# Hack to Exam: Time Series Analysis

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# 1 Retrospective

#### 1.1 Chapter 1

#### 1.1.1 The decomposition of Time Series

$$X_t = m_t + s_t + Y_t$$

where  $X_t$  is the time series,  $m_t$  is the trend component,  $s_t$  is the seasonal component,  $Y_t$  is a stationary stochastic process.

#### 1.1.2 The Stationary Series

**Definition 1 (Stationary Series)** A time series  $\{X_t, t \in T\}$  is said to be a stationary series, satisfying the condition as follow:

- 1.  $E(X_t)^2 < \infty$  for all  $t \in T$
- 2.  $E(X_t) = \mu$  for all  $t \in T$
- 3. let  $\gamma_X(r,s) := E(X_r \mu)(X_s \mu)$  and  $\gamma_X(r,s) = \gamma_X(r+t,s+t)$  for all  $t \in T$

**Definition 2** A process  $\{X_t, t \in \mathbb{Z}\}$  is said to be a white noise with mean  $\mu$  and variance  $\sigma^2$ , written

$$\{X_t\} \sim WN(\mu, \sigma^2)$$
 if  $EX_t = \mu$  and  $\gamma_X(h) = \begin{cases} \sigma^2 & h = 0\\ 0 & h \neq 0 \end{cases}$ 

#### 1.1.3 Linear Stationary Series and Linear Filter

#### Definition 3

# 1.1.4 Normal Time Series

# 1.1.5 Strict Stationary Series

**Definition 4** The time series  $\{X_t, t \in \mathbb{Z}\}$  is said to be strictly stationary if the distribution of

$$(X_{t_1}, \cdots, X_{t_k})$$
 and  $(X_{t_1+h}, \cdots, X_{t_k+k})$ 

are the same for all k, and all  $t_1, \dots, t_k, h \in \mathbb{Z}$ 

# 1.1.6 The Spectral Density