高数公式

> 答案版

不定积分公式

$$\int \tan x \, dx = -\ln|\cos x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int \frac{dx}{\cos x} = \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sin x} = \int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C\ (a>0)$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C\ (a>0)$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right) + C\ (\text{常见}\ a=1)$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x+\sqrt{x^2-a^2}\right| + C\ (|x|>|a|)$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + C\ (a>|x|\geq 0)$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C\ \ (\sin^2 x = \frac{1-\cos 2x}{2})$$

$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C\ \ (\cos^2 x = \frac{1+\cos 2x}{2})$$

$$\int \tan^2 x\,dx = \tan x - x + C\ \ (\tan^2 x = \sec^2 x - 1)$$

$$\int \cot^2 x\,dx = -\cot x - x + C\ \ (\cot^2 x = \csc^2 x - 1)$$

$\int a^x \, dx = \frac{a^x}{\ln a} + C \, , a > 0 \text{且} a \neq 1$

三角函数微分

$d \tan x = \sec^2 x \, dx$

$d \cot x = -\csc^2 x \, dx$

$d \sec x = \sec x \tan x \, dx$

$d \csc x = -\csc x \cot x \, dx$

$\frac{1}{a} d \arctan \frac{x}{a} = \frac{1}{a^2 + x^2} \, dx$

$d \arcsin \frac{x}{a} = \frac{1}{\sqrt{a^2 - x^2}} \, dx$

泰勒公式

1.泰勒级数

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

2.麦克劳林级数

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

3.重要展开式

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\dots+\frac{x^n}{n!}+\dots,-\infty<x<+\infty.$$

$$\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n=1-x+x^2-x^3+\dots+(-1)^n x^n+\dots,-1<x<1.$$

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n=1+x+x^2+x^3+\dots+x^n+\dots,-1<x<1.$$

$$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots+(-1)^{n-1}\frac{x^n}{n}+\dots,-1<x\le 1.$$

$$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\dots,-\infty<x<+\infty.$$

$$\arcsin x=x+\frac{x^3}{3!}+o(x^3).$$

$$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots+(-1)^n\frac{x^{2n}}{(2n)!}+\dots,-\infty<x<+\infty.$$

$$\tan x=x+\frac{x^3}{3}+o(x^3).$$

$$\arctan x=x-\frac{x^3}{3}+o(x^3).$$

$$(1+x)^a=1+ax+\frac{a(a-1)}{2!}x^2+\dots+\frac{a(a-1)\dots(a-n+1)}{n!}x^n+\dots,\begin{cases} x\in(-1,1) & 当 a\le -1\,, \\ x\in(-1,1] & 当 -1<a<0\,, \\ x\in[-1,1] & 当 a>0, a\notin N_+\,, \\ x\in R & 当 a\in N_+\,. \end{cases}$$