MeEn 537 Homework #1 Solution

1-2

- **Forward Kinematics:** Determines position and orientation of the end effector in terms of the joint variables
- **Inverse Kinematics:** Determines joint variables that will give a specified end effector position/orientation or pose
- **Trajectory Planning:** Planning the time history of the joint variables necessary for the robot to execute a given task
- **Workspace:** The total volume swept out by the end-effector as the manipulator exectues all possible motions
- **Accuracy:** A measure of how close the manipulator can come to a given point within its workspace
- **Repeatability:** A measure of how close a manipulator can return to a previously commanded point
- **Resolution:** The smallest increment of motion that can be sensed or controlled. The resolution is a function of the distance travelled and the number of bits of encoded accuracy.
- **Joint Variables:** The relative displacement between adjacent links, denoted θ_i for revolute joints and d_i for prismatic joints.
- **Spherical Wrist:** RRR wrist configuration with joint axes intersecting at a common point.
- **End Effector:** A gripper or tool used to perform the robot's task

1-8

1. handling fragile objects (glass, eggs, etc.)
2. grinding
3. assembly (peg-in-hole problem)
4. defusing explosives
5. human-robot ineraction for assistance

1-12

Any application that requires dexterous manipulation within the robot's reachable workspace. This can include reaching around or behind objects, assembly of complex parts, interaction with humans, artificial limbs.

1-14

Resolution can be defined as:

$$\frac{l\theta}{2^n} = \frac{(50cm)(\pi)}{2^8} = 0.6136cm$$

1-16

This is because the end-effector or tool tip is not measured directly but is calculated with the encoder measurements for joint position and the forward kinematic model. This means that accuracy is affected by measurement errors, computational errors, machining accuracy of parts, etc.

1-20

If the two links are of equal length, then the origin (or x=0 and y=0) can be reached by infinitely many configurations by setting $\theta_2 = 180°$ and letting θ_1 be any arbitrary number.

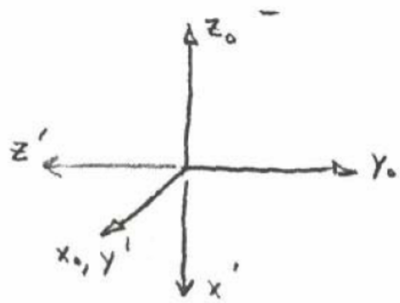
1-21

Controlling a distal link with a large mass will require more torque from the motors for all previous joints. Additionally, momenutm is mass times velocity, so additional mass at the end effector makes it harder to control position when moving quickly. Finally, when using a robot with compliance at the joints, having less mass at the distal end can protect a person from accidental contact at the end-effector. Two major ways to reduce mass of the distal links include by mounting motors or drive trains on previous links (or even the base in some cases) and by using less or lighter material for the links.

2-10 - $R = R_{y,\psi} R_{x,\phi} R_{z,\theta}$
2-11 - $R = R_{z,\theta} R_{x,\phi} R_{x,\psi}$
2-12 - $R = R_{z,\alpha} R_{x,\phi} R_{z,\theta} R_{x,\psi}$
2-13 - $R = R_{z,\alpha} R_{z,\theta} R_{x,\phi} R_{x,\psi}$
2-14 -

2-14

$$R = R_{y,\frac{\pi}{2}} R_{x,\frac{\pi}{2}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



2-15 -

$$R_3^2 = R_1^2 R_3^1$$

given that we know that $R_1^2 = (R_2^1)^T$, we can show the following:

$$R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$$

2-22 -

2-22

$R_{x,\theta} R_{y,\phi} R_{z,\pi} R_{y,-\phi} R_{x,-\theta}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\phi) & 0 & -\sin(\phi) \\ 0 & 1 & 0 \\ \sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} -\cos(2\phi) & -2\cos(\phi)\sin(\phi)\sin(\theta) & \cos(\theta)\sin(2\phi) \\ -2\cos(\phi)\sin(\phi)\sin(\theta) & -\cos(\theta)^2 - \cos(2\phi)\sin(\theta)^2 & -\cos(\phi)^2\sin(2\theta) \\ \cos(\theta)\sin(2\phi) & -\cos(\phi)^2\sin(2\theta) & \cos(\phi)^2\cos(\theta)^2 - \cos(\theta)^2\sin(\phi)^2 - \sin(\theta)^2 \end{bmatrix}$$

2-23 -

2-23

$$R = R_{y,90}R_{z,45} \quad = \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

$$\theta \quad = \quad \cos^{-1}\left(\frac{Tr(R)-1}{2}\right) = \cos^{1}\left(\frac{\frac{\sqrt{2}}{2}-1}{2}\right) = 98.42^\circ$$

2-24 -

The answer for ZYZ Euler angles rotated by $\frac{\pi}{2}$, 0, and $\frac{\pi}{4}$ is the following:

$$R_1^0 = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the direction of the x-axis is just the first column of the rotation matrix$[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]^T$.