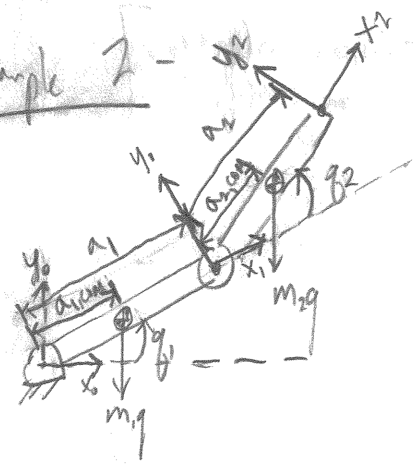
Example 2 -



find - EOM ⇒

define -

$$T_1^0 = A_1(q)$$

$$T_{1,com}^0 = A_{1,com}(q) = \begin{bmatrix} R_{1,com}^0 & t_{1,com}^0 \\ 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1(q)A_2(q)$$

$$T_{2,com}^0 = A_1(q)A_{2,com}(q) = \begin{bmatrix} R_{2,com}^0 & t_{2,com}^0 \\ 0 & 1 \end{bmatrix}$$

start by finding kinetic Energy to get $D(q)$

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q} = \frac{1}{2}\dot{q}^T\left\{\sum_{i=1}^{2} m_i J_{v_{com,i}}^T J_{v_{com,i}} + J_{\omega_{com,i}}^T R_{i,com}^0 I_i (R_{i,com}^0)^T J_{\omega_{com,i}}\right\}\dot{q}$$

$$J_{com,1} = \begin{bmatrix} J_{v_{com,1}} \\ J_{\omega_{com,1}} \end{bmatrix} = \begin{bmatrix} Z_0^0 \times (O_{com,1}^0 - O_0^0) & \begin{matrix}0\\0\\0\end{matrix} \\ Z_0^0 & \begin{matrix}0\\0\\0\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} -a_{1,com}\sin(q_1) & 0 \\ a_{1,com}\cos(q_1) & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_{com,2} = \begin{bmatrix} J_{v_{com,2}} \\ J_{\omega_{com,2}} \end{bmatrix} = \begin{bmatrix} Z_0^0 \times (O_{com,2}^0 - O_0^0) & Z_1^0 \times (O_{com,2}^0 - O_1^0) \\ Z_0^0 & Z_1^0 \end{bmatrix}$$

$$J_{com,2} = \begin{bmatrix} -a_1 \sin(q_1) - a_{2,com}\sin(q_1+q_2) & -a_{2,com}\sin(q_1+q_2) \\ a_1\cos(q_1) + a_{2,com}\cos(q_1+q_2) & a_{2,com}\cos(q_1+q_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

look at rotational kinetic Energy first $\Rightarrow$

$$K_{rot} = \frac{1}{2}\dot{q}^T \left\{ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} R^0_{1,com} \begin{bmatrix} I_{xx,1} & 0 & 0 \\ 0 & I_{yy,1} & 0 \\ 0 & 0 & I_{zz,1} \end{bmatrix} R^{1,com}_0 \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} + \right.$$

$$\left. \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} R^0_{2,com} \begin{bmatrix} I_{xx,2} & 0 & 0 \\ 0 & I_{yy,2} & 0 \\ 0 & 0 & I_{zz,2} \end{bmatrix} R^{2,com}_0 \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \right\} \dot{q}$$

$\Rightarrow$ R's have no effect in-plane like this $\Rightarrow$

$$K_{rot} = \frac{1}{2}\dot{q}^T \begin{bmatrix} I_{zz,1} + I_{zz,2} & I_{zz,2} \\ I_{zz,2} & I_{zz,2} \end{bmatrix} \dot{q}$$

, still need $K_{translational}$ $\Rightarrow$

$$K_{translational} = \frac{1}{2}\dot{q}\left[m_1 J_{v_{com,1}}^T J_{v_{com,1}} + m_2 J_{v_{com,2}}^T J_{v_{com,2}}\right]\dot{q}$$

evaluating this, and adding $K_{rot} + K_{trans} = K_{total}$, then (I would do this on a computer)

$$D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \Rightarrow$$

$$d_{11} = m_1(a_{1,com})^2 + m_2\left(a_1^2 + (a_{2,com})^2 + 2a_1 a_{2,com}\cos(q_2)\right) + I_{zz,1} + I_{zz,2}$$

1 $$d_{12} = d_{21} = m_2\left((a_{2,com})^2 + a_1 a_{2,com}\cos(q_2)\right) + I_{zz,2}$$

$$d_{22} = m_2(a_{2,com})^2 + I_{zz,2}$$

<u>note:</u>

- $D(q)$ or $M(q)$ will always be symmetric
- using the trig identities in the book is not necessary

now for $C(q, \dot{q}) \Rightarrow$

(2)

$$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = -m_2 a_1 a_{2,com} \sin(q_2) = h$$ (just an intermediate variable for simplicity)

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \cancel{\frac{1}{2} \frac{\partial d_{22}}{\partial q_1}}^{\,0} = -m_2 a_1 a_{2,com} \sin(q_2) = h$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = m_2 a_1 a_{2,com} \sin(q_2) = -h$$

$$c_{122} = c_{212} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_2} = 0$$

next look at $P(q)$ -

$$P = \sum_{i=1}^{2} P_i \Rightarrow$$

$$P_1 = m_1 g^T r_{1,com} \Rightarrow$$

$$= m_1 \begin{bmatrix} 0 & 9.81 & 0 \end{bmatrix} \begin{bmatrix} a_{1,com} \cos(q_1) \\ a_{1,com} \sin(q_1) \\ 0 \end{bmatrix} = m_1 (9.81) a_{1,com} \sin(q_1)$$

$$P_2 = m_2 g^T r_{2,com} \Rightarrow$$

$$= m_2 \begin{bmatrix} 0 & 9.81 & 0 \end{bmatrix} \begin{bmatrix} a_1 \cos(q_1) + a_{2,com} \cos(q_1 + q_2) \\ a_1 \sin(q_1) + a_{2,com} \sin(q_1 + q_2) \\ 0 \end{bmatrix}$$

$$= m_2 (9.81) \left(a_1 \sin(q_1) + a_{2,com} \sin(q_1 + q_2) \right)$$

$$P = (m_1 a_{1,com} + m_2 a_1)(9.81) \sin(q_1) + m_2 a_{2,com} (9.81) \sin(q_1 + q_2)$$

(3)

$$g_1 = \frac{\partial P}{\partial q_1} = (m_1 a_{1,com} + m_2 a_1)(9.81) \cos(q_1)$$

$$g_2 = \frac{\partial P}{\partial q_2} = m_2 a_{2,com} (9.81) \cos(q_1 + q_2)$$

combine ①, ② & ③ into eqn 7.62 to get ⇒

$$\boxed{\begin{aligned} &d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + g_1 = \tau_1 \\ &d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + g_2 = \tau_2 \end{aligned}}$$

can be written in matrix form with

$$C(q,\dot{q}) = \begin{bmatrix} h\dot{q}_2 & h(\dot{q}_2+\dot{q}_1) \\ -h\dot{q}_1 & 0 \end{bmatrix}$$