TSA by JG

Intro

Stationarity

Trends & Seasonal Components

No Seasonal Components

Trend and Seasonality

Autocovariance

Strict stationarity

spectral density

Time Series Model TSM

Estimation

Estimation of μ

Estimation of $\gamma(\cdot)$ and $\rho(\cdot)$

Prediction

inference

Prediction of Random Variables PRV

Further Prediction

Further PRV

Predition for stationary time series PSTS

wold decomposition

Partial Correlation

PA Partial Autocorrelation

ARMA Processes

ACVF and how to cal

Prediction of ARMA

Spectral Analysis

Spectral Distribution

Spectral Representation of a time series

Predition in the frequency domain PiFD

interpolation and detection

The Itˆo intergral

Estimation of the spectral density

periodogram

Smoothing the Periodogram SP

Linear Filters

ARMA Processes

Estimation for ARMA models

Yule-Walker estimation

Burg's algorithm

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Maximum Likelihood and Lead Square Estimation

Order Selection

unit roots

Multivariate Time Series MTS

Financial Time Series

Kalman Filtering

State-Space Representation

Prediction of Multivariate random variables

Appendix

A.1 Stochastic Processes

Definition (Stochastic Process) A stochastic process is a *family of random variables* $\{X_t\ , t \in T\}$ *defined on a probability space* $(\Omega, \mathcal{F}, P)$ as follows:

- Ω is a set
- $\mathcal{F}$ is a σ-field i.e.
  - (a) $\emptyset\ \in\ \mathcal{F}$
  - (b) $A_i\ \in\ \mathcal{F}\quad \forall\, i\ \in\ I$ then $\bigcup_i A_i\ \in\ \mathcal{F}$
  - (c) $A\ \in\ \mathcal{F}$ then $A^c\ \in\ \mathcal{F}$
- P is a function $\mathcal{F} \rightarrow [0, 1]$ satisfying:
  - $P(\Omega) = 1$
  - $A_i\ \in\ \mathcal{F}\quad \forall\, i\ \in\ I$ and $A_i \cap A_j = \emptyset\quad \forall\, i\ \in\ I$ then $P(\bigcup_i A_i) = \sum_i P(A_i)$

proposition $P(A) + P(A^c) = 1$

There are definitions on Cholton's Style:

(X is *random variable*) iff ((X is a function $\Omega \to \mathbb{R}$) and ($\{\omega \in \Omega : X(\omega) \leq x\} \in \mathcal{F}$ for all $x \in \mathbb{R}$))

($\{X_t\ ,\ t \in T\}$ is *stochastic process*) iff (X_t is *random variable* for all $t \in T$)

and T is called *index* or *parameter set*

($\{X_t\ ,\ t \in T\}$ is a *time series*) iff ($T \subset \mathbb{Z}$)

Definition (sample-path) The functions $\{X_t(\omega)\ ,\ \omega \in \Omega\}$ on T are called realizations or *sample-path* of the process $\{X_t\ ,\ t \in T\}$

$F_X, x \to P(X \leq x)$ is called the distribution function of a random variable X $F_{(\cdot)} : (\Omega \to \mathbb{R}) \to (\mathbb{R} \to [0,1])$ is called the distribution

The case in higher demension is similar.

$X = (X_1, \cdots, X_n)^\top$ is a n-dim *random variable*, X_i is a *random variable* for $1 \leq i \leq n$

Definition (The distribution of a stochastic process) let $\mathcal{T} := \{t \in T^n\ :\ t_i < t_j\}$ The (finitie-dimensional) distribution function are the family $\{F_t(\cdot)\ ,\ t \in \mathcal{T}\},\quad F_t(x) = P(X_{t_1} \leq x_1, \cdots, X_{t_n} \leq x_n)\quad t \in T^n\ ,\ x \in \mathbb{R}$ The distribution of $\{X_t\ ,\ t \in T\}$ is the family $\{F_t(\cdot)\ ,\ t \in \mathcal{T}\}$

obviously $\mathcal{T}$ is a simplex In a way, we can say:"$t \in T$ is a *series of time*". So the F_t is a distribution of a *series of time* of random variable i.e. the distribution of *stochastic process*. In conviencity, $F_t \sim \mathcal{T} \sim X_t \sim n$, where the symbol $\sim$ readed as 'related to'.

Theorem (Kolmogorov's existence theorem) The family $F_t(\cdot)\ ,\ t \in \mathcal{T}$ are the distribution functions of some stochastic process iff for any $n, t = (t_1, \cdots, t_n) \in \mathcal{T}, x \in \mathbb{R}^n$ and $1 \leq k \leq n$

$\lim_{x_k \to \infty} F_t(x) = F_{t(k)}(x(k))$

A.2 Hilbert Spaces