Limits

Limits
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Large numbers and small numbers

$$\lim_{x \to \infty} f(x) = L$$

What is a limit

Let f have domain $R/\{2\}$

$\lim_{x \to 2} f(x) = 1$ or $f(x) \to 1$ as $x \to 2$

Let f have domain R and $f(2)$ is exist

Left-hand, right-hand, and two-sided limits

$\lim_{x \to 3^-} h(x) = 1$ and $\lim_{x \to 3^+} h(x) = -2$

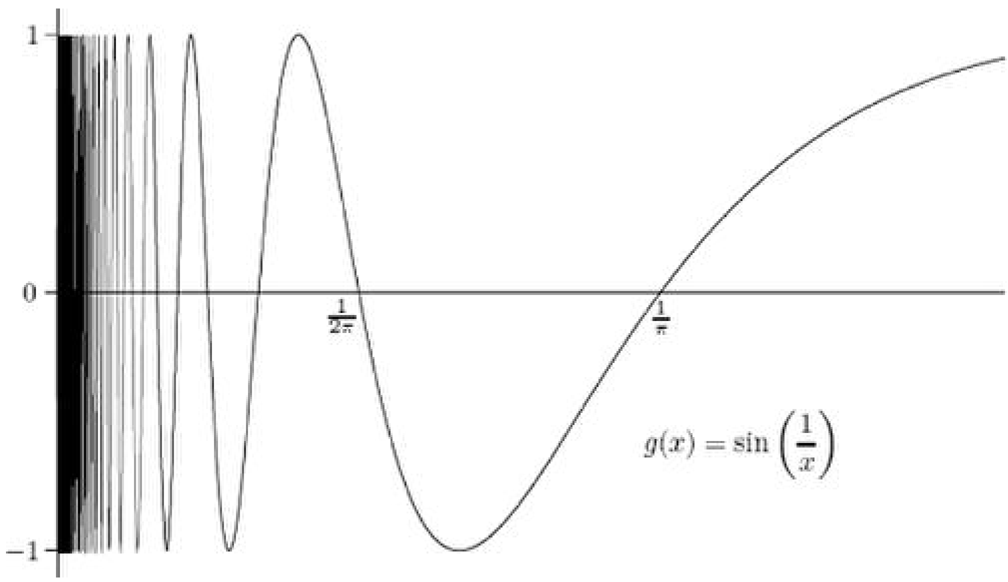
$\lim_{x \to 3} h(x) = ?$

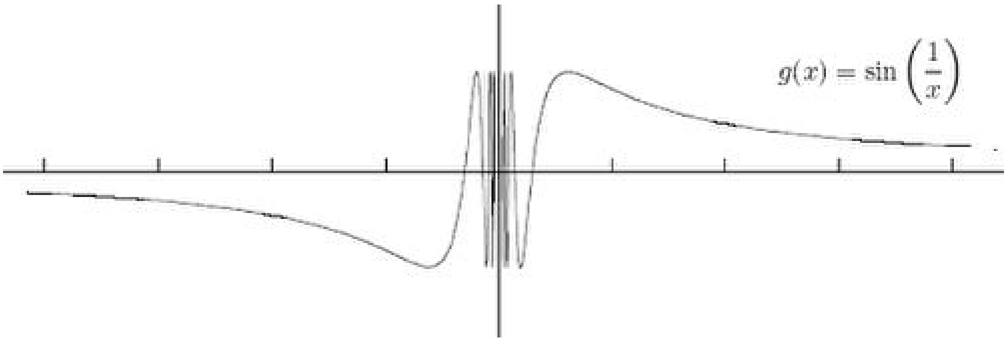
The regular 2-sided limit at $x = a$ exists **exactly when** both left-hand and right-hand limits at $x = a$ exist **and are equal to each other**

Limits at ∞ and $-\infty$

“vertical asymptote"

$f(x) = sin(\frac{1}{x})$





The Sandwich Principle

$\forall x$ near a, we have $g(x) \leq f(x) \leq h(x)$, suppose that $\lim_{x \to a} g(x) = L$, $\lim_{x \to a} h(x) = L$, then we can conclude that $\lim_{x \to a} f(x) = L$

proof

Proofs

$\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$

$+-$

$\times \div$

∞

2 Important Limits

$\lim_{x \to 0} \frac{sinx}{x} = 1$

$\lim_{x \to \infty} (1 + \frac{1}{n})^n = e$