



# Data Mining

## -- Sequential Pattern

Instructor: Jen-Wei Huang

Office: 92528 in the EE building  
jwhuang@mail.ncku

### Example

SID	Sequences
100	<(1,5) (2) (3) (4)>
200	<(1) (3) (4) (3,5)>
300	<(1) (2) (3) (4)>
400	<(1) (3) (5)>
500	<(4) (5)>

min\_support = 2

- ▶ Some frequent sequential patterns
  - {1,2,3,4}, {1,3,5}, {4,5}

# Sequential Pattern

- ▶ “Mining of frequently occurring patterns related to time or other sequences.”
  - J. Han, *Data Mining – Concepts and Techniques*
- ▶ “Given a set of sequences, find the complete set of frequent subsequences”
  - J. Pei, *PrefixSpan*
- ▶ Ex) What items one will buy if he/she has bought some certain items

## Time-related data

- ▶ Customers’ buying behavior
- ▶ Stock price changes
- ▶ Natural phenomena
- ▶ Sensor network data
- ▶ Web access patterns
- ▶ DNA sequence applications

# Definition

- ▶ Let  $I = \{x_1, x_2, \dots, x_n\}$  be a set of different items.
- ▶ An element  $e$ , denoted by  $(x_i x_j \dots)$ , is a subset of items  $\subseteq I$  of which items appear in a sequence at the same time.
- ▶ A sequence  $s$ , denoted by  $\langle e_1, e_2, \dots, e_m \rangle$ , is an ordered list of elements.
- ▶ A sequence database  $Db$  contains a set of sequences and  $|Db|$  represents the number of sequences in  $Db$ .

# Definition

- ▶ A sequence  $\alpha = \langle a_1, a_2, \dots, a_n \rangle$  is a subsequence of another sequence  $\beta = \langle b_1, b_2, \dots, b_m \rangle$  if there exists a set of integers,  $1 \leq i_1 < i_2 < \dots < i_n \leq m$ , such that  $a_1 \subseteq b_{i_1}$ ,  $a_2 \subseteq b_{i_2}$ , ..., and  $a_n \subseteq b_{i_n}$ .
- ▶ The sequential pattern mining can be defined as "Given a sequence database,  $Db$ , and a user-defined minimum support,  $\text{min\_sup}$ , find the complete set of subsequences whose occurrence frequencies  $\geq \text{min\_sup} * |Db|$ ."

# How?

- ▶ Apriori-like algorithms
  - AprioriAll – by Agrawal *et al*
  - GSP – by Srikant *et al*
- ▶ Vertical format algorithms
  - SPADE – by Zaki *et al*
  - SPAM – by Ayres *et al*
- ▶ Partition-based algorithms
  - FreeSpan – by Han *et al*
  - PrefixSpan – by Pei *et al*

## Apriori-like Algorithms

- ▶ 1. Sort phase
  - Sort the database
  - Customer id as the primary key and time as the second key
- ▶ 2. Litemset phase
  - Count the frequency of the itemset
  - The fraction of customers who bought the itemset

# Apriori-like Algorithms

## ► 3. Transformation phase

- Transform each tx to all itemsets in the form of

C01: <(1,5) (2) (3) (4)>

C02: <(1) (3) (4) (3,5)>

C03: <(1) (2) (3) (4)>

C04: <(1) (3) (5)>

C05: <(4) (5)>

CID	Items
2	10 20
5	90
2	30
2	40 60 70
4	30
3	30 50 70
1	30
1	90
4	40 70
4	90
3	10
5	10
1	40 70
5	20
2	90
3	20



CID	Items
1	30 90 {40 70}
2	{10 20} 30 {40 60 70} 90
3	{30 50 70} 10 20
4	30 {40 70} 90
5	90 10 20



Itemset	#
10	3
20	3
30	4
40	3
50	1
60	1
70	4
90	4
{10 20}	1
{40 60}	1
{40 70}	3
{60 70}	1
{40 60 70}	1
{30 50}	1
{30 70}	1
{50 70}	1
{30 50 70}	1

Itemset	#	New
10	3	1
20	3	2
30	4	3
40	3	4
70	4	5
90	4	6
{40 70}	3	7

CID	Items
1	30 90 {40 70}
2	{10 20} 30 {40 60 70} 90
3	{30 50 70} 10 20
4	30 {40 70} 90
5	90 10 20



CID	Items
1	3 6 {4, 5, 7}
2	{1, 2} 3 {4, 5, 7} 6
3	{3, 5} 1 2
4	3 {4, 5, 7} 6
5	6 1 2

## Apriori-like Algorithms

- ▶ 4. Mining phase
  - Apriori-like algorithm
- ▶ 5. Maximal phase
  - Find the maximum patterns

CID	Items
1	3 6 {4, 5, 7}
2	{1, 2} 3 {4, 5, 7} 6
3	{3, 5} 1 2
4	3 {4, 5, 7} 6
5	6 1 2

Itemset	#
1 2	2
1 3	1
1 4	1
1 5	1
1 6	1
1 7	1
2 1	0
2 3	1
2 4	1
2 5	1
2 6	1
2 7	1
3 1	1
3 2	1

Itemset	#
3 4	3
3 5	3
3 6	3
3 7	3
4 1	0
4 2	0
4 3	0
4 5	0
4 6	2
4 7	0
5 1	1
5 2	1
5 3	0
5 4	0

Itemset	#
5 6	2
5 7	0
6 1	1
6 2	1
6 3	0
6 4	1
6 5	1
6 7	1
7 1	0
7 2	0
7 3	0
7 4	0
7 5	0
7 6	2

CID	Items
1	3 6 {4, 5, 7}
2	{1, 2} 3 {4, 5, 7} 6
3	{3, 5} 1 2
4	3 {4, 5, 7} 6
5	6 1 2

Itemset	#
3 4 6	2
3 5 6	2
3 7 6	2

Itemset	#	
10	3	1
20	3	2
30	4	3
40	3	4
70	4	5
90	4	6
{40 70}	3	7

Therefore, frequent sequential patterns are:

<1 2> <3 4> <3 5> <3 6> <3 7> <4 6> <5 6> <7 6>  
 <3 4 6> <3 5 6> <3 7 6>

According to mappings, original frequent sequential patterns are:

<10 20> <30 40> <30 70> <30 90> <30 {40 70}>  
 <40 90> <70 90> <{40 70} 90> <30 40 90> <30 70 90>  
 <30 {40 70} 90>

# Maximal Sequential Patterns

<{1 5} {2} {3} {4}>
<{1} {3} {4} {3 5}>
<{1} {2} {3} {4}>
<{1} {3} {5}>
<{4} {5}>

Customer Sequences

Sequence	Support
<1>	4
<2>	2
<3>	4
<4>	4
<5>	4

Large 1-Sequences

Sequence	Support
<1 2>	2
<1 3>	4
<1 4>	3
<1 5>	2
<2 3>	2
<2 4>	2
<3 4>	3
<3 5>	2
<4 5>	2

Large 2-Sequences

Sequence	Support
<1 2 3>	2
<1 2 4>	2
<1 3 4>	3
<1 3 5>	2
<2 3 4>	2

Sequence	Support
<1 2 3 4>	2

Large 4-Sequences

Sequence	Support
<1 2 3 4>	2
<1 3 5>	2
<4 5>	2

Maximal Large Sequences

## Drawbacks

- ▶ A huge set of candidate sequences generated.
  - Especially 2-item candidate sequence.
- ▶ Multiple Scans of database needed.
  - The length of each candidate grows by one at each database scan.
- ▶ Inefficient for mining long sequential patterns.
  - A long pattern grow up from short patterns
  - The number of short patterns is exponential to the length of mined patterns.



# Vertical Format Algorithms

- ▶ Transform the database into vertical format
- ▶ Use vertical list format, for each sequential pattern
- ▶ Grow the subsequences (patterns) one item at a time by Apriori candidate generation

	<A>	<B>	<(AB)>	<AB>
C01,T01	1	1	1	0
C01,T02	1	0	0	0
C01,T03	0	1	0	1
C02,T01	1	1	1	0
C02,T02	0	1	0	1
C02,T03	1	0	0	0

SID	EID	Items
1	1	a
1	2	abc
1	3	ac
1	4	d
1	5	cf
2	1	ad
2	2	c
2	3	bc
2	4	ae
3	1	ef
3	2	ab
3	3	df
3	4	c
3	5	b
4	1	e
4	2	g
4	3	af
4	4	c
4	5	b
4	6	c

a		b		...
SID	EID	SID	EID	...
1	1	1	2	
1	2	2	3	
1	3	3	2	
2	1	3	5	
2	4	4	5	
3	2			
4	3			

ab			ba			...
SID	EID (a)	EID(b)	SID	EID (b)	EID(a)	...
1	1	2	1	2	3	
2	1	3	2	3	4	
3	2	5				
4	3	5				

aba				...
SID	EID (a)	EID(b)	EID(a)	...
1	1	2	3	
2	1	3	4	

# Drawbacks

- ▶ Still a huge set of candidate sequences to be examined.
- ▶ Inefficient for mining long sequential patterns.
- ▶ Large working space is needed.

## Partition-based Algorithms

- ▶ A sequence  $P=\{p_1, p_2, \dots, p_k\}$  is a prefix of another sequence  $S=\{s_1, s_2, \dots, s_n\}$  if for all  $i=1 \sim k$ ,  $p_i \subseteq s_i$ , and  $k \leq n$
- ▶  $\langle a \rangle$ ,  $\langle aa \rangle$ ,  $\langle a(ab) \rangle$  and  $\langle a(abc) \rangle$  are all prefixes of sequence  $\langle a(abc)(ac)d(cf) \rangle$

# Partition-based Algorithms

- Given a sequence  $\langle a(abc)(ac)d(cf) \rangle$

Prefix	Projection
$\langle a \rangle$	$\langle (abc)(ac)d(cf) \rangle$
$\langle aa \rangle$	$\langle (\_bc)(ac)d(cf) \rangle$
$\langle ab \rangle$	$\langle (\_c)(ac)d(cf) \rangle$

# Partition-based Algorithms

- ▶ Step 1: find length-1 sequential patterns
  - $\langle a \rangle$ ,  $\langle b \rangle$ ,  $\langle c \rangle$ ,  $\langle d \rangle$ ,  $\langle e \rangle$ ,  $\langle f \rangle$
- ▶ Step 2: divide the original database into the projections of each 1-sequential patterns
  - 6 sub-databases:
  - The ones having prefix  $\langle a \rangle$ ,  $\langle b \rangle$ , .....,  $\langle f \rangle$

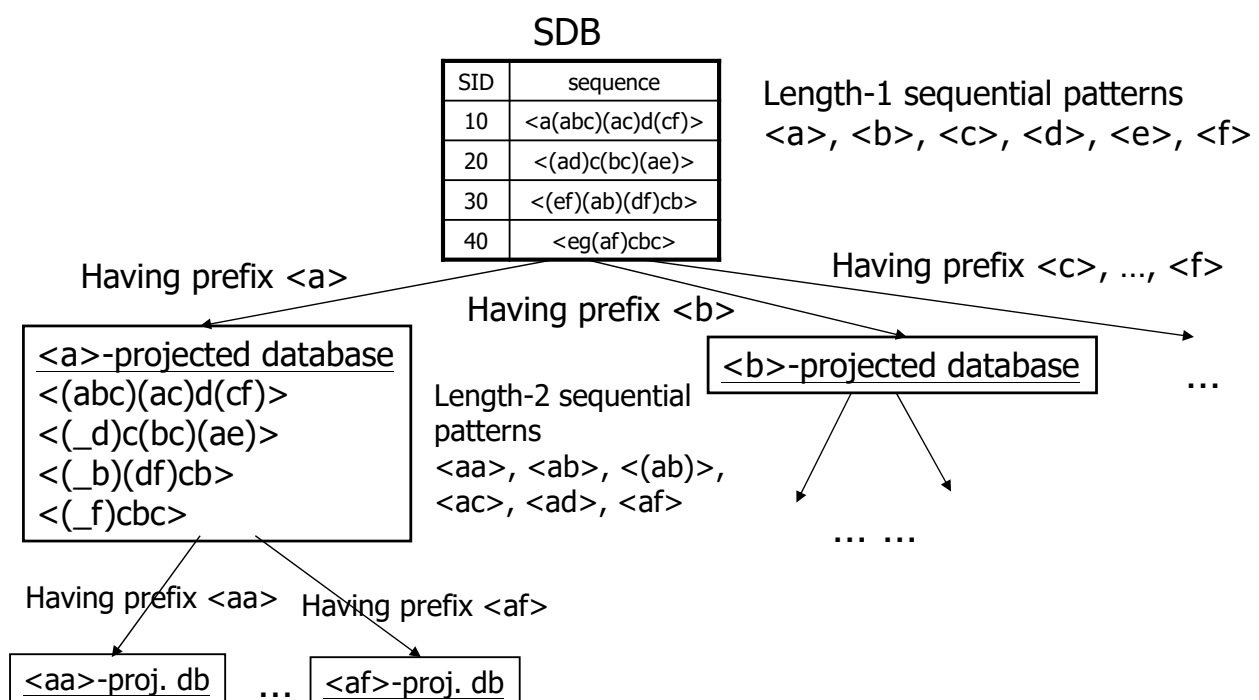
SID	sequence
10	$\langle a(abc)(ac)d(cf) \rangle$
20	$\langle (ad)c(bc)(ae) \rangle$
30	$\langle (ef)(ab)(df)cb \rangle$
40	$\langle eg(af)cbc \rangle$

# Partition-based Algorithms

- ▶ For  $\langle a \rangle$ -projected database:
- ▶ Find all length-2 seq. pat. having prefix  $\langle a \rangle$ 
  - $\langle aa \rangle$ ,  $\langle ab \rangle$ ,  $\langle (ab) \rangle$ ,  $\langle ac \rangle$ ,  $\langle ad \rangle$ ,  $\langle af \rangle$
- ▶ Then, partition into sub-sub-databases
  - Having prefix
  - $\langle aa \rangle$ ,  $\langle ab \rangle$ ,
  - $\langle (ab) \rangle$ ,  $\langle ac \rangle$ ,
  - $\langle ad \rangle$ ,  $\langle af \rangle$

$\langle a \rangle$ -projected database  
 $\langle (abc)(ac)d(cf) \rangle$   
 $\langle (_d)c(bc)(ae) \rangle$   
 $\langle (_b)(df)cb \rangle$   
 $\langle (_f)cbc \rangle$

# Partition-based Algorithms



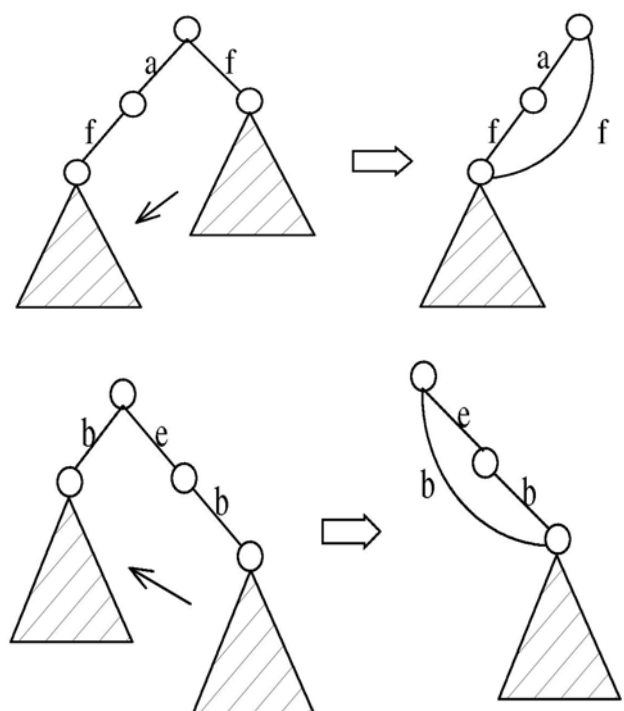
\*Adapted from Prof. Jiawei Han's slides

# Drawbacks

- ▶ Construction of projected databases.
- ▶ Large working space is needed.

## Closed Sequential Patterns

- ▶ A **closed sequential pattern**  $s$ : there exists no superpattern  $s'$  such that  $s' \supset s$ , and  $s'$  and  $s$  have the same support
- ▶ Motivation: reduces the number of (redundant) patterns but attains the same expressive power
- ▶ Using Backward Subpattern and Backward Superpattern pruning to prune redundant search space



# References

- ▶ [1] R. Srikant and R. Agrawal. Mining sequential patterns: Generalizations and performance improvements. EDBT'96.
- ▶ [2] M. Zaki. SPADE: An Efficient Algorithm for Mining Frequent Sequences. Machine Learning, 2001.
- ▶ [3] J. Pei, J. Han, H. Pinto, Q. Chen, U. Dayal, and M.-C. Hsu. PrefixSpan: Mining Sequential Patterns Efficiently by Prefix-Projected Pattern Growth. ICDE'01 (TKDE'04).
- ▶ [4] X. Yan, J. Han, and R. Afshar. CloSpan: Mining Closed Sequential Patterns in Large Datasets. SDM'03

# References

- ▶ Slides from Prof. J.-W. Han, UIUC
- ▶ Slides from Prof. M.-S. Chen, NTU
- ▶ Slides from Prof. W.-Z. Peng, NCTU