

1 Private Deep Learning

1.1 Two hidden layer neural network

Let $\phi : \mathbb{R} \rightarrow \mathbb{R}$ denote the ReLU activation, i.e., $\phi(u) = \max\{u, 0\}$.

We consider a two hidden layer neural network $f : \mathbb{R}^d \rightarrow \mathbb{R}$ (which can be decomposed into two functions $h : \mathbb{R}^d \rightarrow \mathbb{R}^{m_a}$ and $g : \mathbb{R}^{m_a} \rightarrow \mathbb{R}^{m_b}$) as follows:

$$\begin{aligned} h(x) &= \phi(W_A x), \\ g(z) &= W_C^\top \phi(z), \\ f(x) &= g(h(x)) = W_C^\top \phi(W_B \phi(W_A x)), \end{aligned}$$

where $W_A \in \mathbb{R}^{m_a \times d}$, $W_B \in \mathbb{R}^{m_b \times m_a}$ and $W_C \in \mathbb{R}^{m_b}$.

Let $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\} \subset \mathbb{R}^d \times \mathbb{R}$ denote a set of n input data points. We can think of x_i s are images and y_i s are the corresponding labels.

In classical deep learning training, the task is to find weights W_A, W_B and W_C such that \mathcal{L} is minimized.

$$\mathcal{L} = \sum_{i=1}^n \|f(x_i) - y_i\|_2^2$$

In this lecture, we describe a slightly different goal. The purpose is to find some W_A, W_B and W_C such that

1. Utility: $f(x_i) \approx y_i, \forall i \in [n]$
2. Privacy: Given $h(x_i)$ and W_A , it is “hard” recover x_i