MEEn 537

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HW#1

1 2. Forward Kinematics: it is to find the position and orientation of the end effector in terms of the joint variables.

Inverse Kinematics: to find joint variables when the end effec tor's position and orientations are defined.

Trajectory planning: instead of just one point for the end effector to go, assign a specific path to get to that point.

Workspace: all possible points that robot can reach.

Accuracy: the error between desired point and actual point of the end effector.

Repeatability: the variance of the end effector's position given the same joint command.

Resolution: the smallest increment of motion that the controller can sense.

Joint variables: variables to joints. θ for resolute joint, d for prismatic joint.

Spherical wrist: wrists whose three joint axes intersect at a com mon point.

End effector: a functional effector that is attached at the end of manipulator.

1 8. Manipulator to grab a fragile object, Handshaking robot, Painting robot,

1 12. When a robot manipulator has to work in a limited space, it is useful to go to a point of interest with many different configurations.

1 14. 180/256=0.7031degree

1 16. Because sensor biases on joints are usually greater than sensor noises.

1 20. There are only two solutions for each point except for the case that the manipulator is fully stretched.

1 21. So the power to operate the manipulator is less. Also to make the manipulator more agile. Use lighter material. Place power source at the base.

2-10. $R = R_{y,\psi} R_{x,\phi} R_{z,\theta}$

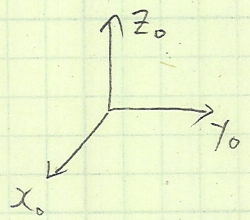
2-11. $R = R_{z,\theta} R_{x,\phi} R_{x,\psi}$

2-12. $R = R_{z,\alpha} R_{x,\phi} R_{z,\theta} R_{x,\psi}$

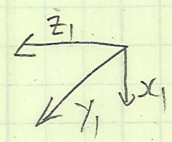
2-13. $R = R_{z,\alpha} R_{z,\theta} R_{x,\phi} R_{x,\psi}$

2-14. $R_0^1 = R_{y,\frac{\pi}{2}} R_{x,\frac{\pi}{2}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

Initial



Final



2-15. $R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$R_3^1 = R_2^1 R_3^2$

$R_3^1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$R_3^2 = R_2^{1\,T} R_3^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$

2-22. $R_{x,\theta} R_{y,\phi} R_{z,\pi} R_{y,-\phi} R_{x,-\theta}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C{-\phi} & 0 & S{-\phi} \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C{-\theta} & -S{-\theta} \\ 0 & S{-\theta} & C{-\theta} \end{bmatrix}$$

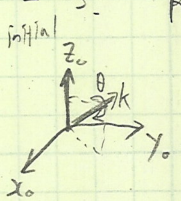
$$= \begin{bmatrix} C\phi & 0 & S\phi \\ S\theta S\phi & C\theta & -S\theta C\phi \\ -C\theta S\phi & S\theta & C\theta C\phi \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -C\phi & 0 & S\phi \\ -S\theta S\phi & -C\theta & -S\theta C\phi \\ C\theta S\phi & -S\theta & C\theta C\phi \end{bmatrix} \begin{bmatrix} C{-\phi} & 0 & S{-\phi} \\ 0 & 1 & 0 \\ -S{-\phi} & 0 & C\phi \end{bmatrix}$$

$$= \begin{bmatrix} S^2\phi - C^2\phi & 0 & 2S\phi C\phi \\ -2S\theta S\phi C\phi & -C\theta & S\theta S^2\phi - S\theta C^2\phi \\ 2C\theta C\phi S\phi & -S\theta & -C\theta S^2\phi + C\theta C^2\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} = \begin{bmatrix} S^2\phi - C^2\phi & -2S\theta S\phi C\phi & 2S\phi C\phi C\theta \\ -2S\theta S\phi C\phi & -C^2\theta + S^2\theta C^2\phi - S\theta S^2\phi C\theta & -S\theta C\theta + S^2\theta C^2\phi - S^2\theta S^2\phi \\ 2C\theta C\phi S\phi & -S\theta C\theta + C\theta S^2\phi S\theta - S\theta C\theta C^2\phi & -S^2\theta - C^2\theta S^2\phi + C^2\theta C^2\phi \end{bmatrix}$$

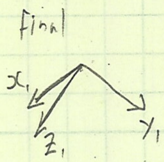
2-23. $R = R_{y_0,90^\circ} R_{z_1,45^\circ} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$

initial



$\theta = \cos^{-1}\left(\dfrac{\frac{\sqrt{2}}{2} - 1}{2}\right) = 1.7178$ or 98.42°

Final



$k = \dfrac{1}{2\sin\theta} \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 1 + \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 0.3497 \\ 0.8443 \\ 0.3497 \end{bmatrix}$ $\quad R_{k,\theta}$

2-24. $R = R_{z,\frac{\pi}{2}} R_{y,0} R_{z,\frac{\pi}{4}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

135° relative to the base frame.