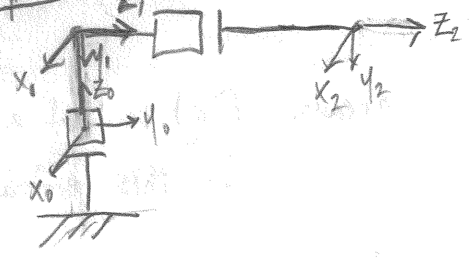
Example 1 – figure 7.7



assume:

- COM for link 1 is a distance $d_{1,com}$ from the frame 0 in direction z_0
- COM for link 2 is a distance $d_{2,com}$ from frame 1 in direction z_1
- g = direction of gravity NOT negative = $\begin{bmatrix} 0 \\ 0 \\ 9.81 \end{bmatrix}$

find: Equations of Motion

start with kinetic energy to get $D(q)$ & $C(q,\dot{q})$

$$K = \frac{1}{2}\dot{q}^T \sum_{i=1}^{2} \left(m_i J_{v_{ci}}^T J_{v_{ci}} + J_{\omega_{ci}}^T R_{c_i}^0 I (R_{c_i}^0)^T J_{\omega_{ci}} \right) \dot{q}$$

$$J_{c_1} = \begin{bmatrix} J_{v_{c1}} \\ J_{\omega_{c1}} \end{bmatrix} = \begin{bmatrix} z_0^0 & 0 \\ & 0 \\ & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{c_2} = \begin{bmatrix} J_{v_{c2}} \\ J_{\omega_{c2}} \end{bmatrix} = \begin{bmatrix} z_0^0 & z_1^0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow$$

$$K = \frac{1}{2}\dot{q}^T \underbrace{\left\{ m_1 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} + m_2 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} + 0 \right\}}_{D(q)} \dot{q} \Rightarrow$$

$$D(q) = \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix}$$

for $C(q,\dot{q})$

$$c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)$$

; because $D(q)$ is not a function of q for this problem, these terms will all be zero.

<u>now look at $P(q)$ to get g_k (see eqn 7.61 in book)</u>

$$P = m_1 g^T r_{c_1} + m_2 g^T r_{c_2}$$

$$r_{c_1} = \begin{bmatrix} 0 \\ 0 \\ q_1 + d_{1,com} \end{bmatrix}$$

~ got this from inspection, but could have pulled from forward kinematics.

$$r_{c_2} = \begin{bmatrix} 0 \\ q_2 + d_{2,com} \\ q_1 + d_{1,com} \end{bmatrix}$$

$$P = m_1\left(9.81(q_1 + d_{1,com})\right) + m_2\left(9.81(q_1 + d_{1,com})\right) \Rightarrow$$

$$= (m_1 + m_2)(9.81)(q_1 + d_{1,com})$$

$$\frac{\partial P}{\partial q_1} = g_1(q) = (m_1 + m_2)(9.81)$$

$$\frac{\partial P}{\partial q_2} = g_2(q) = 0$$

<u>combine all into eqn 7.62 or 7.63 -</u>

$$\boxed{\begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} (m_1 + m_2)9.81 \\ 0 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}}$$

to double check, if at equilibrium, $\ddot{q} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$$\left.\begin{aligned} f_1 &= (m_1 + m_2)9.81 \\ f_2 &= 0 \end{aligned}\right\}$$

– this makes sense and tells us the force required at the joints to keep the system in equilibrium