} \\ M_{32} \\ M_{33} \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

If we are calibrating, what is known? What is unknown?

For a given point on a known object, we know (X, Y, Z) and the corresponding pixel coordinate (u, v), but want to find the M's.

Calibration

Which we can repeat for N different known real-world points as follows:

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -u_2X_2 & -u_2Y_2 & -u_2Z_2 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -v_2X_2 & -v_2Y_2 & -v_2Z_2 \\ & & & & & & & \vdots & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \\ M_{21} \\ M_{22} \\ M_{23} \\ M_{24} \\ M_{31} \\ M_{32} \\ M_{33} \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ u_N \\ v_N \end{bmatrix}$$

Now we can solve for M using least squares techniques (pseudo-inverse or SVD)

Resources for working with camera images and robotics

- http://petercorke.com/Machine_Vision_Toolbox.html - toolbox that you'll need for your homework

- https://www.vision.caltech.edu/bouguetj/calib_doc/ - toolbox for finding a camera's intrinsic and extrinsic camera parmeters

Common Ways of Getting Real-time 3D Data

- Could also use these instead of a single camera to possibly make the tracking and control problem easier or more robust.

Stereo Vision – Two cameras calibrated in a very specific way

Structured Light– Sending out a light pattern (often IR light) and using that to get 3D

Time of Flight– Sending out a laser or sound signal and measuring the time it takes to come back