Analytical Jacobian

transforms velocities in joint space to velocities in Cartesian (or task or Operational) Space. $\Rightarrow$

example

if ψ, θ, ϕ are Euler angles describing orientation $\Rightarrow$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = J_A(q)\,\dot{q}$$

$\rightarrow$ for Geometric Jacobian, there were $\begin{matrix} \omega_x \\ \omega_y \\ \omega_z \end{matrix}$

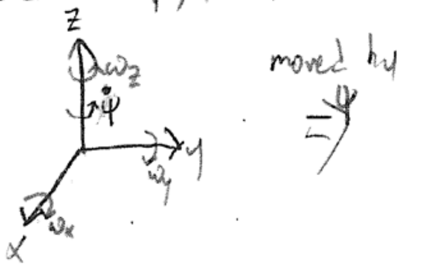
so if we have a function $f(q)$, which is the forward kinematics, parameterizing rotation by ψ, θ, ϕ, then

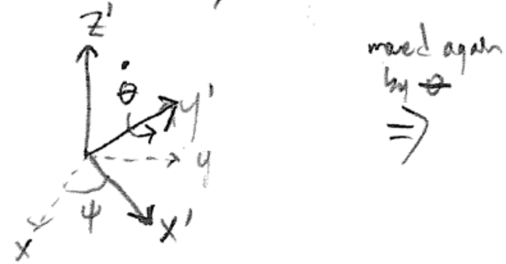
$$\dot{X} = \frac{\partial f(q)}{\partial q}\frac{\partial q}{\partial t} \Rightarrow$$

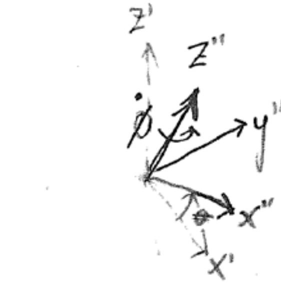
$$= \underbrace{\frac{\partial f(q)}{\partial q}}_{J_A(q)}\dot{q}$$

, but finding it like this would be a pain! $\Rightarrow$ so can we easily relate $\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$ to something we already know how to calculate $\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$?

Let ψ, θ, ϕ, be ZYZ rotations $\Rightarrow$







$\dot\psi$ is related to $[\omega_x, \omega_y, \omega_z]$ as $\dot\psi[0, 0, 1]$

$\dot\theta$ is related to $[\omega_x, \omega_y, \omega_z]$ as $\dot\theta[-S_\psi \; C_\psi \; 0]$

$\dot\phi$ is related to $[\omega_x, \omega_y, \omega_z]$ is $\dot\phi[C_\psi S_\theta \;\; S_\psi S_\theta \;\; C_\theta]$

<u>remember that</u>

- ψ, θ, ϕ will come from R which is a function of q (your forward kinematics)

so we can relate $\begin{bmatrix} \dot\psi \\ \dot\theta \\ \dot\phi \end{bmatrix}$ to $\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$ like this $\Rightarrow$

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -S_\psi & C_\psi S_\theta \\ 0 & C_\psi & S_\psi S_\theta \\ 1 & 0 & C_\theta \end{bmatrix}}_{T_A(q)^{-1}} \begin{bmatrix} \dot\psi \\ \dot\theta \\ \dot\phi \end{bmatrix} \qquad \text{can solve for} \quad \begin{bmatrix} \dot\psi \\ \dot\theta \\ \dot\phi \end{bmatrix} = T_A \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$\hookrightarrow$ last three rows of $J(q)$ times $\dot q$

$$J_A = \begin{bmatrix} I & 0 \\ 0 & T_A(q) \end{bmatrix} J(q)$$ this is generally easier to calculate than $\frac{\partial f(q)}{\partial q}$

Look at paper about pose on learning suite. required reading.

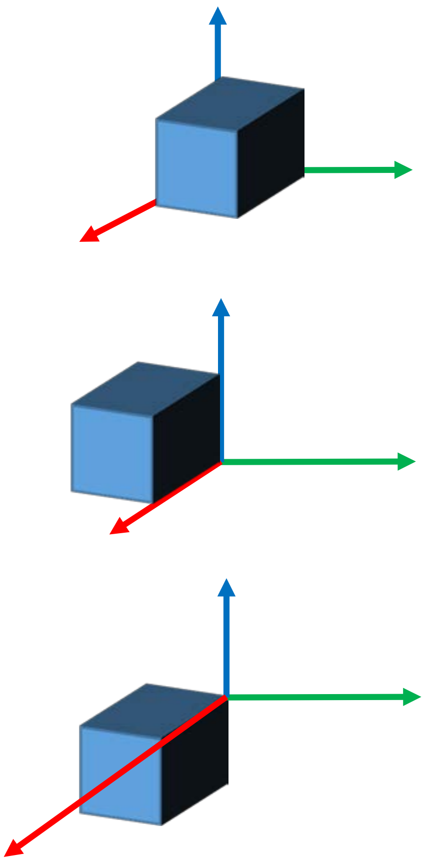
Why?

1) we often care about velocities (or displacements) in Cartesian space even though angular velocities at the end effector may be more intuitive.

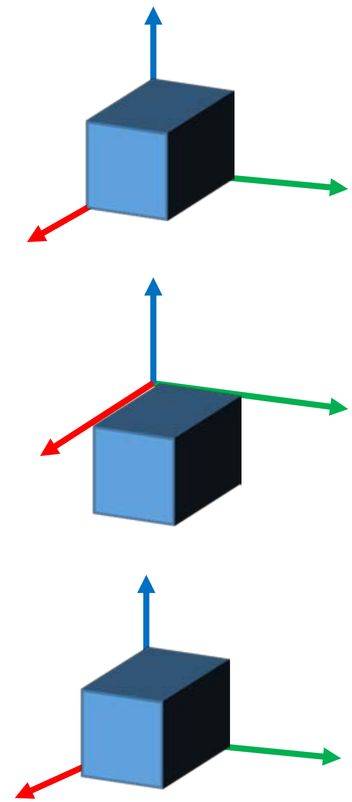
2) we can integrate $\begin{bmatrix} \dot\psi \\ \dot\theta \\ \dot\phi \end{bmatrix}$ to get $\begin{bmatrix} \psi \\ \theta \\ \phi \end{bmatrix}$, whereas integrating $\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$ gives non-unique solutions and unclear physical interpretation.

Integrating the Geometric Jacobian Eqns

$\omega = [\frac{\pi}{2}, 0, 0]^T, \ 0 \le t \le 1$
$\omega = [0, \frac{\pi}{2}, 0]^T, \ 1 < t \le 2$



$\omega = [0, \frac{\pi}{2}, 0]^T, \ 0 \le t \le 1$
$\omega = [\frac{\pi}{2}, 0, 0]^T, \ 1 < t \le 2$



$$t = 0$$

$$t = 1$$

$$t = 2$$

but $\int_0^2 \omega dt = [\frac{\pi}{2}, \frac{\pi}{2}, 0]^T$ for both cases!!

Analytical Inverse Kinematics

See link 2 example in your book and Section 3.3 for more complicated robots

See these links for examples:

- http://www.diag.uniroma1.it/~deluca/rob1_en/10_InverseKinematics.pdf - slides 14-19
- http://www.dis.uniroma1.it/~deluca/rob1_en/Article_KinInvPuma600.pdf
- http://hades.mech.northwestern.edu/images/7/7f/MR.pdf - page 215