



POLITECNICO  
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IMAGE AND SOUND  
**ISPG**  
PROCESSING GROUP

# CREATIVE PROGRAMMING AND COMPUTING

Grammar- and Automata-based agents

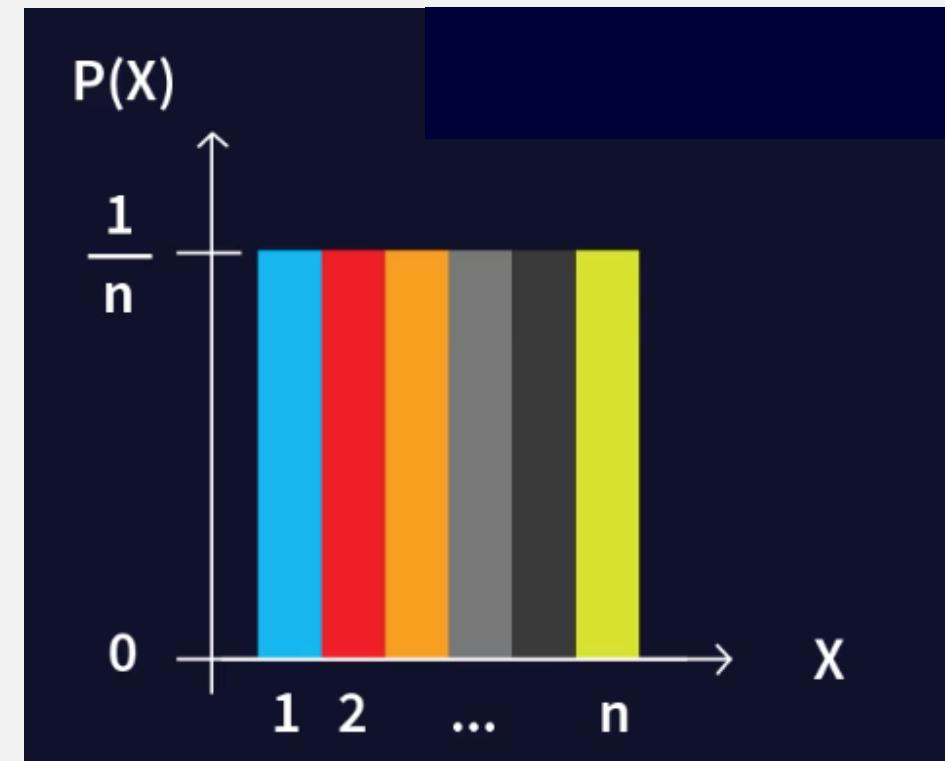
## MODELLING CREATIVITY - A RANDOM PROCESS

- The oldest, simplest and most common way to model **creativity** is through the introduction of uncertainty (**randomness**)
- The artist selects a space of parameters where values can take values in a certain ranges
- Values are then randomly selected
- **Definition:**
  - **Given a variable  $X$  defined over a discrete values space (the domain  $D$ ), a random draw is giving equal probability to all the option from  $D$**

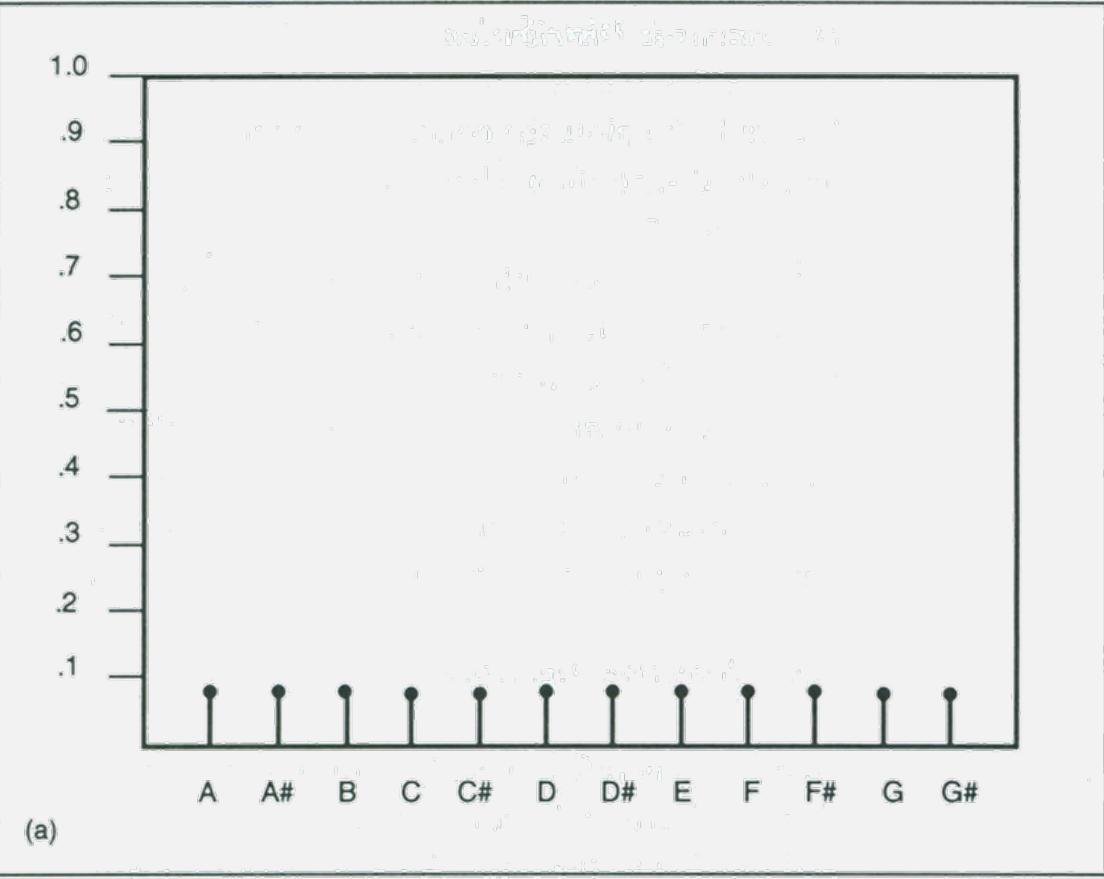
# MODELLING CREATIVITY - A RANDOM PROCESS

- **Probability variable  $X$ :** variable that can assume values according to a **probability distribution**
- Probability distribution gives the likelihood  **$P(X)$**  of each possible values  $X$  can assume

- Uniform distribution
- It is the distribution we describe as random process



# UNIFORM DISTRIBUTION IN MUSIC



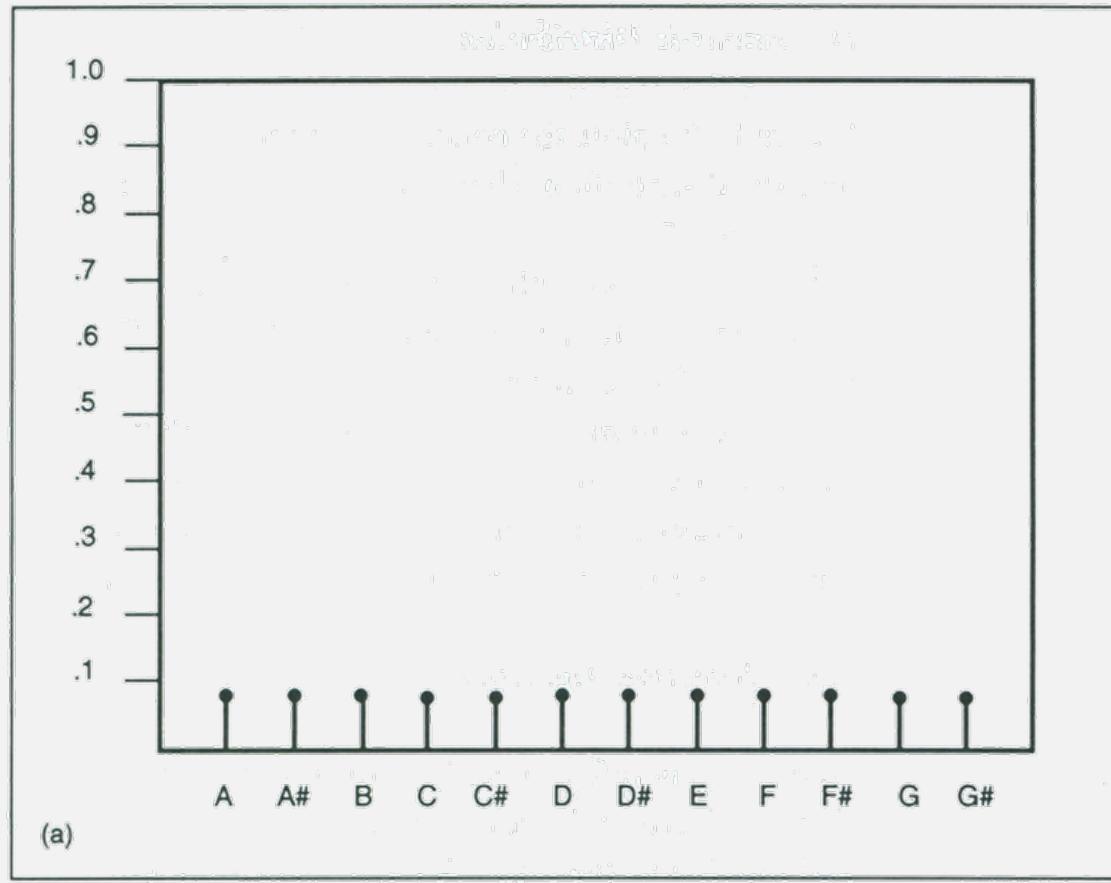
From [ROADS 96]: Each line on the vertical axis represents the probability of a corresponding value on the horizontal axis. In this case, the horizontal axis represents the pitches of a chromatic scale. Uniform distribution: every pitch has the same probability: 0.8

In SuperCollider: **Prand**, **Pxrand**

**Randomness\_uniform**

# UNIFORM DISTRIBUTION IN MUSIC

- Uniform distribution for random walk
- Generates a sequence of notes where each note is randomly selected



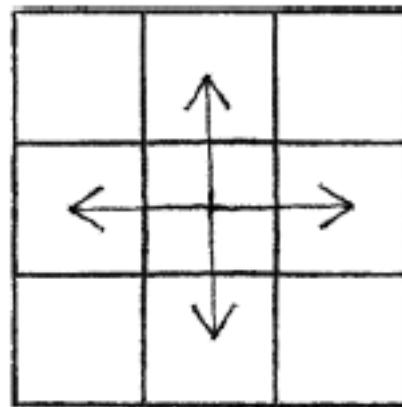
- Tare processes in which, while the next note in a sequence is random, but this value depends on the current note
- Random walk: given the current note, the value of the next note is a half-step higher or a half-step lower depending on the flip of a fair coin (uniform distribution)

In SuperCollider: **Pwalk**

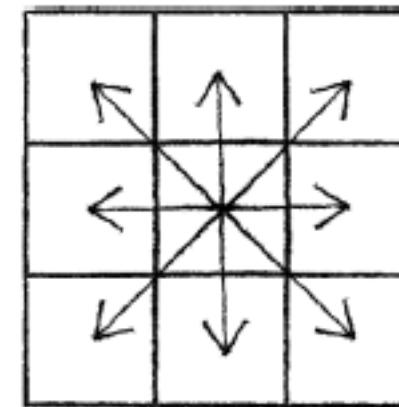
RandomWalk

# UNIFORM DISTRIBUTION IN COMPUTER GRAPHICS

- Uniform distribution for random walk
- Generates a sequence of graphic points where the location is randomly selected
- Given the location of a point, the next point will be randomly selected according to some linking rules



4 possible steps



8 possible steps

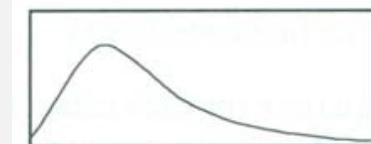
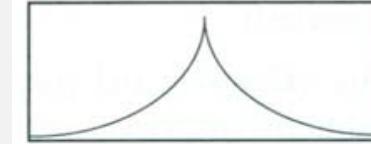
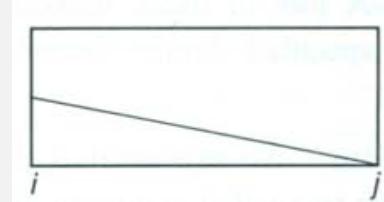
RandomWalkTrail

# OTHER DISTRIBUTIONS

- It is possible to have other probability distributions
  - Linear distributions
  - Exponential distributions
  - Bell shaped distribution
  - Asymmetrical distributions
  - U-shaped distributions

In SuperCollider: **Pwrand**

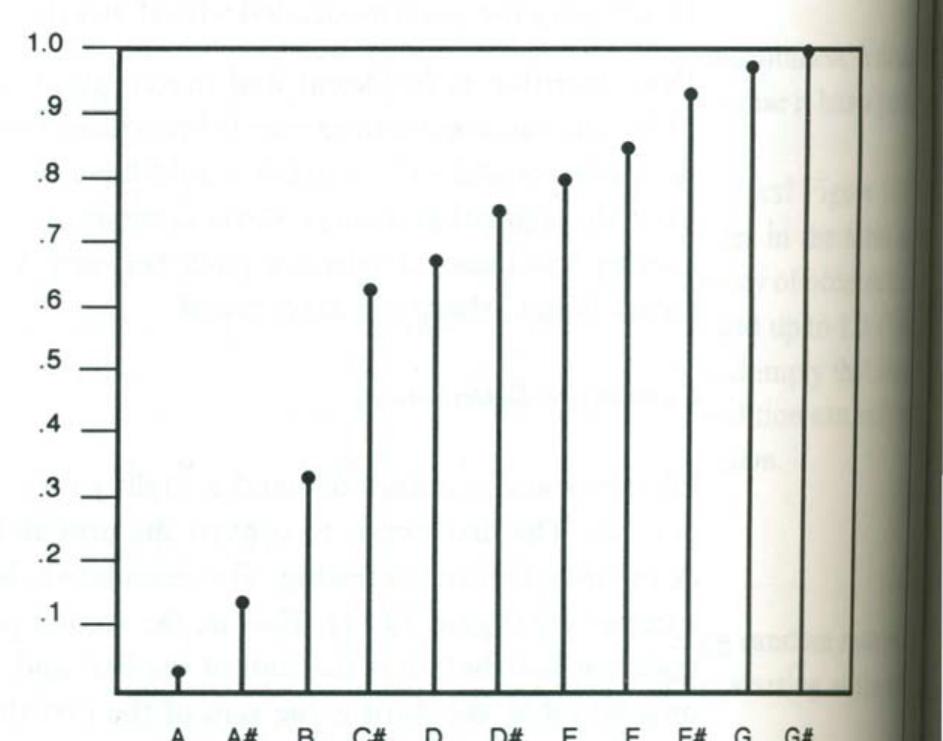
**Randomness\_no\_uniform**



# CUMULATIVE DISTRIBUTION IN MUSIC

- Cumulative distribution is interesting
- 1°step: Convert the probability distribution into a cumulative distribution by summing the values successively
- 2°step: Compare the pseudorandom number with the numbers in the cumulative distribution:

```
for i=A to G#
  if random_num is less than ptable(i)
  then
    play(i)
end
```



## FURTHER CONSIDERATIONS

- There are many extensions of the straightforward probability-based walk
- Allow the possibility to change probabilities over time: the piece starts with one distribution at the beginning and with another at the end (e.g. through some sort of interpolation).
- The selection process can be replicated to other
  - musical parameters such as register, duration, dynamic intensity,....
  - graphical parameter like brightness or colour

# THE LEVY FLIGHT

- Monte Carlo method
  - pick two random numbers
  - the second one is a “qualifying random value.” It will assess whether to use the first one or throw it away and pick another one
- A specific case is when the random number is the probability itself
  1. *Pick a random number:  $R_1$*
  2. *Compute a probability  $P$  that  $R_1$  should qualify. Let’s try:  $P = R_1$ .*
  3. *Pick another random number:  $R_2$*
  4. *If  $R_2$  is less than  $P$ , then we have found our number— $R_1$ !*
  5. *If  $R_2$  is not less than  $P$ , go back to step 1 and start over.*

## THE LEVY FLIGHT

- Here we are saying that the likelihood that a random value will qualify is equal to the random number itself
- Let's say we pick 0.1 for RI. This means that RI will have a 10% chance of qualifying. If we pick 0.83 for RI then it will have a 83% chance of qualifying
- The higher the number, the greater the likelihood that we will actually use it
- In our example:  $probability = pow(1.0 - r1, 8)$
- Smaller jump are preferred

## A SONIFICATION EXAMPLE THE LEVY FLIGHT

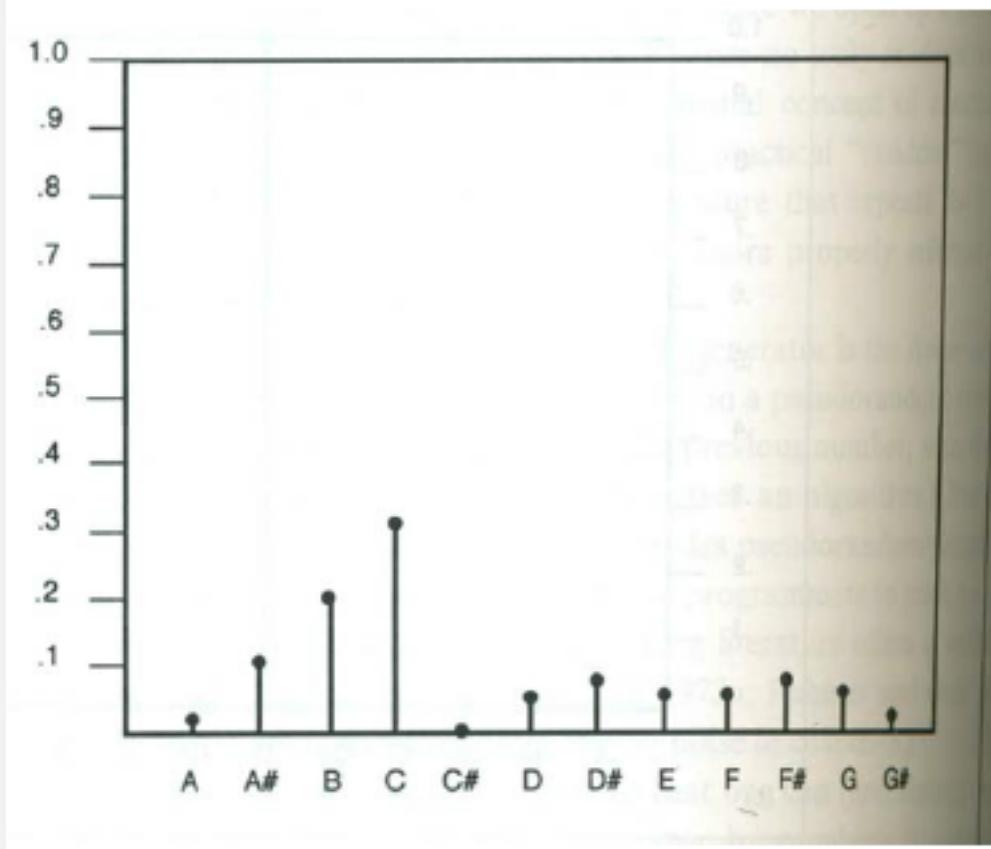
- Computer graphics random walk based on Monte Carlo probability model
- Uncertainty is not only applied to the direction of the next point but also to the distance
- Current position (x,y) sent to the Wobble Bass via OSC
  - **x/(width/2) -> cutoff multiplier**
  - **map((y+x),0,height+width,60,200) -> fundamental frequency**

**RandomWalkLevy**

**Multi voice Levy Flight**

## FURTHER CONSIDERATIONS

- Probability distribution can be learnt from a corpus of data



- E.g. Analysing the frequencies of notes in a set of composition we can retrieve the probability for each note
- The generated walk can follow the learnt probability distribution

# NOISE

- Specifically, a sequence of random values is called **noise**
- Probability distribution of data generation allows to define the correlation between consequent values in the noise
- Uniform distribution -> White noise -> no correlation between consequent values

In SuperCollider: **Pwhite**, **Pbrown**

Noise

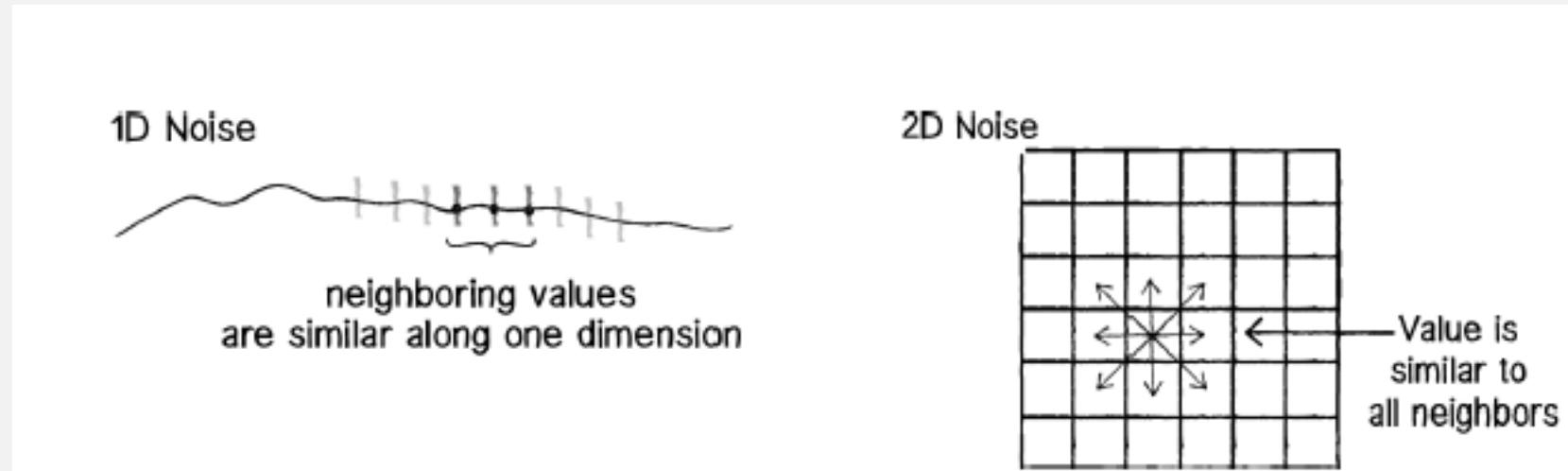
# NOISE AND COMPUTER GRAPHICS

- Noise is a fundamental elements in creative computing, however the standard definition is not suitable to model natural events
- It seem that in nature very few phenomena can be really described by uniform probability distribution
- In the early 1980s, Ken Perlin was working on the original *Tron* movie
- His task: build mathematical models to generate various effects with natural qualities, such as clouds, landscapes, and patterned textures like marble
- Those qualities exhibits patterns but at the same time a certain degree of randomness



# NOISE AND COMPUTER GRAPHICS

- The **perlin noise** tends to keep a certain degree of similarity between sequent values



- In Processing -> **noise()**
- Perlin noise is defined in an infinite n-dimensional space, in which each pair of coordinates corresponds to a fixed semi-random value (fixed only for the lifespan of the program)

**PerlinNoise2D**

**PerlinNoiseWalkAcceleration**

## GRAMMAR-BASED AGENTS



# GRAMMARS

- Let's think about natural language
- Human languages has three dimension: **syntax**, semantics and pragmatics
- Syntax
  - The language is composed by language **elements** (verbs, adjectives, etc.)
  - Elements are combined in sentences following **rules**
  - The set of elements and the set of rules compose a **grammar**
  - The **syntax** of a Language can be formalized using a formal grammar
- With a grammar:
  - Possible to assert if a sentence is part of the language or not
  - Generate new sentences following the rules

# GRAMMARS

- A formal grammar is a set of rules to expand high-level symbols (**non-terminal symbols** or **sentences**) into more detailed sequences of symbols (**terminal symbols** or **words**) representing elements of formal languages.
- Words are generated by repeatedly applying rewriting rules, in a sequence of so-called **derivation steps**.
- Grammars are suited to represent systems with hierarchical structure, which is reflected in the recursive application of the rules
- Rules are: **LEFT SIDE -> RIGHT SIDE**
  - The left side gets replaced by the right side
  - The left and right side can be either non-terminal or terminal symbols

# GRAMMARS - EXAMPLE

## A set of non-terminal symbols, V

- S :Sentence
- VP :Verbal Phrase
- NP :Noun Phrase
- V :Verb
- PP :Prepositional Phrase
- AP :Adjective Phrase
- Adv :Adverb
- P :Preposition
- Det :Determinant
- N :Noun

## A set of rules, R

- $S \rightarrow NP\ VP$
- $VP \rightarrow V\ (NP)\ (PP)$
- $AP \rightarrow (Adv)\ A\ (PP)$
- $PP \rightarrow P\ NP$
- $NP \rightarrow (Det)\ (AP)\ N\ (PP)$

$()$  = optional

S = initial symbols

## GRAMMARS - EXAMPLE

### A terminal symbols, $V_T$

- $V_T = \{\text{"Dolly"}, \text{"Bob"}, \text{"cat"}, \text{"dog"}, \text{"house"}, \text{"a"}, \text{"the"}, \text{"follows"}, \text{"likes"}, \text{"knows"}, \text{"small"}, \text{"big"}, \text{"nice"}, \text{"hungry"}, \text{"very"}, \text{"extremely"}, \text{"in"}, \text{"for"}, \text{"to"}\}$

### Non-terminal to terminal rewrite rules

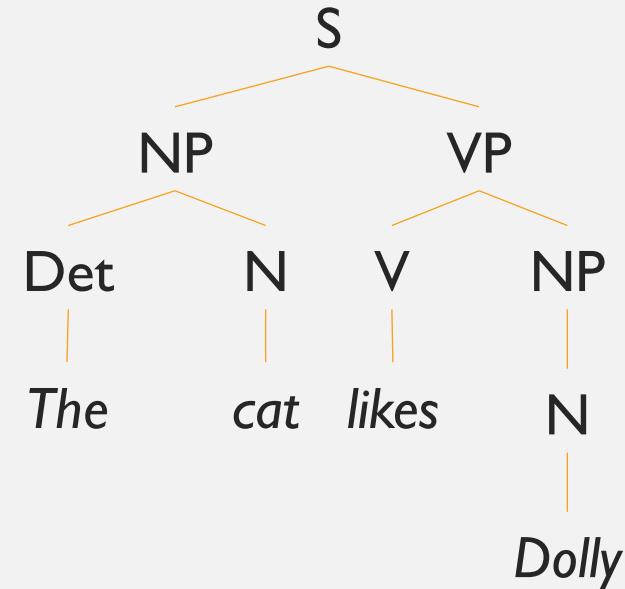
- N → “Dolly”, “Bob”, “cat”, “dog”, “house”
- Det → “a”, “the”
- V → “follows”, “likes”, “knows”
- A → “small”, “big”, “nice”, “hungry”
- Adv → “very”, “extremely”
- P → “in”, “for”, “to”

# GRAMMARS - DEFINITION

- A grammar is defined as  $G=[V, V_T, R, S]$
- $V$  = Set of non terminal symbols
- $V_T$  = Set of terminal symbols
- $R$  = Set of rules
- $S$  = Initial symbol

**Check if a sentence belong to the language**

*The cat likes Dolly*



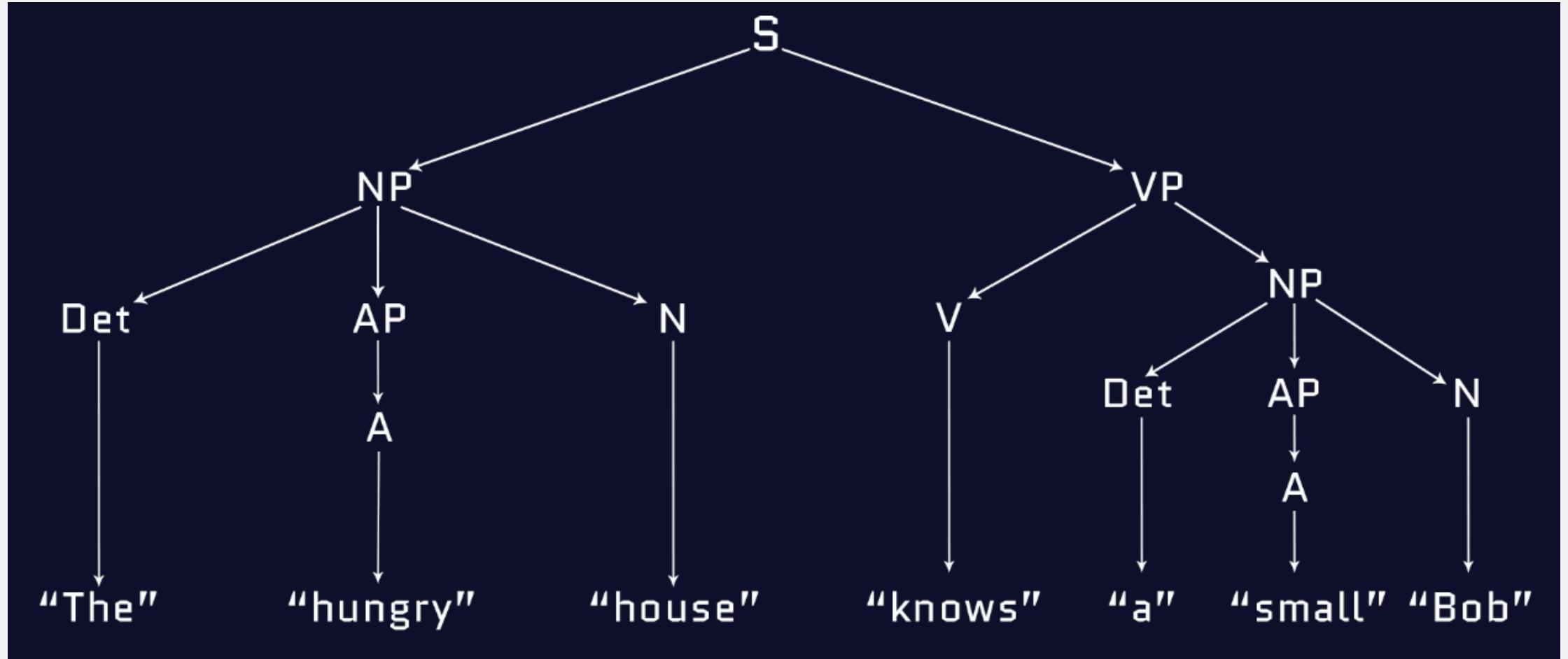
# GRAMMARS - DEFINITION

## Generate new sentences

|   |                                       |
|---|---------------------------------------|
| S   | :initial symbol                       |
| NP VP   | :applying rule $S \rightarrow NP\ VP$ |
| Det N VP  | :applying rule                        |
| “the” N VP  | :applying $Det \rightarrow “the”$     |
| “the” “dog” VP  | :applying $N \rightarrow “dog”$       |
| “the” “dog” V NP PP   | :applying $VP \rightarrow V\ NP\ PP$  |
| “the” “dog” “follows” NP PP                                     | :applying $V \rightarrow “follows”$   |
| “the” “dog” “follows” Det AP N PP                               | :applying $NP \rightarrow Det\ AP\ N$ |
| “the” “dog” “follows” “a” AP N PP                               | :applying $Det \rightarrow “a”$       |
| “the” “dog” “follows” “a” Adv A N PP                            | :applying $AP \rightarrow Adv\ A$     |
| “the” “dog” “follows” “a” “very” A N PP                         | :applying $A \rightarrow “big”$       |
| “the” “dog” “follows” “a” “very” “big” N PP                     | :applying $A \rightarrow “big”$       |
| “the” “dog” “follows” “a” “very” “big” “cat” PP                 | :applying $N \rightarrow “cat”$       |
| “the” “dog” “follows” “a” “very” “big” “cat” P NP               | :applying $PP \rightarrow N\ P$       |
| “the” “dog” “follows” “a” “very” “big” “cat” “in” NP            | :applying $P \rightarrow “in”$        |
| “the” “dog” “follows” “a” “very” “big” “cat” “in” Det N         | :applying $NP \rightarrow Det\ N$     |
| “the” “dog” “follows” “a” “very” “big” “cat” “in” “the” N       | :applying $Det \rightarrow “the”$     |
| “the” “dog” “follows” “a” “very” “big” “cat” “in” “the” “house” | :applying $N \rightarrow “house”$     |

# GRAMMARS - DEFINITION

Syntax correct but not the semantics



But ... in some context not-semantically correct sentences are creative sentences

## GRAMMARS - DEFINITION

- In the case more rules shares the left-side it is possible to define the probabilities to each rule to be applied

$R_1: S \rightarrow NP\ VP\ NP$

$p(R_1) = 0.1, \text{ rare}$

$R_2: S \rightarrow NP\ VP$

$p(R_2) = 0.3, \text{ occasional}$

$R_3: S \rightarrow NP$

$p(R_3) = 0.6, \text{ common}$

- The Grammar and the probability can be learnt by a corpus of data
- Many algorithms has been proposed

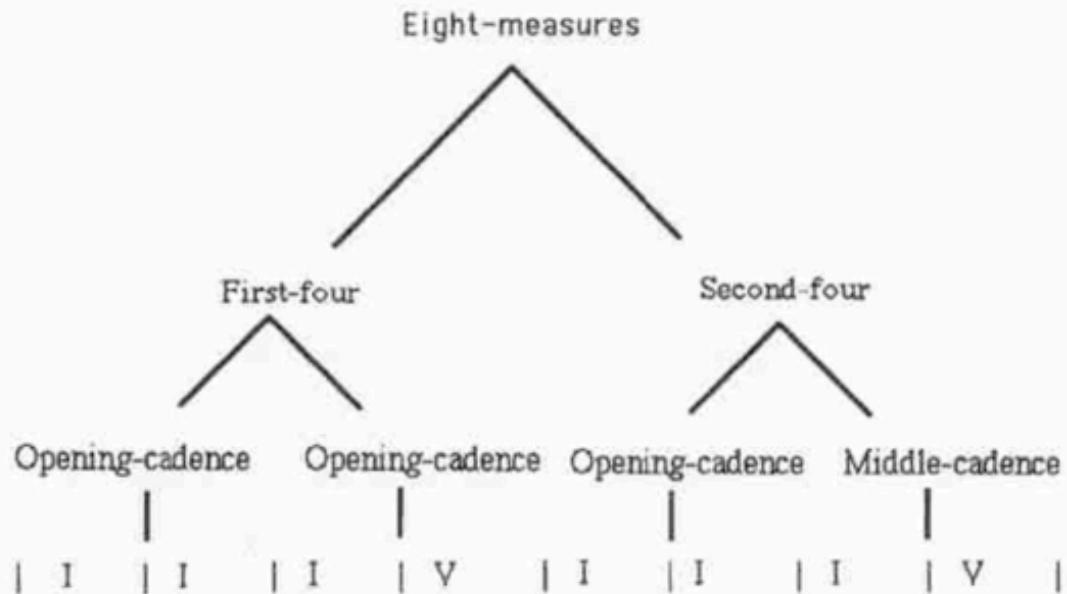
## GRAMMARS – MUSIC COMPOSITION

- Grammars are suited to represent systems with hierarchical structure, which is reflected in the recursive application of the rules
- As hierarchical structures can be recognized in most styles of music, it is hardly surprising that formal grammar theory has been applied to analyse and compose music for a long required for music composition
- First attempts to use grammars for music and relies on hand-made definition of symbols and rules

# GRAMMARS – MUSIC COMPOSITION

A Context-Free Grammar for Generating 8 Measure Tonal Chord Sequences

|                  |   |                  |                 |
|------------------|---|------------------|-----------------|
| Eight measures   | → | First four       | Second four     |
| First four       | → | Opening cadence  | Opening cadence |
|                  | → | Opening cadence' | Opening cadence |
| Second four      | → | Opening cadence  | Middle cadence  |
|                  | → | Opening cadence' | Closing cadence |
|                  | → | Middle cadence   | Middle cadence  |
|                  | → | Middle cadence   | Closing cadence |
| Opening cadence  | → | I I              | I I             |
|                  | → | I I              | V V             |
| Opening cadence' | → | I I              | III III         |
|                  | → | I I              | IV IV           |
| Middle cadence   | → | I I              | V V             |
|                  | → | V V              | V V             |
| Closing cadence  | → | I I              | I I             |
|                  | → | IV IV            | I I             |
|                  | → | V V              | I I             |



## IMPRO-VISOR

- In 2007 Keller and Morrison created the Impro-visor
- Educational grammar-based software for automatic Jazz solo melody creation
- *“Impro-Visor was designed to assist the soloist, who may not yet have a strong theory background, in the process of creating solos”*
- The grammar was built manually based on jazz music theory
- Later on they introduced a tool for an evaluation from user to adjust probability distributions in rules selection

[https://www.youtube.com/watch?v=I\\_AadO\\_A0BU](https://www.youtube.com/watch?v=I_AadO_A0BU)

## IMPRO-VISOR - RHYTHMIC SEQUENCES

- **I° Step – generate coherent skeletal rhythmic sequences**

- Terminal alphabet is { h, q }, representing half-notes and quaternotes
- Non-terminal alphabet is { S, M, H }
  - S represents a sequence of one or more measures
  - M represents a measure
  - H represents a half-measure
- The last two roles allow the syncopation

- $S \rightarrow M$
- $S \rightarrow MS$
- $M \rightarrow HH$
- $H \rightarrow h$
- $H \rightarrow qq$
- $S \rightarrow HS$
- $S \rightarrow qS$

## IMPRO-VISOR - RHYTHMIC SEQUENCES

- The complete set of terminal symbols
- There is a complex sequence of rules to generate the terminal symbols
  - **1** a whole-note
  - **2** a half-note
  - **4** a quarter-note
  - **4.** a dotted quarter-note
  - **8** an eighth-note
  - **16** a sixteenth-note
  - **4/3** a quarter-note triplet
  - **8/3** an eighth-note triplet
  - **16/3** a sixteenth-note triplet

## IMPRO-VISOR - RHYTHMIC SEQUENCES

- An example of a rhythmic sequence

- $S \rightarrow M \rightarrow HH \rightarrow hH \rightarrow hh$
- $S \rightarrow M \rightarrow HH \rightarrow hH \rightarrow hqq$
- $S \rightarrow M \rightarrow HH \rightarrow qqH \rightarrow qqh$
- $S \rightarrow MS \rightarrow HHS \rightarrow qqHS \rightarrow qqhS \rightarrow qqhHH \rightarrow qqhqqH \rightarrow qqhqqqq$

## IMPRO-VISOR - MELODY SEQUENCES

- **2° Step – once generated a rhythmic skeleton, we can provide a tone for each chord**
- Give a chord, notes can be:
- Terminal alphabet:
  - 1) C a chord tone.
  - 2) L a color tone.
  - 3) A an approach tone.
  - 4) H a “helpful” tone, defined to be any of one of the above.
  - 5) S a scale tone, a member of a scale designated as being compatible with the chord.
  - 6) X an arbitrary tone.
  - 7) R a rest.
- **Chord tones:** tones of the current chord
- **Color tones:** tones that are not in the chord, but which are individually sonorous with it.: “tensions”
- **Approach tones:** non-chord tones that make good transitions to chord-tones or color tones
- **Other:** tones that do not fall into one of the categories above

A8/3 designates an approach tone with a duration of a eighth note triplet.

## IMPRO-VISOR - MELODY SEQUENCES

1 is a duration string representing a whole-note

2 is a duration string representing a half-note

4 is a duration string representing a quarter-note

8 is a duration string representing an eighth-note

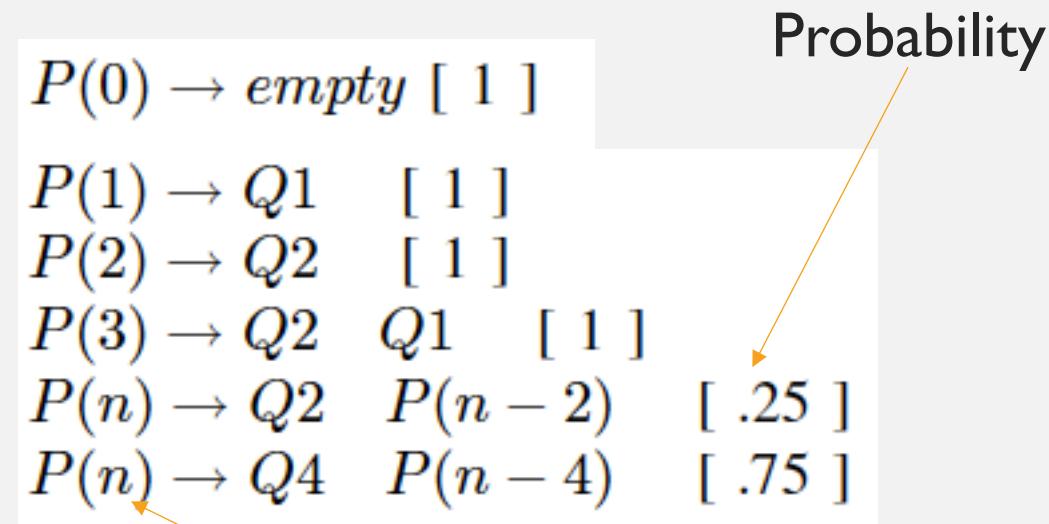
16 is a duration string representing a sixteenth-note

A8/3 designates an approach tone with a duration of a eighth note triplet.

- 1) **C** a chord tone.
- 2) **L** a color tone.
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- 5) **S** a scale tone, a member of a scale designated as being compatible with the chord.
- 6) **X** an arbitrary tone.
- 7) **R** a rest.

## IMPRO-VISOR - MELODY SEQUENCES

- The grammar for melody generation is like



Constraint: Number of quarter notes

Probability

## IMPRO-VISOR - MELODY SEQUENCES

- The remaining set of rules

$Q4 \rightarrow Q2 \quad V4 \quad V4 \quad [ 0.52 ]$   
 $Q4 \rightarrow V8 \quad N4 \quad N4 \quad N4 \quad V8 \quad [ 0.01 ]$   
 $Q4 \rightarrow V4 \quad Q2 \quad V4 \quad [ 0.47 ]$   
 $Q2 \rightarrow N2 \quad [ 0.06 ]$   
 $Q2 \rightarrow V4 \quad V4 \quad [ 0.6 ]$   
 $Q2 \rightarrow V8 \quad N4 \quad V8 \quad [ 0.12 ]$   
 $Q2 \rightarrow H4. \quad N8 \quad [ 0.16 ]$   
 $Q2 \rightarrow H4/3 \quad H4/3 \quad H4/3 \quad [ 0.06 ]$   
 $Q1 \rightarrow C4 \quad [ 1 ]$   
 $V4 \rightarrow N4 \quad [ 0.22 ]$   
 $V4 \rightarrow V8 \quad V8 \quad [ 0.72 ]$   
 $V4 \rightarrow H8/3 \quad H8/3 \quad H8/3 \quad [ 0.05 ]$   
 $V4 \rightarrow H8/3 \quad H8/3 \quad A8/3 \quad [ 0.01 ]$   
 $V8 \rightarrow N8 \quad [ 0.99 ]$   
 $V8 \rightarrow H16 \quad A16 \quad [ 0.01 ]$

$N2 \rightarrow C2 \quad [ 1 ]$   
 $N4 \rightarrow C4 \quad [ 0.5 ]$   
 $N4 \rightarrow L4 \quad [ 0.2 ]$   
 $N4 \rightarrow S4 \quad [ 0.5 ]$   
 $N4 \rightarrow A4 \quad [ 0.01 ]$   
 $N4 \rightarrow R4 \quad [ 0.25 ]$   
 $N8 \rightarrow C8 \quad [ 0.4 ]$   
 $N8 \rightarrow L8 \quad [ 0.2 ]$   
 $N8 \rightarrow S8 \quad [ 0.4 ]$   
 $N8 \rightarrow A8 \quad [ 0.01 ]$   
 $N8 \rightarrow R8 \quad [ 0.1 ]$

## IMPRO-VISOR - MELODY SEQUENCES

- A possible sequence

A8 L8 S8 C8 H4. S8

*approach, color, scale, helpful, scale*

- Give a C major 9



CM9

The image shows three staves of musical notation, each labeled "CM9". The notation consists of vertical stems with small horizontal dashes above them, indicating pitch. The first staff has a green dash at the top. The second staff has a blue dash at the top. The third staff has a blue dash at the top. The stems are mostly vertical, with some slight variations in angle.

CM9

CM9

## GRAMMAR-BASED COMPOSITION - EXAMPLE

To make the composition more intriguing, we need to have more variety in words and rules

- **Terminal symbols (words)**
  - **w:** whole note
  - **h:** half-note, i.e., half measure
  - **q:** quarter-note
  - **o:** quaver/ octave-note
  - **\$\{w, h, q, o\}:** corresponding pause
  - **t\{h, q, o\}** corresponding note in a triplet
- **Non-Terminal symbols (sentences)**
  - **S → M, SM**
  - **M → HH, QHQ, w, \$w, th th th**
  - **H → QQ, OQO, h, \$h, tq tq tq**
  - **Q → q, oo, to to to, \$q**
  - **O → o, \$o**
- **5 instruments**

Multitrack.mp3

# L-SYSTEMS

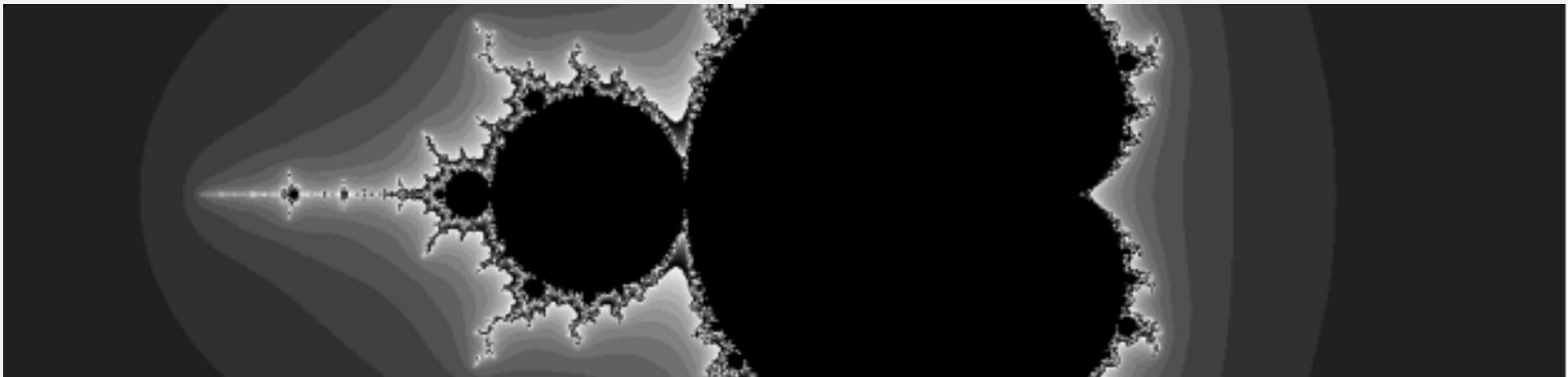


## L-SYSTEMS

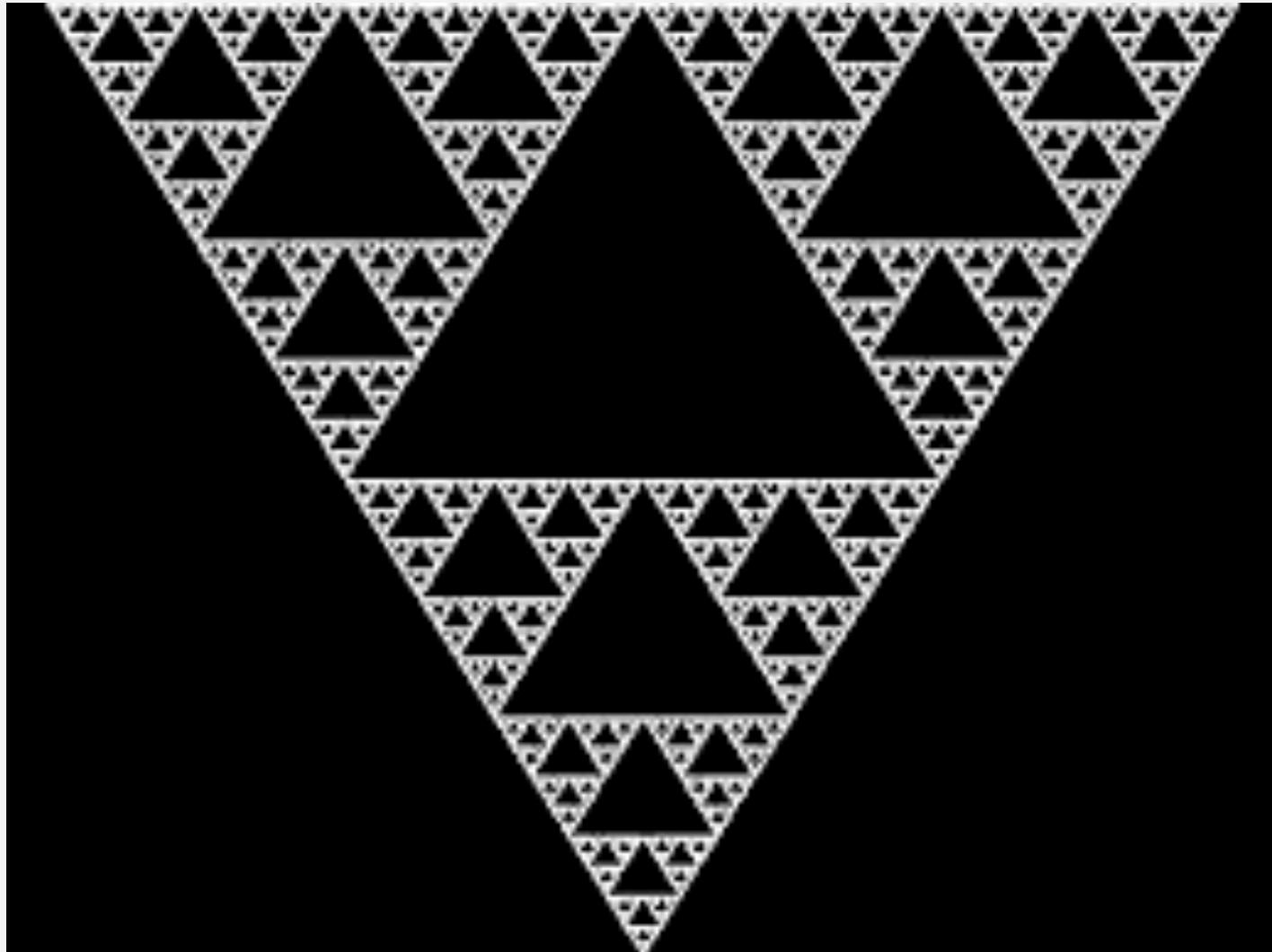
- In 1968, Hungarian botanist Aristid Lindenmayer developed a grammar-based system to model the growth patterns of plants
  - In L-system there is no difference between terminal symbols and non-terminal symbols
  - All the symbols corresponds to an action
- 
- L-System is defined as: **L=(v, w, P)**
  - v = an alphabet of a finite set of symbols
    - $v^*$  = set of all possible sequences generated from v (Kleene Closure)
    - $v^+ = v^* / \{\} = v^*$  without the empty sequence
  - w = seed/initiator  $\in v^+$
  - P = finite set of rules  $Z \rightarrow X$  where  $Z \in v$ , and the successor  $X \in v^*$

## L-SYSTEMS

- L-System are very power in generating fractals
- The fractals is defined “**a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole.**”



# L-SYSTEMS

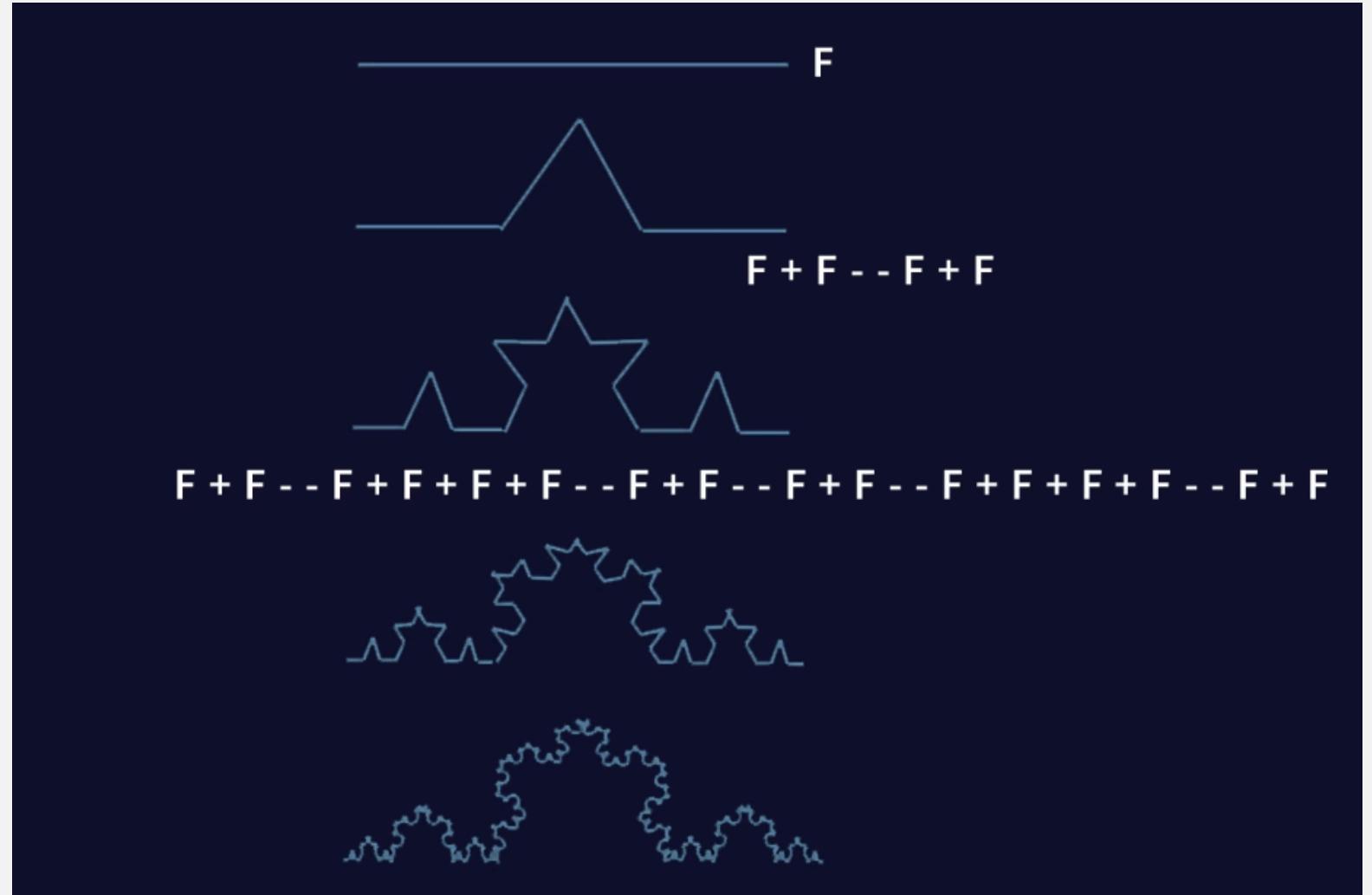


# L-SYSTEMS

- A classical L-System alphabet
  - F: Draw a line and move forward
  - G: Move forward (without drawing a line)
  - +: Turn left
  - -: Turn right
  - [: Save current location
  - ]: Restore previous location
- There is the need to define how the actions corresponds in terms of drawing.  
Example:
    - Turn left and turn right:  $60^\circ$
    - Move: 10 pixels

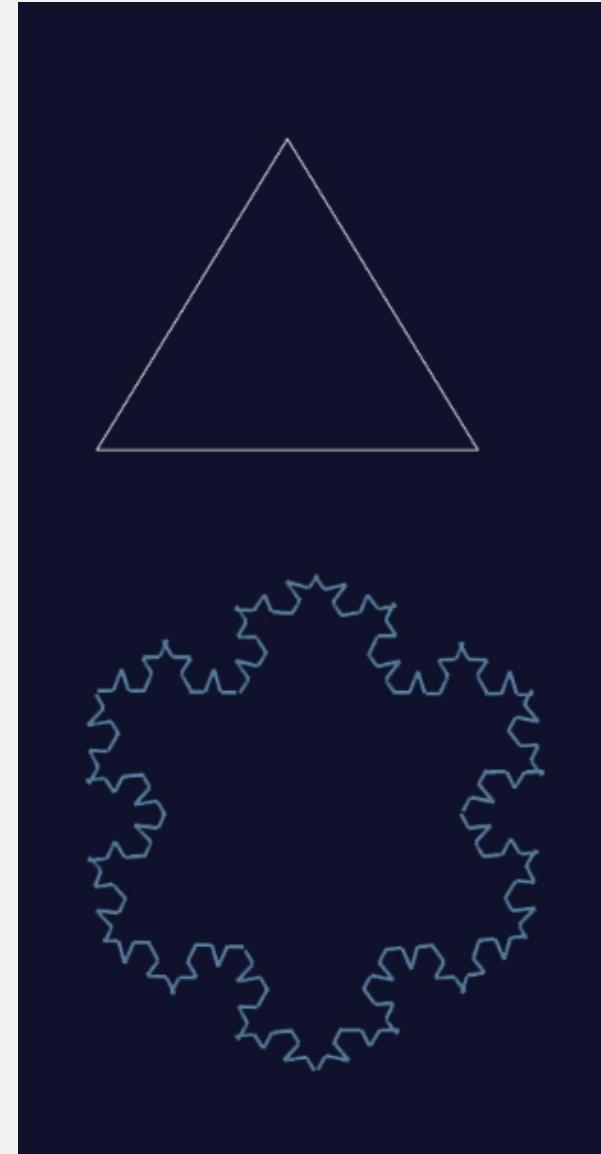
# THE KOCH CURVE

- $v^* = \{F, +, -, \{\}\}$
- Rule:  $F \rightarrow F+F- -F+F$
- Seed  $w = F$
- $F$ : Draw a line and move forward (length 2)
- $+$ : Turn left ( $60^\circ$ )
- $-$ : Turn right ( $60^\circ$ )



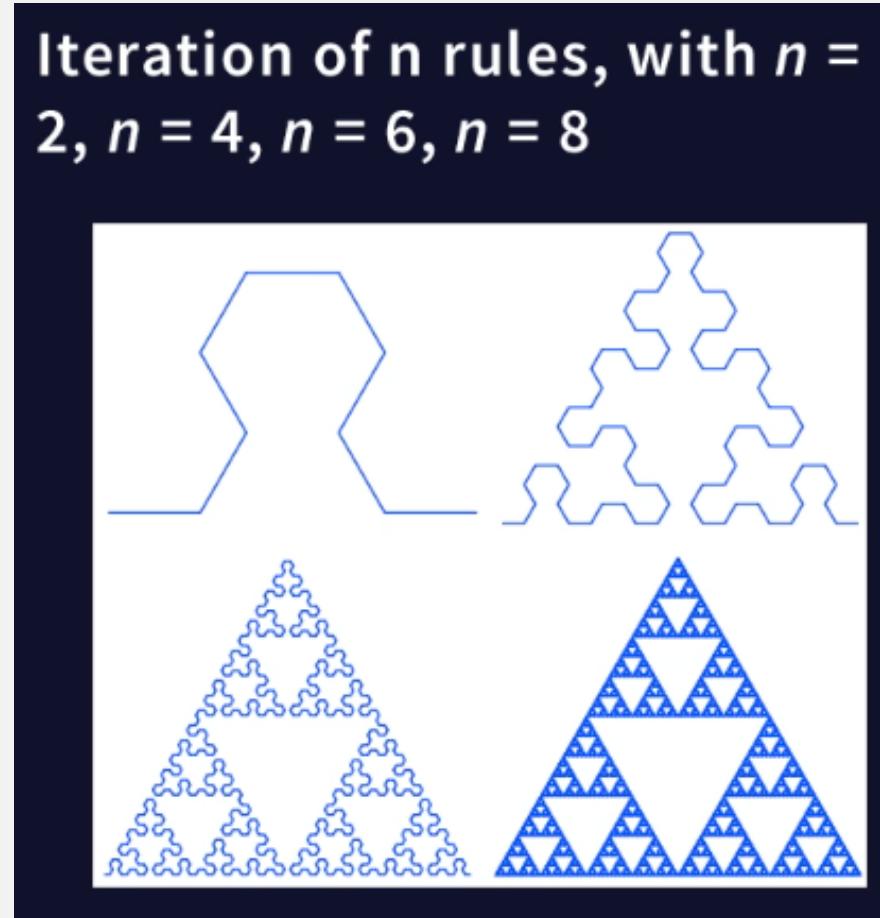
# THE KOCH CURVE

- It is possible to take a complex shape a Seed
- Rule:  $F \rightarrow F+F- -F+F$
- Seed =  $F - - F - - F$
- $\Phi = 60^\circ$  (angle)
- $L = 2$  (length)



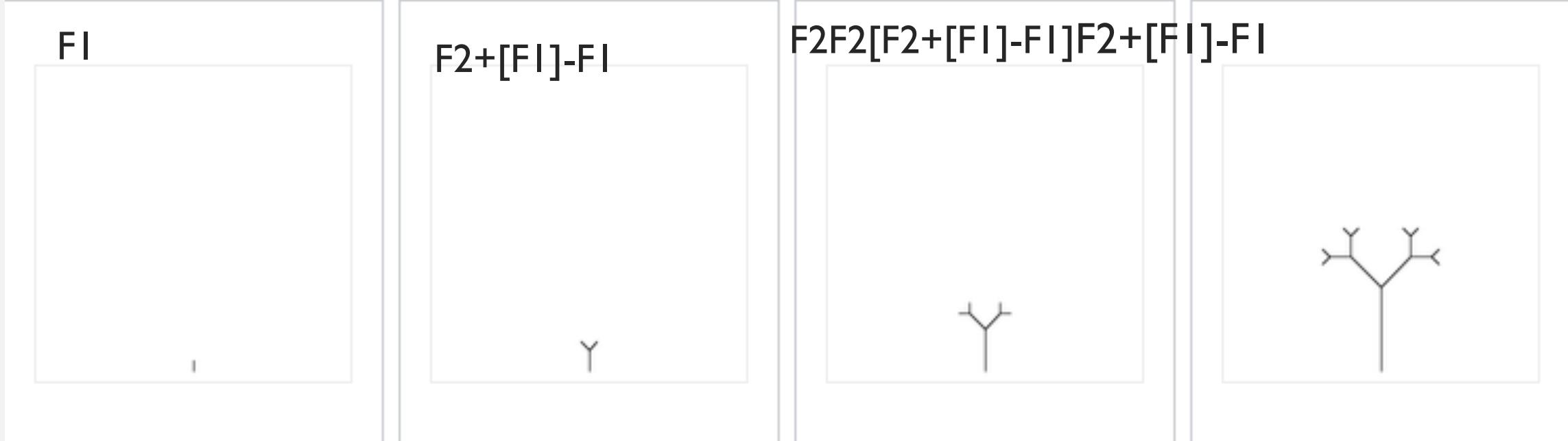
# SIERPINSKI TRIANGLE

- Vocabulary: F1, F2, +, -
- F1, F2: draw forward by L
- Rules:
  - $F1 \rightarrow +F2-F1-F2+$
  - $F2 \rightarrow -F1+F2+F1-$
- Seed = F
- $\Phi = 60^\circ$  (angle)
- $L = 2$  (length)
- For n=2
- $F1 \rightarrow +F2-F1-F2+ \rightarrow + -F1+F2+F1- - +F2-F1-F2+ - -F1+F2+F1-$



## L-SYSTEMS – BINARY TREE

- F1: draw a line segment ending in a leaf
- F2: draw a line segment
- Rules:
  - $F2 \rightarrow F2\ F2$
  - $F1 \rightarrow F2 +[F1]-F1$
- Seed = F1
- Phi =  $45^\circ$  (angle)
- L = 2 (length)



## L-SYSTEMS – FRACTAL PLANT

- Alphabet: X, F
  - X is not an action
- Rules:
  - $(X \rightarrow F+[[X]-X]-F[-FX]+X)$
  - $(F \rightarrow FF)$
- Seed = X
- Phi =  $25^\circ$  (angle)
- L = 2 (length)

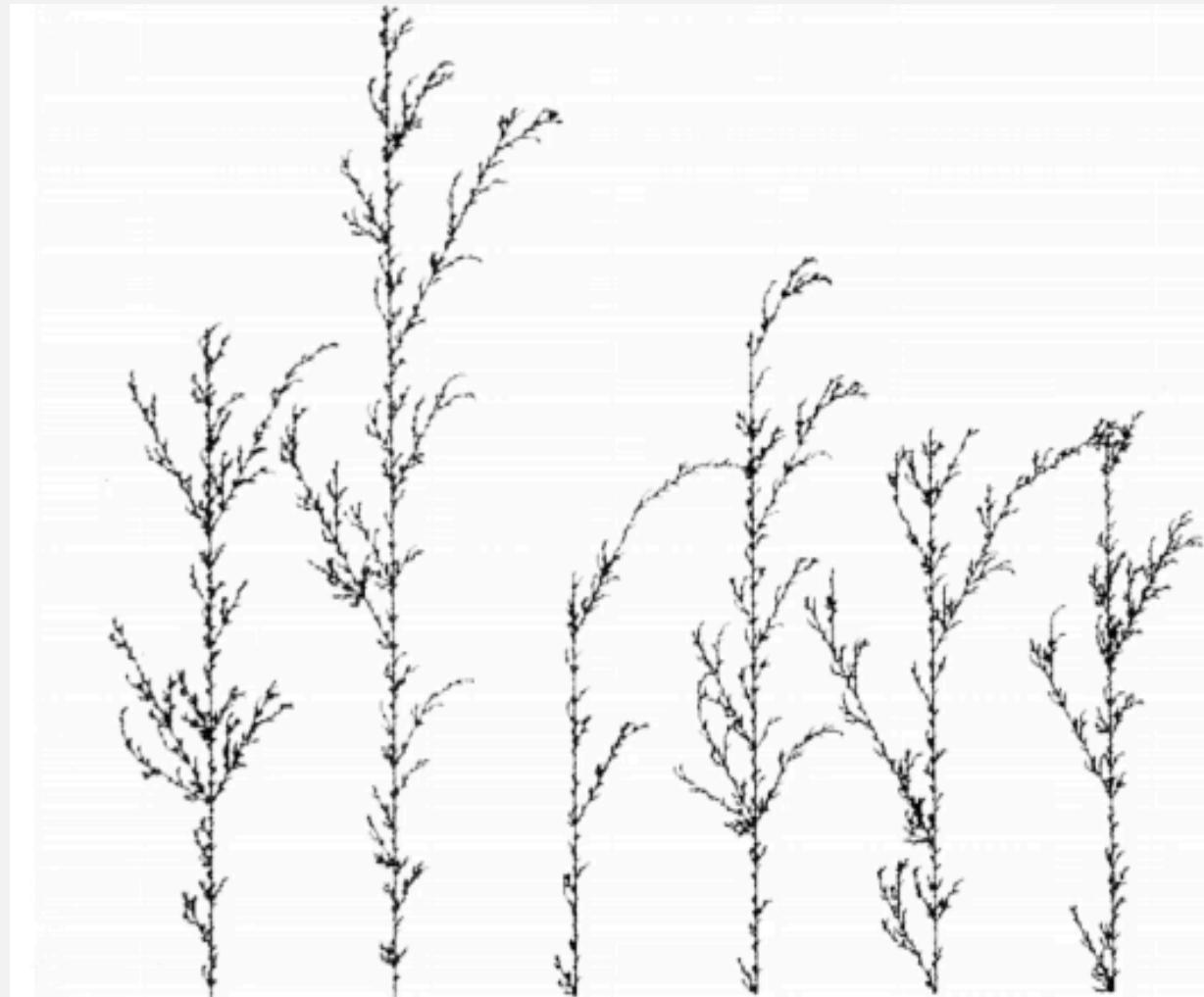


LSystem\_Tree

## STOCHASTIC L-SYSTEMS

- In deterministic L-Systems there is one rule per symbol
- In stochastic L-Systems there can be more production for each symbols
- In this case a likelihood is associated to each rule

|                                     |              |
|-------------------------------------|--------------|
| P1: $F \rightarrow F[[+F]]F[[-F]]F$ | $p(P1)=0.33$ |
| P2: $F \rightarrow F[[+F]]F$        | $p(P2)=0.33$ |
| P3: $F \rightarrow F[[-F]]$         | $p(P3)=0.34$ |



## L-SYSTEM - EXAMPLE

Jon McCormack - Bloom



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# MATERIALS

- **References**

- Robert M. Keller, David R. Morrison - A grammar approach to improvisation, Proceedings SMC'07, 4th Sound and Music Computing Conference, 11-13 July 2007, Lefkada, Greece
- Jose David Fernandez, Francisco Vico, AI Methods in Algorithmic Composition:A Comprehensive Survey, Journal of Artificial Intelligence Research 48 (2013)
- Chien-Hung Liu and Chuan-Kang Ting, Computational Intelligence in Music Composition:A Survey, IEEE Transactions on Emerging Topics in Computational Intelligence, December 2016,

- **Further readings**

- Terzidis - Algorithms for Visual Design Using the Processing Language, Wiley Publishing, 2009 – Chapter 7
- Daniel Shiffman – Nature of code, 2012 – Chapter 8
- Curtis Roads – The Computer music Tutorial, 1996 – Chapter 19