CMRM Homework Assignment No. 2

December 10, 2020

1 Description

The purpose of this homework assignment is to use the Autocorrelation Phase Matrix as a tool for rhythmic analysis.

Given a novelty function $x : \mathbb{Z} \to \mathbb{R}$ of finite length N, the Autocorrelation Phase Matrix (APM) is defined as:

$$P(\phi, k) = \sum_{i=0}^{\lceil \frac{N-\phi}{k} \rceil - 1} x(ki + \phi)x(k(i+1) + \phi) \tag{1}$$

where k is the "lag" and ϕ is the phase, which is constrained so that $\phi < k$, therefore the APM turns out to be a triangular matrix. Additional information has been presented in class during the laboratories on rhythmic analysis. The original paper is:
Douglas Eck, "Beat tracking using an autocorrelation phase matrix", *IEEE International Conference on Acoustics, Speech and Signal Processing* – ICASSP 2007.

2 Input preparation

- Generate an audio signal x with a simple rhythmic signature of 4/4 where each quarter note has duration 0.5 seconds and at $F_s = 22050$. The total duration of a measure is therefore 2 seconds. In order to create the signal you can simply use `librosa.clicks` function.
- Generate a second audio signal y with a simple rhythmic signature of 3/4 where each quarter note has duration 2/3 seconds at $Fs = 22050$. The total length of the measure is, again, 2 seconds. In order to create the signal you can simply use `librosa.clicks` function.
- Generate a third audio signal z as the superposition of the two previous ones.
- Compute the novelty curves for x, y and z, obtaining nov_x, nov_y and nov_z. Choose one of the algorithms shown in class or the one used in the APM Notebook (complex based novelty function). For example, you can use complex based novelty function with window length $N_{nov} = 512$ and hop size $H_{nov} = 256$. Normalize the novelty function so that its magnitude ranges between 0 and 1.
- **Question 1**: given N_{nov} and H_{nov} parameters, what is the relationship between the original sampling frequency and the sampling frequency of the novelty signal? Can you briefly explain why?

3 Autocorrelation Phase Matrix Parameters

Prepare all the parameters for APM computation:

- **Question 2**: Assume the considered BPMs goes from 40 BPM to 260 BPM, what is the set of lags $k_0, k_1, \dots, k_{K-1}$ that need to be considered in the autocorrelation phase matrix computation expressed in seconds? How can you express this range in samples?
- **Question 3**: Given the set of lags considered, what is the minimum length N_{apm} for the computation of the autocorrelation phase matrix?
- Define the vector `lags` $= [k_0, k_1, \dots, k_{K-1}]$

4 Autocorrelation Phase Matrix Computation

Compute the APM for the prepared input signals, using $N_{apm} = 2$ and using the results obtained in Question 2 :

- Apply smoothing to nov_x, nov_y and nov_z, using a modified version of the smooth function shown in class. In particular:
  - normalize the output of the function
  - guarantee that the input and the output have the same length
  - the length of the boxcar window used for smoothing is appropriate (try different values).
- Add to nov_x, nov_y and nov_z a constant noise of small amplitude (e.g., approximately 2% of the maximum amplitude of each novelty curve).
- Compute APM for x, y, z obtaining P_x, P_y, P_z. Each one is a matrix with size $K \times k_{K-1}$.

5 Autocorrelation Phase Matrix Tempo Analysis

Extract the tempo information from the autocorrelation phase matrix:

- **Question 4**: how would you extract tempo information from the autocorrelation phase matrix? Propose and motivate the procedure.
- From P_x and P_y, estimate the tempo BPM_x and BPM_y expressed in BPM correspondent to audio signal x and y using the proposed procedure.

6 Autocorrelation Phase Matrix Rhythmic Analysis

Extract rhythmic information from the autocorrelation phase matrix:

- **Question 5**: What can you learn from P_z? Comment on peaks locations and their meaning in relation to the rhythmic content.
- The file `apm_dict.npy` contains an APM matrix (together with k and phi axes) obtained analysing a 6 second audio recording containing a polyrhythm. Load the data from the file (see attached notebook) and analyse the APM (see Figure 1).
  - **Question 6** How many rhythms it is possible to identify? Explain your answer.
  - Define a possible train of pulses `pulses` that explains the APM. Hint: if more than one rhythm is present, define more than one train of pulses and then combine them together
  - Generate a `click_track` starting from the computed pulses using `librosa.clicks` function. This track will be an approximation of the original polyrhythm.

7 Files to be delivered

You are required to deliver the following files:

1. a report, containing answers to the questions and, if necessary, additional information on your implementation. Include you surname/surnames in the title of the report (e.g.,`Rossi.docx` or `Rossi_Bianchi.docx`).
2. the attached Jupyter Notebook file, `HomeworkTemplate.ipynb` completed with all the necessary code. Rename the notebook with your surname/surnames (e.g., `Rossi.ipynb` or `Rossi_Bianchi.ipynb`). Please, follow the given structure and do not change the name of the variables. Remember to plot, print or play all intermediate results. In particular, try to be precise in plot, for example specifying title, axis labels, etc.

Put both the report and the notebook in a folder, named using your surname/surnames and compress the folder in a zip file.

Figure 1: APM contained in file `apm_dict.npy`