泰勒公式

1.泰勒级数

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

2.麦克劳林级数

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

3.重要展开式

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ..., -\infty < x < +\infty.$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + ... + (-1)^n x^n + ..., -1 < x < 1.$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ... + x^n + ..., -1 < x < 1.$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... + (-1)^{n-1} \frac{x^n}{n} + ..., -1 < x \leq 1.$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + ..., -\infty < x < +\infty.$$

$$\arcsin x = x + \frac{x^3}{3!} + o(x^3).$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots, \ -\infty < x < +\infty.$$

$$\tan x = x + \frac{x^3}{3} + o(x^3).$$

$$\arctan x = x - \frac{x^3}{3} + o(x^3).$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!} x^2 + \ldots + \frac{a(a-1)\ldots(a-n+1)}{n!} x^n + \ldots, \begin{cases} x \in (-1,1) & \text{当} a \le -1, \\ x \in (-1,1] & \text{当} -1 < a < 0, \\ x \in [-1,1] & \text{当} a > 0, a \notin N_+, \\ x \in R & \text{当} a \in N_+. \end{cases}$$