Jacobian (Kinematic) Singularities

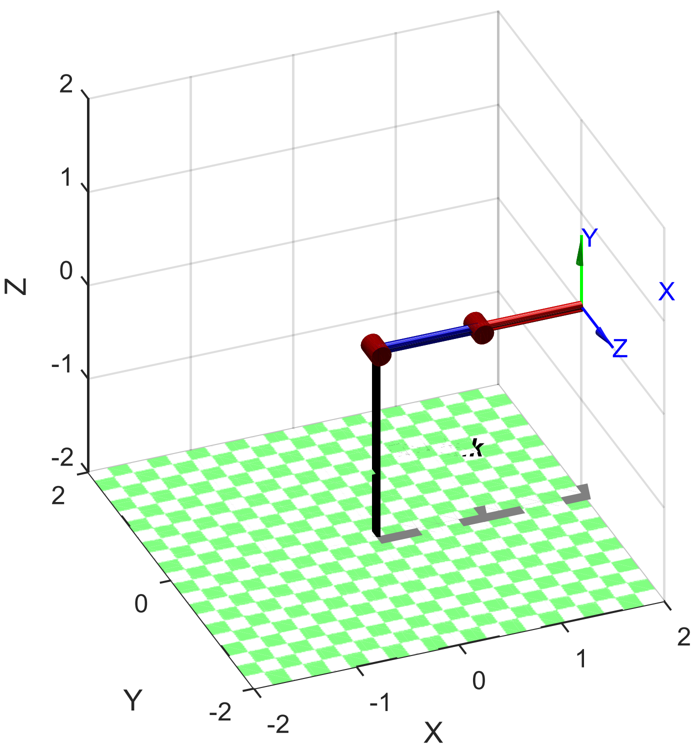
$$\xi = J(q)\dot{q}$$

- Configurations of an arm where motion of the structure is limited or reduced (i.e. can’t move the end-effector in an arbitrary direction) and Jacobian may decrease in rank
  - Remember that “rank” is the minimum between the number of linearly independent rows and columns
  - Can look at singular values (or eigenvalues if square) and compare biggest and smallest to understand if near singular configuration.
- When at a singularity, infinite solutions to the inverse kinematics problem may exist
- Near a singularity, small velocities in the operational space may cause large velocities in joint space (or in other words, may require large velocities in the joint space)
- Two types:
  - Boundary singularities
  - Internal singularities

Jacobian (Kinematic) Singularities

Boundary Singularities

- Occur when manipulator is outstretched or retracted and near boundaries of its workspace
- Can be avoided by not commanding the arm to go to boundaries

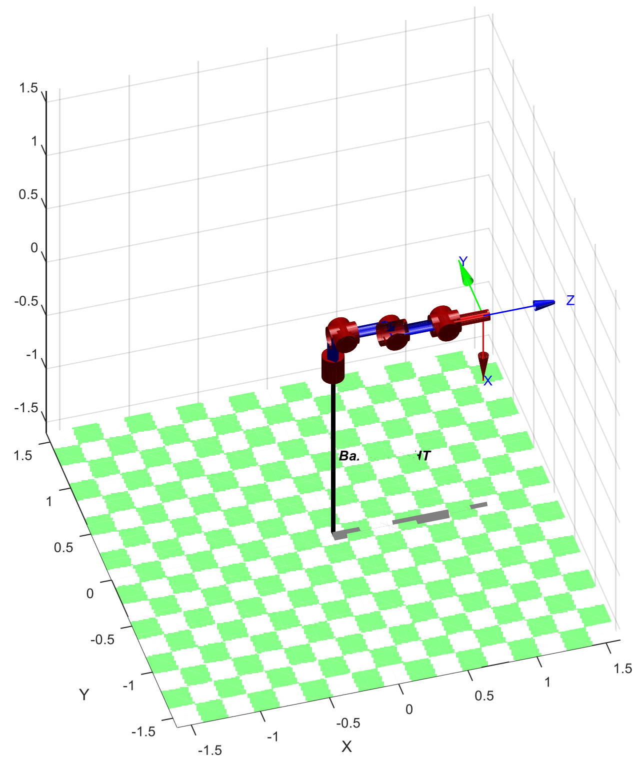


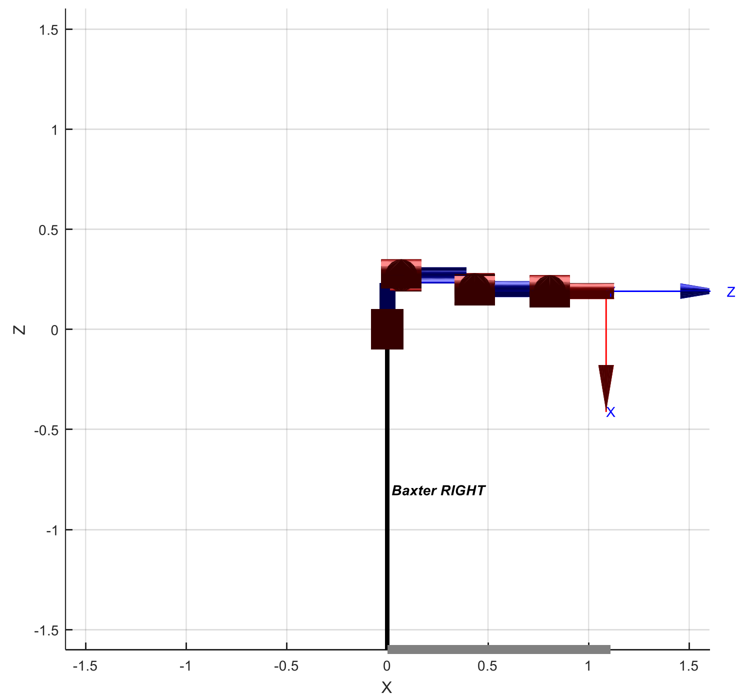
$$J(q) = \begin{matrix} 0 & 0 \\ 0.0000 & 0.0000 \\ 2.0000 & 1.0000 \\ 0 & 0 \\ -1.0000 & -1.0000 \\ 0.0000 & 0.0000 \end{matrix}$$

Jacobian (Kinematic) Singularities

Internal Singularities

- Occurs inside the reachable workspace
- Generally caused by the alignment of two or more axes of motion or specific end-effector poses
- More serious than boundary singularities
- See MATLAB example





Jacobian (Kinematic) Singularities

Take Aways:

- Be aware of singularities for a given manipulator and avoid controlling near those configurations (at least with certain methods that use the inverse of the Jacobian)
- No general purpose way (or analytical solution) to find when the Jacobian will be singular unless can decouple the arm like is done on pages 142-148 of the book (requires a spherical wrist)
- Also affects directions in which you can apply force (although not in the same way as velocity)

Applying Force and Rank of J(q)

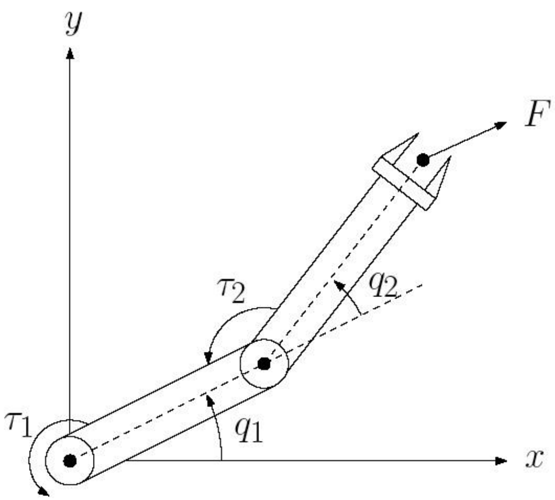


Figure 4.11: Two-link planar robot.

$$\tau = J^T F$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & a_1 c_1 + a_2 c_{12} & 0 & 0 & 0 & 1 \\ -a_2 s_{12} & a_2 c_{12} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$$

Reciprocal Wrenches

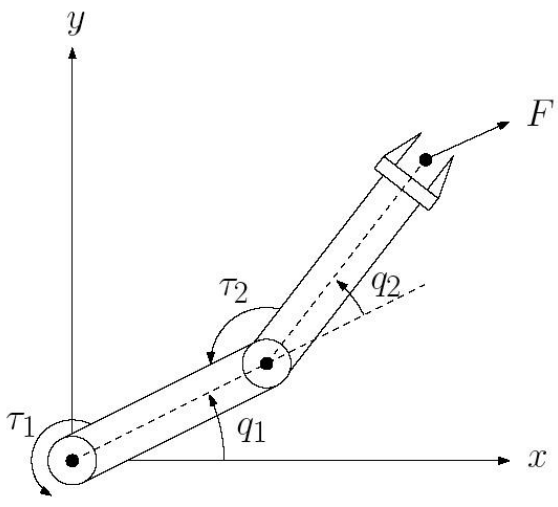


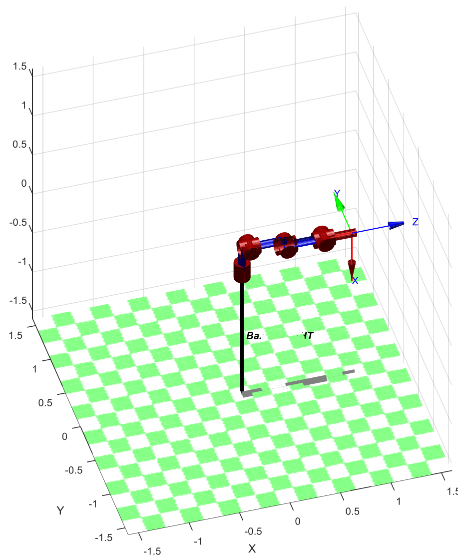
Figure 4.11: Two-link planar robot.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = J^T F$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & a_1 c_1 + a_2 c_{12} & 0 & 0 & 0 & 1 \\ -a_2 s_{12} & a_2 c_{12} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$$

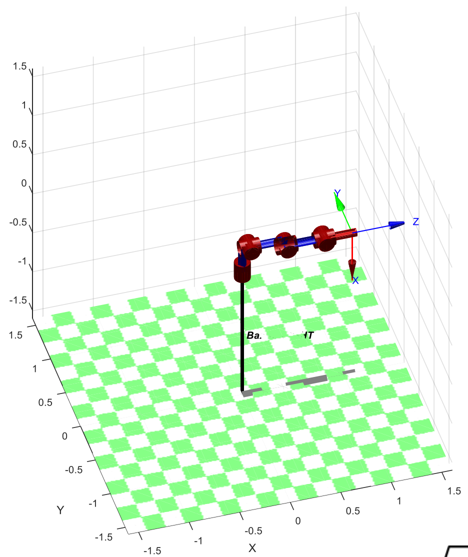
- When there are directions that we can apply forces or torques on a rigid body attached to the end effector that cause no joint effort (torque or force), these are reciprocal wrenches (or non-trivial solutions the equations above).
- What are the reciprocal wrench directions for this arm?
  - What about if arm is fully extended? Do the number of
- Reciprocal wrenches don’t always align with axes.

Manipulability



When J(q) is full rank, we can calculate a measure of how much the arm can move in different directions at a given configuration.

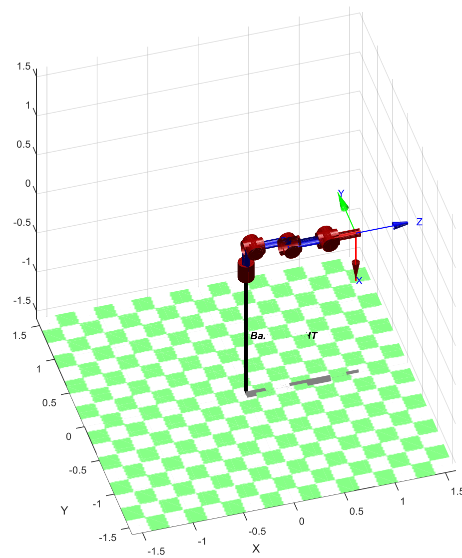
Manipulability

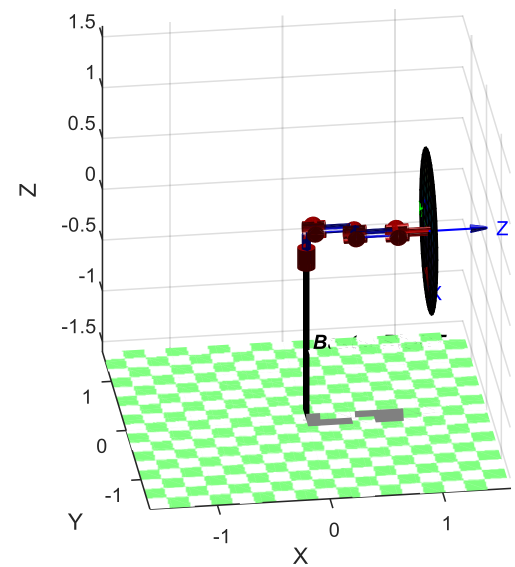


$$\mu = \sqrt{det(JJ^T)}$$

- This is a single number that represents the manipulator's ability to move in any Cartesian direction in that configuration
- Often makes sense to only calculate for the first three rows or last three rows of the Jacobian corresponding with velocity or angular velocity.

Manipulability





$$eig(JJ^T)$$

- Can be used to help evaluate and/or design a robot
- Can also look at the eigenvalues (see equation MATLAB function listed above) to visualize a manipulability ellipsoid (see demonstration)
- Ellipsoids that are almost spherical give the best general performance at a specific point.

Manipulability

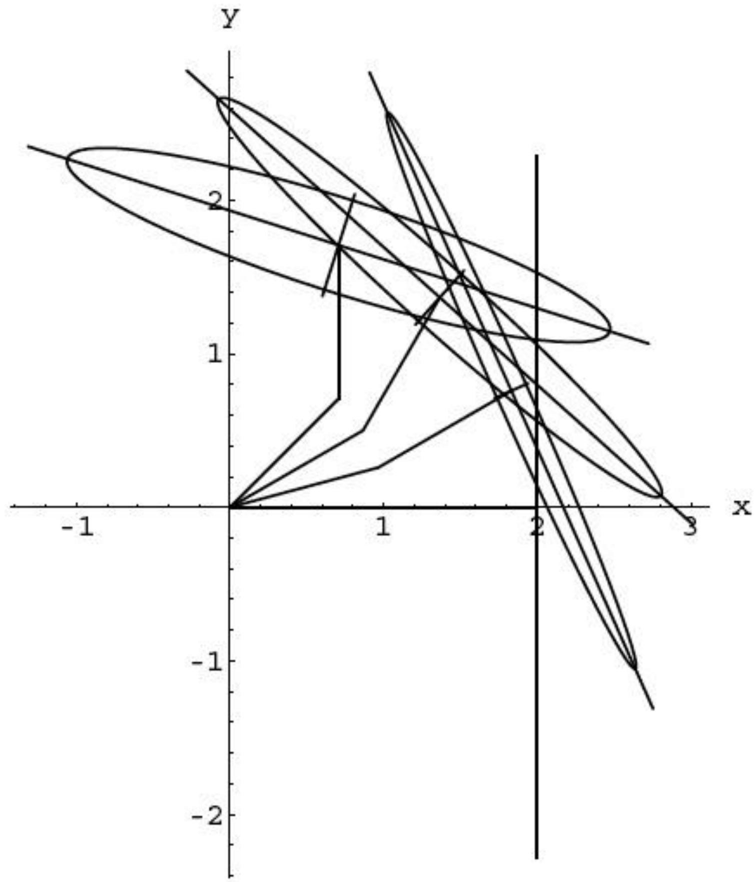


Figure 4.12: Manipulability ellipsoids are shown for several configurations of the two-link arm.