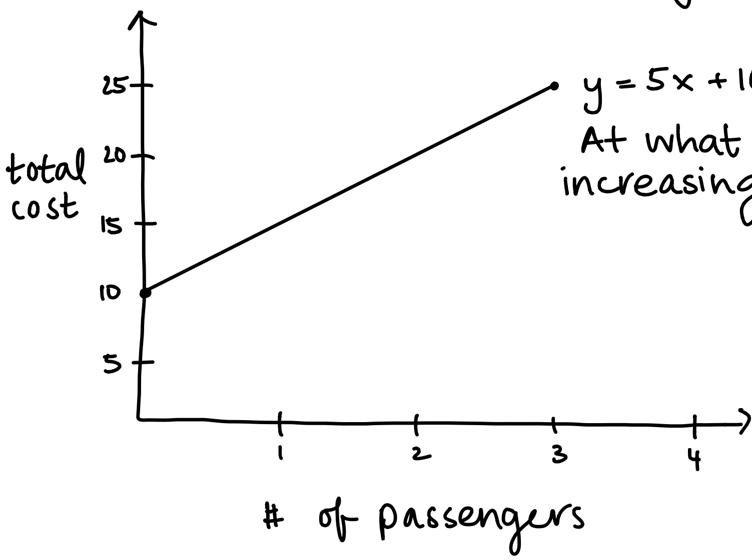
Introductory Calculus

Derivative – rate of change of a function

– at what rate is something increasing/decreasing?



At what rate is total cost increasing?

– this rate is the DERIVATIVE of the function

$$f(x) = 5x + 10$$

$$f'(x) = 5 \leftarrow$$ for a linear function, this is the <u>slope</u>

– formula for a derivative:

$$\frac{d}{dx}(ax^n) = (a \cdot n)x^{n-1}$$

notation for taking a derivative

a is a <u>constant</u>

ex. $d/dx\,(5x) = (5 \cdot 1)(x^{1-1}) = 5x^0 = 5$

$d/dx\,(x^3) = (3)(x^{3-1}) = 3x^2$

$d/dx\,(4x^4 + 3x^2) = (4 \cdot 4)x^{4-1} + (3 \cdot 2)x^{2-1}$

$= 16x^3 + 6x$

- derivative rules:

SUM $\quad d/dx\,(f(x)+g(x)) = f'(x) + g'(x)$

ex. $d/dx\,(4x^4 + 3x^2) = 16x^3 + 6x$

DIFFERENCE $\quad d/dx(f(x) - g(x)) = f'(x) - g'(x)$

ex. $d/dx(4x^4 - 3x^2) = 16x^3 - 6x$

PRODUCT $\quad d/dx\,(f(x)\cdot g(x)) = f(x)g'(x) + f'(x)g(x)$

ex. $d/dx\,((x^2+1)(3x^3-9)) = (x^2+1)(9x^2) + (2x)(3x^3-9)$

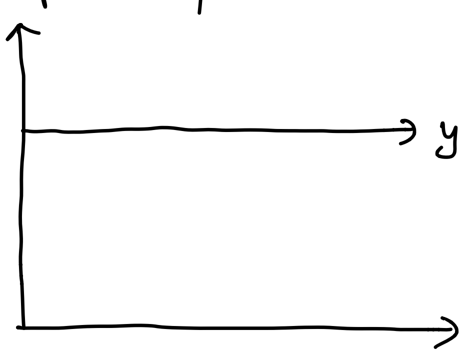
QUOTIENT $\quad d/dx\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

ex. $d/dx\left(\frac{3x^2}{x^2+1}\right) = \frac{(x^2+1)(6x) - (3x^2)(2x)}{(x^2+1)^2}$

- derivative of a constant: <u>ZERO</u>
  - mathematically: constants can be represented as cx^0 because $x^0 = 1$

$d/dx\,(cx^0) = (0\cdot c)x^{-1} = 0 \qquad$ ex. $d/dx(7) = d/dx(7x^0) = 0$

  - graphically:



$y=7$ is a straight line

slope $=0$

there is no rate of change

· higher order derivatives: can take a derivative many times, until you get 0

– written as:

1st derivative	$f'(x)$	$\frac{dy}{dx}$
2nd derivative	$f''(x)$	$\frac{d^2y}{dx^2}$
3rd derivative	$f'''(x)$	$\frac{d^3y}{dx^3}$
4th derivative	$f^4(x)$	$\frac{d^4y}{dx^4}$
...		
nth derivative	$f^n(x)$	$\frac{d^ny}{dx^n}$

– ex. find d^4y/dx^4 of $x^6 + 4x^4 + 3x^3 + 2$

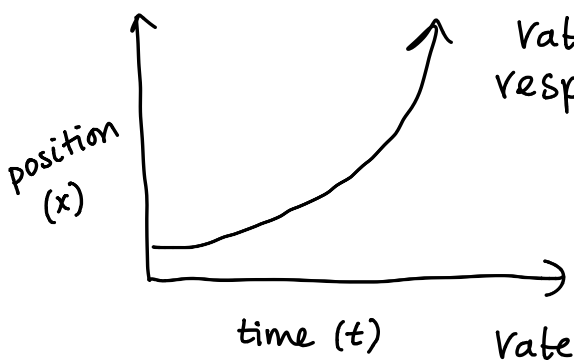
$d/dx\left(x^6 + 4x^4 + 3x^3 + 2\right) = 6x^5 + 16x^3 + 9x^2 \Rightarrow$ 1st derivative

$d/dx\left(6x^5 + 16x^3 + 9x^2\right) = 30x^4 + 48x^2 + 18x \Rightarrow$ 2nd derivative

$d/dx\left(30x^4 + 48x^2 + 18x\right) = 120x^3 + 96x + 18 \Rightarrow$ 3rd derivative

$d/dx\left(120x^3 + 96x + 18\right) = 360x^2 + 96 \Rightarrow$ 4th derivative

· application of derivatives: PHYSICS



rate of change of position with respect to time is also known as

speed / velocity

$\frac{d}{dt}(\text{position}) = \text{velocity} \qquad \frac{dx}{dt} = v$

rate of change of velocity with respect to time is also known as

acceleration

$\frac{d}{dt}(\text{velocity}) = \text{acceleration} \qquad \frac{dv}{dt} = \frac{d^2x}{dt^2} = a$