TSA by JG

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Appendix

A.1 Stochastic Processes

Definition (Stochastic Process) A stochastic process is a *family of random variables* $\{X_t\ ,\ t \in T\}$ *defined on a probability space* $(\Omega, \mathcal{F}, P)$ as follows:

- Ω is a set
- $\mathcal{F}$ is a σ-field i.e.
  - (a) $\emptyset\ \in\ \mathcal{F}$
  - (b) $A_i\ \in\ \mathcal{F}\quad \forall\, i\ \in\ I$ then $\bigcup_i A_i\ \in\ \mathcal{F}$
  - (c) $A\ \in\ \mathcal{F}$ then $A^c\ \in\ \mathcal{F}$
- P is a function $\mathcal{F} \to [0,1]$ satisfying:
  - $P(\Omega) = 1$
  - $A_i\ \in\ \mathcal{F}\quad \forall\, i\ \in\ I$ and $A_i \cap A_j = \emptyset\quad \forall\, i\ \in\ I$ then $P(\bigcup_i A_i) = \sum_i P(A_i)$

proposition $P(A) + P(A^c) = 1$

There are definitions on Cholton's Style:

(X is *random variable*) iff ((X is a function $\Omega \to \mathbb{R}$) and ($\{\omega\ \in\ \Omega : X(\omega) \leq x\}\ \in\ \mathcal{F}$ for all $x\ \in\ \mathbb{R}$))

and we denoted $P(X \leq x) := P(X^{-1}([-\infty, x])$ and $P(X < x) := P(X^{-1}([-\infty, x)$

($\{X_t\ ,\ t \in T\}$ is *stochastic process*) iff (X_t is *random variable* for all $t\ \in\ T$)

and T is called *index* or *parameter set*

($\{X_t\,,\,t \in T\}$ is a *time series*) iff ($T \subset \mathbb{Z}$)

Definition (sample-path) The functions $\{X_t(\omega)\,,\,\omega\,\in\,\Omega\}$ on T are called realizations or *sample-path* of the process $\{X_t\,,\,t\,\in\,T\}$

$F_X, x \to P(X \le x)$ is called the distribution function of a random variable X
$F_{(\cdot)} : (\Omega \to \mathbb{R}) \to (\mathbb{R} \to [0,1])$ is called the distribution

The case in higher demension is similar.

$X = (X_1,\,\cdots\,, X_n)^\top$ is a n-dim *random variable*, X_i is a *random variable* for $1\,\le\,i\,\le\,n$

Definition (The distribution of a stochastic process) let
$\mathcal{T} := \{t \in T^n\,:\,t_i < t_j\}$
The (finitie-dimensional) distribution function are the family $\{F_t(\cdot)\,,\,t\,\in\,\mathcal{T}\}$,
$F_t(x) = P(X_{t_1} \le x_1, \cdots, X_{t_n} \le x_n) \quad t\,\in\,T^n\,,\,x \in \mathbb{R}$
The distribution of $\{X_t\,,\,t \in T\}$ is the family $\{F_t(\cdot)\,,\,t\,\in\,\mathcal{T}\}$

obviously $\mathcal{T}$ is a simplex
In a way, we can say:"$t \in T$ is a *series of time*". So the F_t is a distribution of a *series of time* of random variable i.e. the distribution of *stochastic process*. In conviencity, $F_t \sim \mathcal{T} \sim X_t \sim n$, where the symbol $\sim$ readed as 'related to'.

Theorem (Kolmogorov's existence theorem) The family $F_t(\cdot)\,,\,t\,\in\,\mathcal{T}$ are the distribution functions of some stochastic process iff for any n, $t = (t_1, \cdots, t_n)\,\in\,\mathcal{T}$, $x\,\in\,\mathbb{R}^n$ and $1 \le k \le n$

$$\lim_{x_k \to \infty} F_t(x) = F_{t(k)}(x(k))$$

where $t(k) = (t_1, \cdots, t_{k-1}, t_{k+1}, \cdots, t_n)$ and $x(k) = (x_1, \cdots, x_{k-1}, x_{k+1}, \cdots, x_n)$ (It is to say, $t(k)$ is t *deleted the k-th variable* and $x(k)$ is x *deleted the k-th variable*)

$$\phi_t(u) = \int_{\mathbb{R}^n} e^{iu'x} F_t(dx_1, \cdots, dx_n)$$

be the characteristic function of F. Then KET can be restated as follow:

$$\lim_{u_i \to 0} \phi_t(u) = \phi_{t(i)}(u(i))$$

where $u(i)$ ana $t(i)$

(a n-dim r.v. Y is *normally distributed*) iff (($Y = AX + b$) where $A\,\in\,M_n(\mathbb{R})$, $X \sim N_n(0,1)$ and $b\,\in\,\mathbb{R}^n$)

and apparently

$$\mu_Y := E(Y) := (E(Y_1), \cdots, E(Y_n))^\top = E(AX+b) = E(AX) + E(b) = A \cdot E(X) + b = A \cdot 0 + b = b$$

$$ \Sigma_{YY} := \text{Cov}(Y,Y) := E([Y - E(Y)][Y - E(Y)]^\prime) = E([AX + b - b][AX + b - b]^\prime) = E(AXX^\prime A^\prime) = A \cdot E(XX^\prime) \cdot A^\prime = AA^\prime$

and

$$\phi_Y(u) = E\exp(iu'Y) = E\exp(iu'(AX+b)) = E(\exp(iu\prime b) * \exp(iu\prime AX)) = \exp(iu'b)\prod_k E\exp(i(u'A)_k X_k)$$

where $(u'A)_k$ is the k-th component of the vector $u'A$.

and

$$Eexp(iaX_i = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(iax) \exp(-x^2/2) dx = exp(-a^/2)$$

then

$$\phi_Y(u) = \exp(iu'b - \frac{1}{2} u' \Sigma_{YY} u)$$

Definition (Standard Brownian Motion) A standard Brownian motion or a standard Wiener Process $\{B(t)\,,\, t \geq 0\}$ is a stochastic process satisfying:

- (a) $B(0) = 0$
- (b) for every t and $0 = t_0 < t_1 < \cdots < t_n$, $\Delta_k := B(t_k) - B(t_{k-1})$ are independent
- (c) $B(t) - B(s) \sim N(0, t-s)$ for $t \leq s$

(c) is saying $B(t+1) - B(t) \sim N(0,1)$ basically

Definition (Poisson Process) A Poisson Process $\{N(t)\,,\, t \leq 0\}$ with a **mean rate** λ is a stochastic process statisfying:

- (a) $N(0) = 0$
- (b) for every t and $0 = t_0 < t_1 < \cdots < t_n$, $\Delta_k := N(t_k) - N(t_{k-1})$ are independent
- (c) $N(t) - N(s) \sim \mathrm{Po}(\lambda(t-s))$ for $t \geq s$

Poisson (a) (b) is then same as SBM (a) (b) the diff is SBM(c) is standard but Pois(c) is a Poisson distribution.

A.2 Hilbert Spaces