19/9/'23

Activity 7 – The graph of $\sin^{-1} x$

Objective: To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$)

Pre-requisite knowledge: Knowledge of trigonometric functions and inverse of trigonometric functions and their properties

Materials required: A cardboard of dimensions 30 cm × 30 cm, ruler, coloured pencils, board pins, paper pins and strings

Procedure:

1. Place a chart paper firmly on a cardboard with the help of board pins as shown in Fig. 5.1.

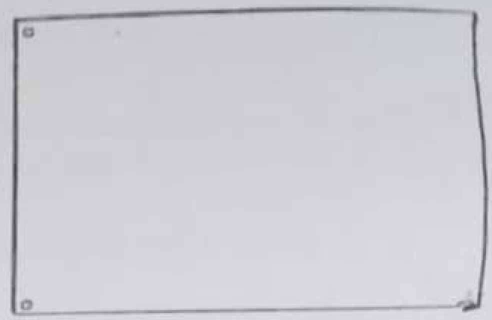

Fig. 5.1

2. On the chart paper draw perpendicular axes X'OX' and YOY' as shown in Fig. 5.2.

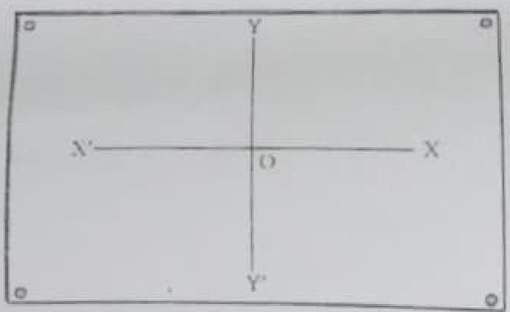

Fig. 5.2

3. Mark the points on y-axis 1, $\frac{1}{2}$, 0, $\frac{-1}{2}$, -1, etc.

4. To sketch the graph of $y = \sin x$, we can make a table of values that we can compute exactly

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin x	0	$\frac{1}{2} = 0.5$	$\frac{1}{\sqrt{2}} = 0.71$	$\frac{\sqrt{3}}{2} = 0.87$	1

5. Fix paper pins in the coordinate plane to represent the points namely $A_1\left(\frac{\pi}{6}, 0.5\right)$, $A_2\left(\frac{\pi}{4}, 0.71\right)$ $A_3\left(\frac{\pi}{3}, 0.87\right)$, $A_4\left(\frac{\pi}{2}, 1\right)$ as shown in Fig 5.3.

6. On the other side of the x-axis, repeat the same process and mark the points given in the table below

x	$\frac{-\pi}{6}$	$\frac{-\pi}{4}$	$\frac{-\pi}{3}$	$\frac{-\pi}{2}$
$\sin x$	$\frac{-1}{2} = -0.5$	$\frac{-1}{\sqrt{2}} = -0.71$	$\frac{-\sqrt{3}}{2} = -0.87$	-1

7. Fix the paper pins on the points namely $A_1\left(\frac{-\pi}{6}, -0.5\right), A_2\left(\frac{-\pi}{4}, -0.71\right), A_3\left(\frac{-\pi}{3}, -0.87\right), A_4\left(\frac{-\pi}{2}, -1\right)$. Fix one paper pin on the origin O, as shown in Fig. 5.3.

8. Join the pins with the help of a string on both side of x-axis. Here we will get a curve which is the graph of $\sin x$ from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$

9. Now plot the points (1, 1), (2, 2), (3, 3), (4, 4), ... etc. on the coordinate plane to draw the graph of line $y = x$.

10. From the marked points A_1, A_2, A_3, A_4, draw perpendiculars on the line $y = x$ and construct lines such that length of perpendicular on both sides of the line $y = x$ are equal. Mark these points as B_1, B_2, B_3, B_4 and fix the pins on them.

11. Repeat the same process on the other side of x-axis and fix the pins on the points namely B_1', B_2', B_3', B_4'.

12. Join the pins on both sides of the line $y = x$ by a string tightly to obtain the graph of $y = \sin^{-1} x$

13. Now place a mirror on the line $y = x$. The mirror image of the graph of $\sin x$ represents the graph of $\sin^{-1} x$, which shows $\sin^{-1} x$ is a reflection of $\sin x$ about the line $y = x$.

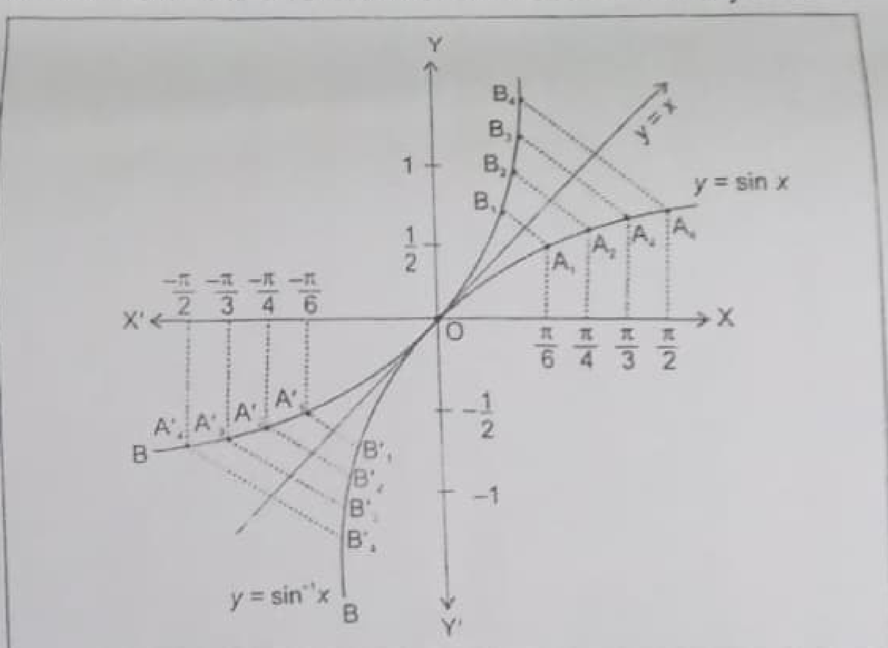

Fig. 5.3

Observations

1. The image of point A_1 in the mirror (the line $y = x$) is B_1.
2. The image of point A_2 in the mirror (the line $y = x$) is B_2.

3. The image of point A_3 in the mirror (the line $y = x$) is B_3.
4. The image of point A_4 in the mirror (the line $y = x$) is B_4.
5. The image of point A'_1 in the mirror (the line $y = x$) is B'_1.
6. The image of point A'_2 in the mirror (the line $y = x$) is B'_2.
7. The image of point A'_3 in the mirror (the line $y = x$) is B'_3.
8. The image of point A'_4 in the mirror (the line $y = x$) is B'_4.

Conclusion

The mirror image of the graph of sinx about the line $y = x$ is the graph of $\sin^{-1}x$, and the mirror image of the graph of $\sin^{-1}x$ about the line $y = x$ is the graph of $\sin x$.

Application : This activity is useful for concept clarity about graphs of inverse trigonometric functions.

Follow up Activity

26/9/'23

Activity 8 - Logarithms

Objective : To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base e) of the number x.

Pre-requisite knowledge : Knowledge of logarithm as a function from positive real numbers to all real numbers and properties of logarithm function.

Materials required : Drawing board, graph paper, log tables or calculator (graphic/scientific)

Procedure :

1. On the drawing board, paste a graph paper.
2. On the graph paper, draw the co-ordinate axes X'OX and YOY'.
3. Find some ordered pairs satisfying the function $y = \log_{10} x$
4. For the function $y = \log_{10} x$, find the value of Y for different values of x. The values may be tabulated as below :

x	1	2	3	4	5	6	7	8	9	10
$y = \log_{10} x$	$\log_{10} 1$	$\log_{10} 2$	$\log_{10} 3$	$\log_{10} 4$	$\log_{10} 5$	$\log_{10} 6$	$\log_{10} 7$	$\log_{10} 8$	$\log_{10} 9$	$\log_{10} 10$
	0	0.3010	0.4771	0.6020	0.6989	0.7781	0.8450	0.9030	0.9542	1

5. Plot the points (1, 0), (2, 0.30), (3, 0.47), (4, 0.60), (5, 0.69), (6, 0.77), (7, 0.84), (8, 0.90), (9, 0.95) and (10, 1). Join these points to get the graph of $y = \log_{10}x$ as shown in figure 8.1.
6. Now, for the function $y' = \log_e x$, find the value of y' for different values of x. The values may be tabulated as below :

x	1	2	3	4	5	6	7	8	9	10
$y' = \log_e x$	$\log_e 1$	$\log_e 2$	$\log_e 3$	$\log_e 4$	$\log_e 5$	$\log_e 6$	$\log_e 7$	$\log_e 8$	$\log_e 9$	$\log_e 10$
	0	0.6931	1.0986	1.3862	1.6094	1.7917	1.9459	2.0794	2.1972	2.3025

7. Now plot the points (1, 0), (2, 0.69), (3, 1.09), (4, 1.38), (5, 1.60), (6, 1.79), (7, 1.94), (8, 2.07) (9, 2.19) and (10, 2.30). Join these points to get the graph of $y' = \log_e x$

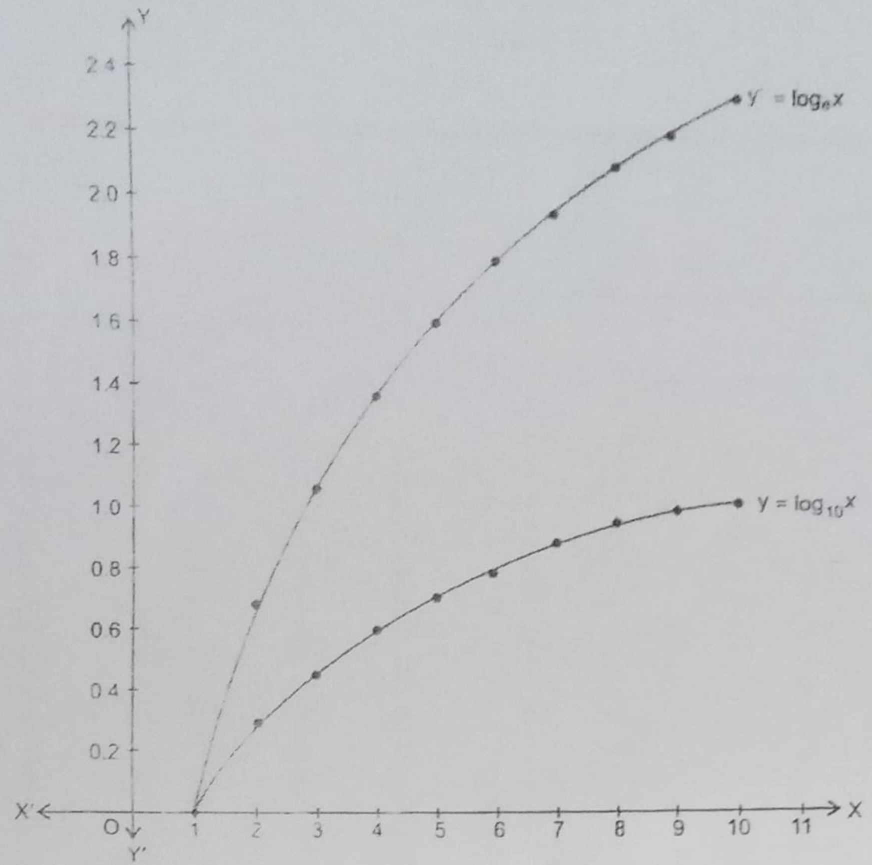

Fig. 8.1

S.No.	Points on the x-axis	$y = \log_{10} x$	$y' = \log_e x$	Ratio $\frac{y}{y'}$ (approximate)
1	$x_1 = 1$	$y_1 = 0$	$y_1' = 0$	undefined
2	$x_2 = 2$	$y_2 = 0.3010$	$y_2' = 0.6931$	0.4342
3	$x_3 = 3$	$y_3 = 0.4771$	$y_3' = 1.0986$	0.4342
4	$x_4 = 4$	$y_4 = 0.6020$	$y_4' = 1.3862$	0.4342
5	$x_5 = 5$	$y_5 = 0.6989$	$y_5' = 1.6094$	0.4342
6	$x_6 = 6$	$y_6 = 0.7781$	$y_6' = 1.7917$	0.4342
7	$x_7 = 7$	$y_7 = 0.8450$	$y_7' = 1.9459$	0.4342
8	$x_8 = 8$	$y_8 = 0.9030$	$y_8' = 2.0794$	0.4342
9	$x_9 = 9$	$y_9 = 0.9542$	$y_9' = 2.1972$	0.4342
10	$x_{10} = 10$	$y_{10} = 1$	$y_{10}' = 2.3025$	0.4343

Activity 21 TO FIND ANGLE IN A SEMICIRCLE Date 29/9/'23

Objective : To verify that angle in a semicircle is a right angle, using vector method.

Pre-requisite knowledge : Knowledge of circle and its properties, knowledge of vectors.

Materials required : Cardboard, white sheets of paper, nails, hammer, thread, gluestick, paper arrowheads, etc

Procedure :

1. On a cardboard of size 25 cm x 25 cm, paste a white sheet of paper.
2. On the white sheet of paper, draw a circle of radius 10 cm, with centre O.
3. Draw a diameter PQ of this circle.
4. Take any point A on the circumference of this circle, as shown in figure 21.1.
5. Fix nails at O, P, Q and A.
6. Join OP, OQ, OA, PA and QA, using thread. Stick arrowheads on threads along OP, OQ, OA, PA and QA, as shown in fig 21.1.

   Arrowheads show that OP, OQ, OA, PA and QA are vectors.

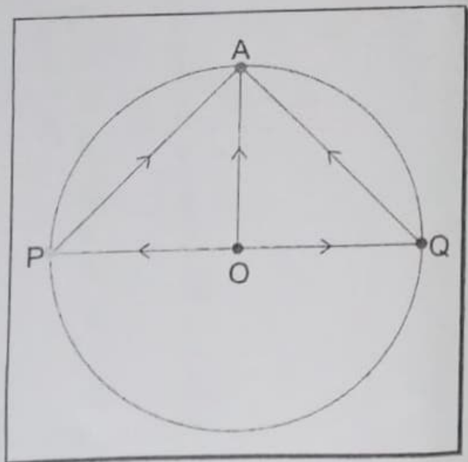

Fig. 21.1

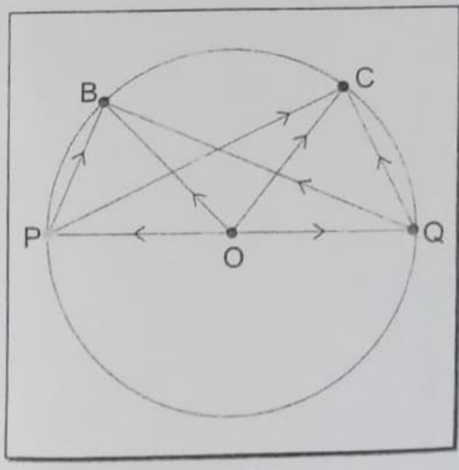

Fig. 21.2

7. Now take another cardboard sheet of dimensions 25 cm × 25 cm and repeat steps 1, 2 and 3 above.
8. Take any two points B and C on the circumference of the circle.
9. Fix nails at O, P, Q, B and C.
10. Join OP, OQ, OB, OC, PB, PC, QB, QC using threads. Stick arrowheads on threads along OP, OQ, OB, OC, PB, PC, QB and QC, as shown in figure 21.2. These arrowheads are to make them vectors.

Observations

1. By actual measurement (Figure 21.1) we have :

$|\overrightarrow{OA}| = 10$ cm, $|\overrightarrow{OP}| = 10$ cm, $|\overrightarrow{OQ}| = 10$ cm

$|\overrightarrow{PA}| = 12$ cm, $|\overrightarrow{QA}| = 16$ cm, $|\overrightarrow{PQ}| = 20$ cm

$|\overrightarrow{PA}|^2 + |\overrightarrow{QA}|^2 = 144 + 256 = 400 = |\overrightarrow{PQ}|^2$

$\Rightarrow \angle PAQ = 90^\circ$ [Pythagoras theorem]

$\Rightarrow \overrightarrow{PA} \cdot \overrightarrow{QA} = |\overrightarrow{PA}|\,|\overrightarrow{QA}| \cos 90^\circ = 0$

2. Similarly by actual measurements (figure 21.2), we have

$|\overrightarrow{OB}| = |\overrightarrow{OC}| = |\overrightarrow{OP}| = |\overrightarrow{OQ}| = 10$ cm

$|\overrightarrow{PB}| = 8$ cm, $|\overrightarrow{QB}| = 18.3$ cm, PQ = 20 cm

$|\overrightarrow{PC}| = 17$ cm, QC = 10.5 cm

$|\overrightarrow{PB}|^2 + |\overrightarrow{QB}|^2 = 8^2 + (18.3)^2 \approx 400 = |\overrightarrow{PQ}|^2$

$\Rightarrow \angle PBQ = 90^\circ$ [Pythagoras theorem]

$\Rightarrow \overrightarrow{PB} \cdot \overrightarrow{QB} = |\overrightarrow{PB}|\,|\overrightarrow{QB}| \cos 90^\circ = 0.$

Also, $|\overrightarrow{PC}|^2 + |\overrightarrow{QC}|^2 = 17^2 + (10.5)^2 \approx 400 = |\overrightarrow{PQ}|^2$

$\Rightarrow \angle PCQ = 90^\circ$.

$\Rightarrow \overrightarrow{PC} \cdot \overrightarrow{QC} = |\overrightarrow{PC}|\,|\overrightarrow{QB}| \cos 90^\circ = 0.$

3. Also using a protractor, if we measure the angle between the vectors $\overrightarrow{PA}$ and $\overrightarrow{QA}$, it comes out to be 90°, ie., $\angle PAB = 90^\circ$

Similarly, on measuring angles between the vectors $\overrightarrow{PB}$ and $\overrightarrow{QB}$, is 90°, i.e, $\angle PBQ = 90^\circ$ and angle between the vectors $\overrightarrow{PC}$ and $\overrightarrow{QC}$ is 90°, i.e., $\angle PCQ = 90^\circ$.

Conclusion

From the above activity, it is verified that the angle in a semicircle is a right angle.

Application : Useful to understand the concept of dot product and perpendicular vectors.

Activity [redacted] – Conditional Probability

Date 3/10/23

Objective : To explain the computation of conditional probability of a given event A when event B has already occurred, through an example of throwing a pair of dice.

Pre-requisite knowledge : Knowledge of probability, terms related to it, (i.e., random experiment, sample space, event, equally likely events, etc.), conditional probability, etc.

Materials required : Cardboard sheet, squared sheet (2 cm × 2 cm), gluestick, etc.

Procedure

1. On a cardboard sheet, paste a squared paper containing 36 squares, each square of size 2 cm × 2 cm, as shown in figure [redacted].

(3, 2) S = {(1,1) (1,2), (1,3), (1,4), (1,5), (1,6) (2,1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3,
(5, 3), (3, 3), (3, 4), (3, 5) (3, 6), (4, 1), (4, 2) (4, 3), (4, 4) (4, 5), (4, 6), (5, 1), (5,
(5, 4), (5, 5) (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

[redacted]

Observations

Case I : *To find the conditional probability of an event A, when B has already occurred, where A is the event a* **number 4 appears on both dice and B is the event 4 has already appeared on one of the dice.** Here we have to find P(A/B)

1. From the figure [redacted]

Outcome favourable to A is (4, 4)

$\therefore$ No. of outcomes favourable to A, i.e., n (A) = 1.

Outcomes favourable to B are

(1, 4), (2, 4), (3, 4), (4, 4) (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)

$\therefore$ No. of outcomes favourable to B, i.e., n(B) = 11

Outcome which is common to A and B is (4, 4)

$\therefore$ No. of outcomes favourable to (A $\cap$ B) i.e., n (A $\cap$ B) = 1

Hence, $P(A/B) = \dfrac{n(A \cap B)}{n(B)} = \dfrac{1}{11}$

Another Method : We can also use $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Total no. of outcomes = 36

$\therefore \quad n(S) = 36$

$P(B) = \dfrac{n(B)}{n(S)} = \dfrac{11}{36}$

$P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{36}$

So. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{11}{36}} = \frac{1}{11}$

Case 2: To find the conditional probability of an event A when B has already occurred, where A is the event getting a sum 10 and B is the event a doublet has already occurred. Here also, we have to find P(A/B)

2. From the figure [redacted]: Outcomes favourable to A are (4, 6), (5, 5), (6, 4)

$\therefore$ No of outcomes favourable to A, i.e., $n(A) = 3$

Outcomes favourable to B are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).

$\therefore$ No of outcomes favourable to B i.e, $n(B) = 6$

Outcome which is common to A and B is (5, 5)

$\therefore$ No of outcomes favourable to $(A \cap B)$, i.e., $n(A \cap B) = 1$.

Hence $P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{6}$

Another method :

We can also use $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$n(S) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$$\text{So. } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

Case 3: To find the conditional probability of an event A when B has already occurred, where A is the event the sum of the numbers on the two dice is 6 *and B is the event* numbers appearing on two dice are different.

Here also we have to find P(A/B).

3. From figure [redacted], we have :

Outcomes favourable to A are (1, 5), (2, 4), (3, 3), (4, 2), (5, 1).

$\therefore$ No of outcomes favourable to A, i.e., $n(A) = 5$

Outcomes favourable to B are

(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4) (6, 5)

$\therefore$ No of outcomes favourable to B, i.e., $n(B) = 30$

Outcomes which are common to A and B are (1, 5), (2, 4), (4, 2), (5, 1)

$\therefore$ No of outcomes favourable to $(A \cap B) = 4$

Hence $P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{4}{30} = \frac{2}{15}$

conclusion: The above activity explains how to compute the conditional probability of an event, when another event has already occured

~~Another Method :~~

~~We can also use $P(A/B) = \frac{P(A \cap B)}{P(B)}$~~

Application: Useful to illustrate the concept of Bayes' Theo