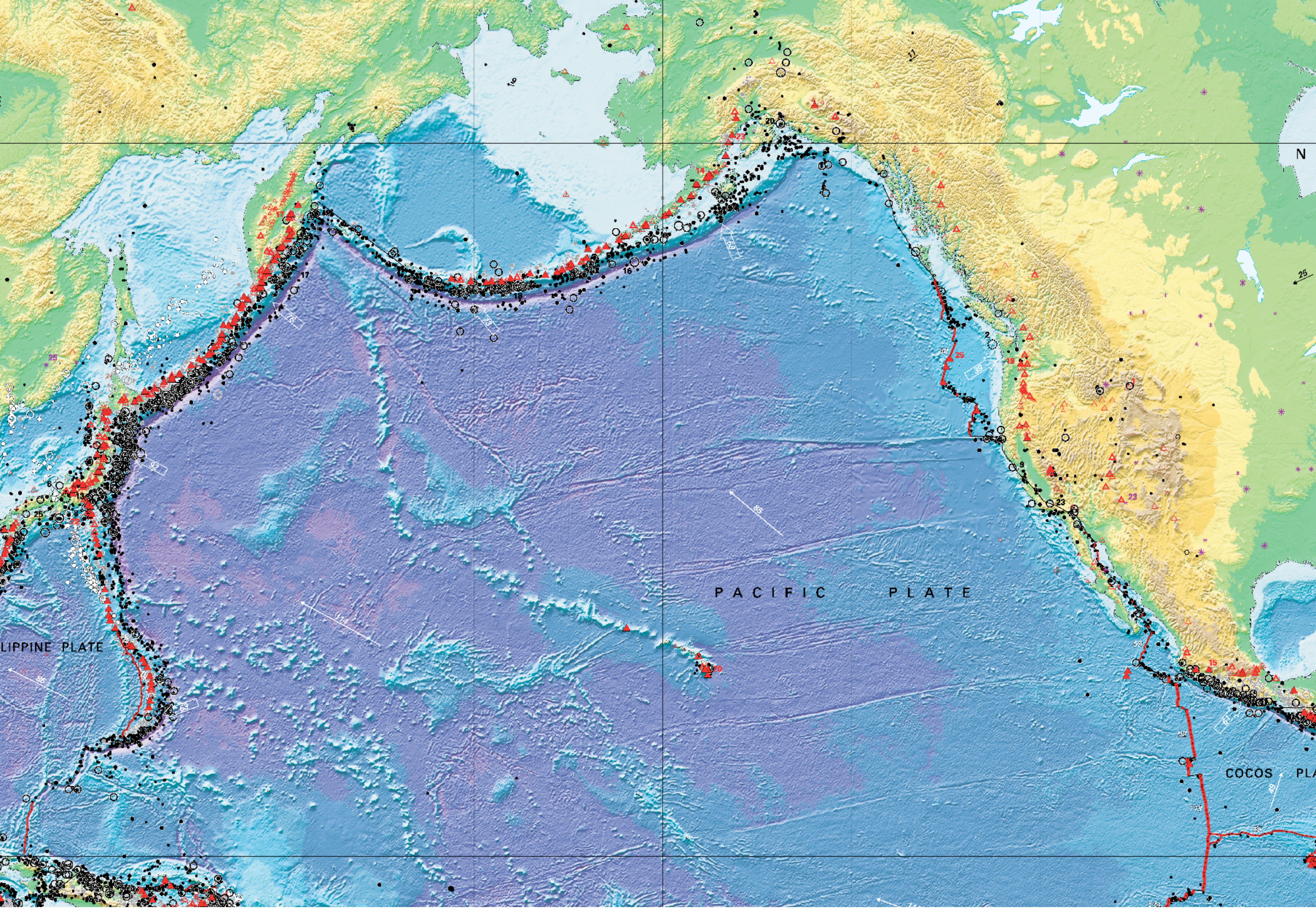
Geodynamics Presentation: from (Turcotte & Schubert, 2014, Sec3.16)

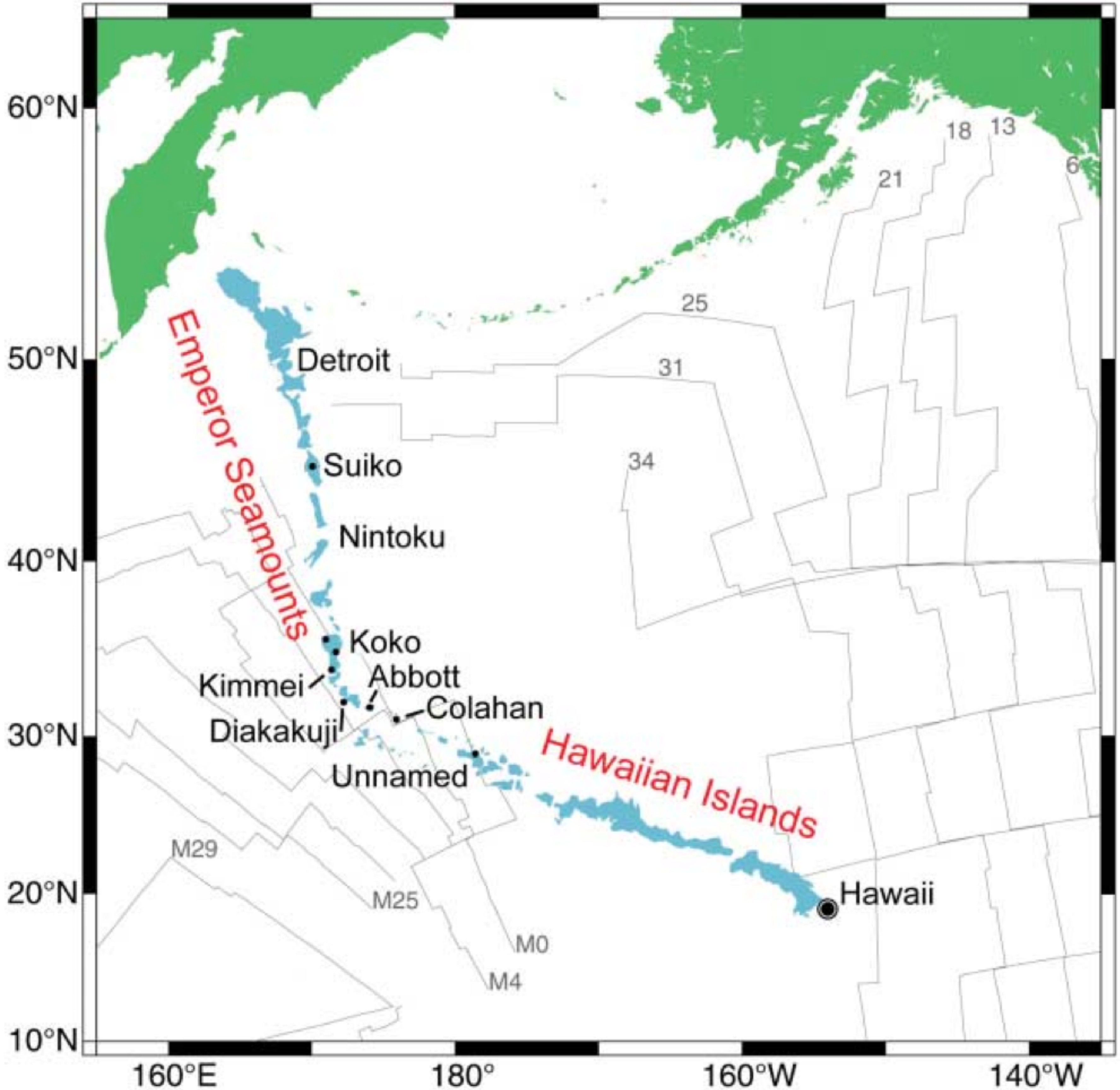
Bending of the Elastic Lithosphere under the Loads of Island Chains

Xiaochuan

2014.09.11



• Simkin et al., 2006   5800km peak: 4207m



Seamount	Age ± 2σ (Ma)
Suiko	60.9 ± 0.3
Koko (north)	52.6 ± 0.8
Koko (south)	50.4 ± 0.1
Kimmei	47.9 ± 0.2
Diakakuji	46.7 ± 0.1
Abbott	41.5 ± 0.3
Colahan	38.7 ± 0.2
Unnamed	31.0 ± 0.2

- apex of bend at Diakakuji
- (volcanoes named after former emperors of Japan)

- (Sharp and Clague, 2006)

improved $^{40}Ar/^{39}Ar$ dating methods

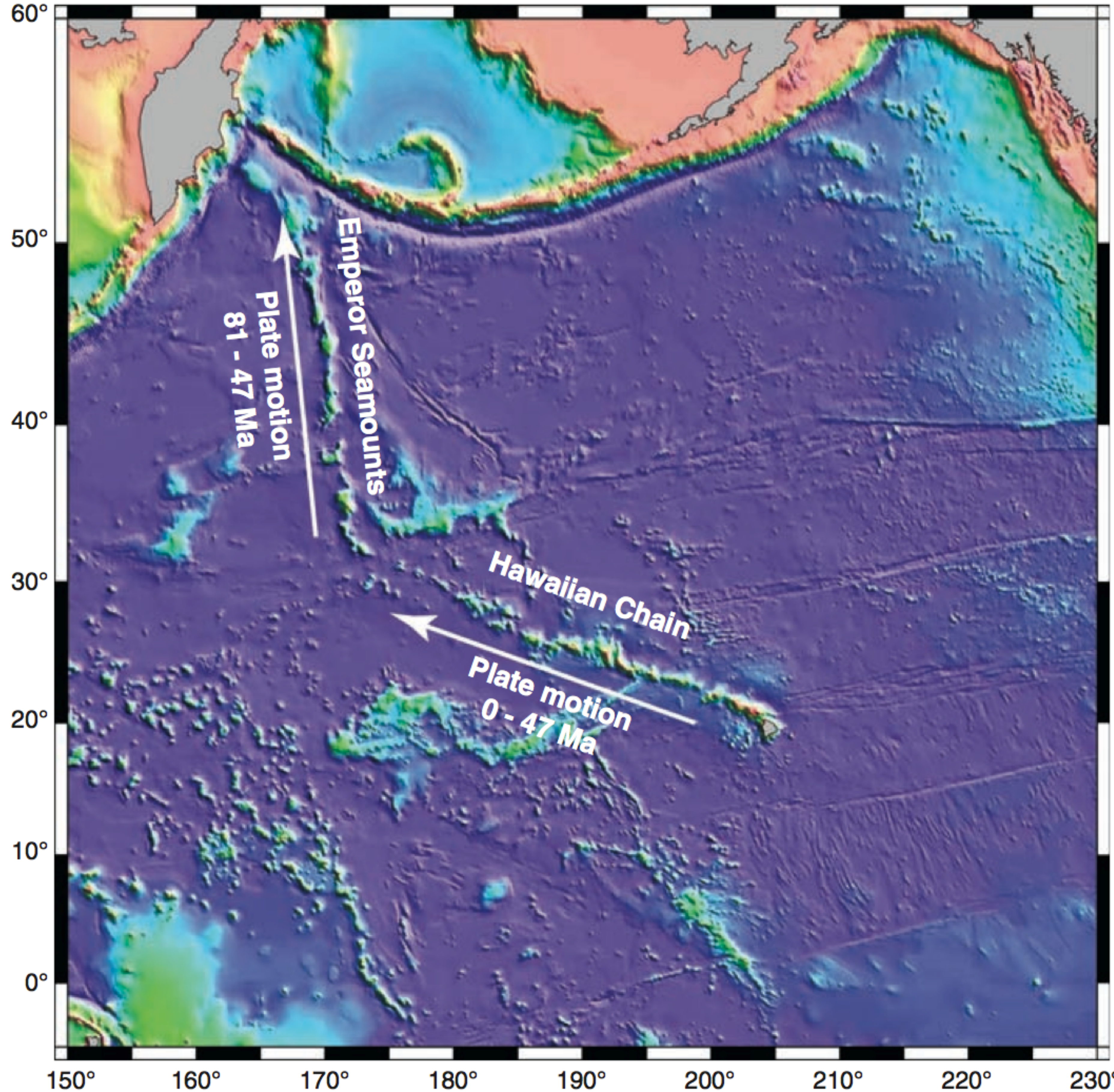
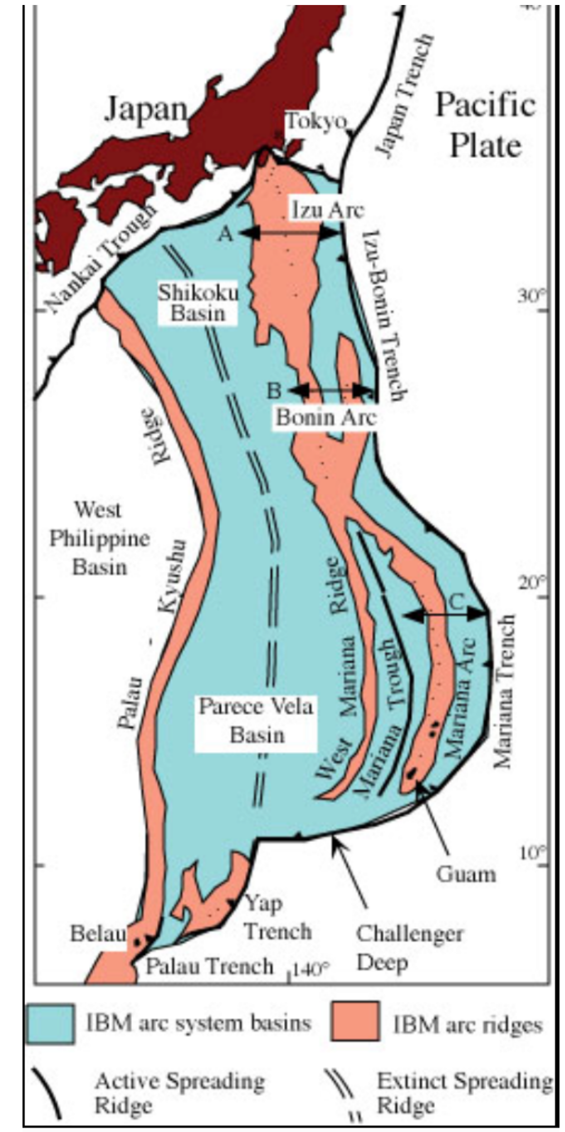


Fig. 1. Bathymetry of the northwest Pacific Ocean basin. The traditional interpretation is highlighted: The Hawaiian volcanic chain records absolute Pacific plate motion to the northwest, whereas the Emperor Seamounts reflect a more northerly plate direction.

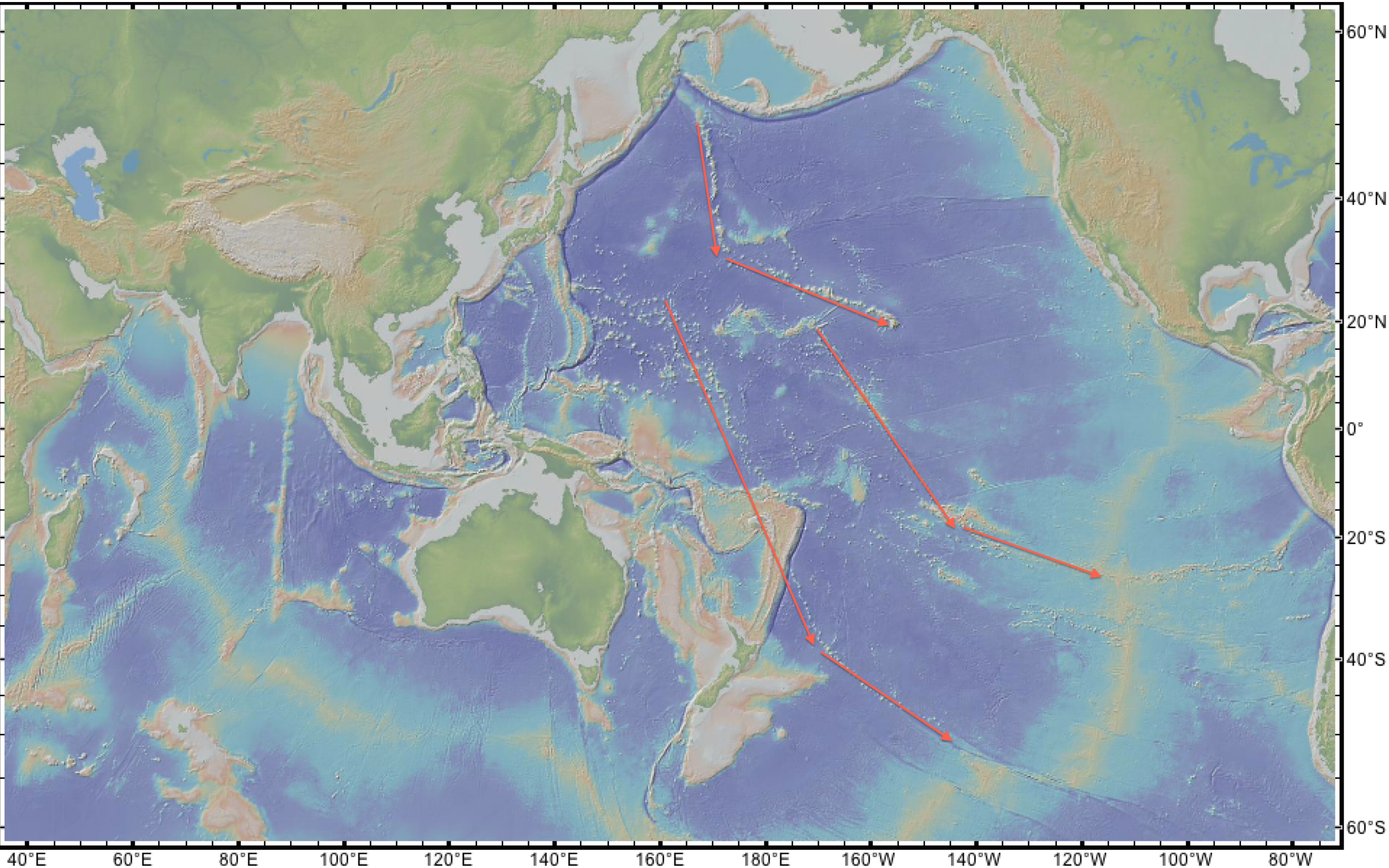
(Tarduno et al., 2009)

50-Ma Initiation of Hawaiian-Emperor Bend Records Major Change in Pacific Plate Motion

Warren D. Sharp[1]* and David A. Clague[2]



- 1) Initiation of magmatism in the **Izu-Bonin-Mariana**(IBM) arc system ~ 50Ma **—>** major shift of Pacific plate motion
- 2) Early Eocene(56Ma) initiation of the >2600km **Tonga-Kermadec arc** **—>** redirection of plate motion
- 3) Lockup of the **India-Eurasia collision** zone. (dated by onset of major crustal shortening in that region at ~50Ma)
- 4) Rifting of Australian plate from Antarctic plate **—> Australian Pacific collision**

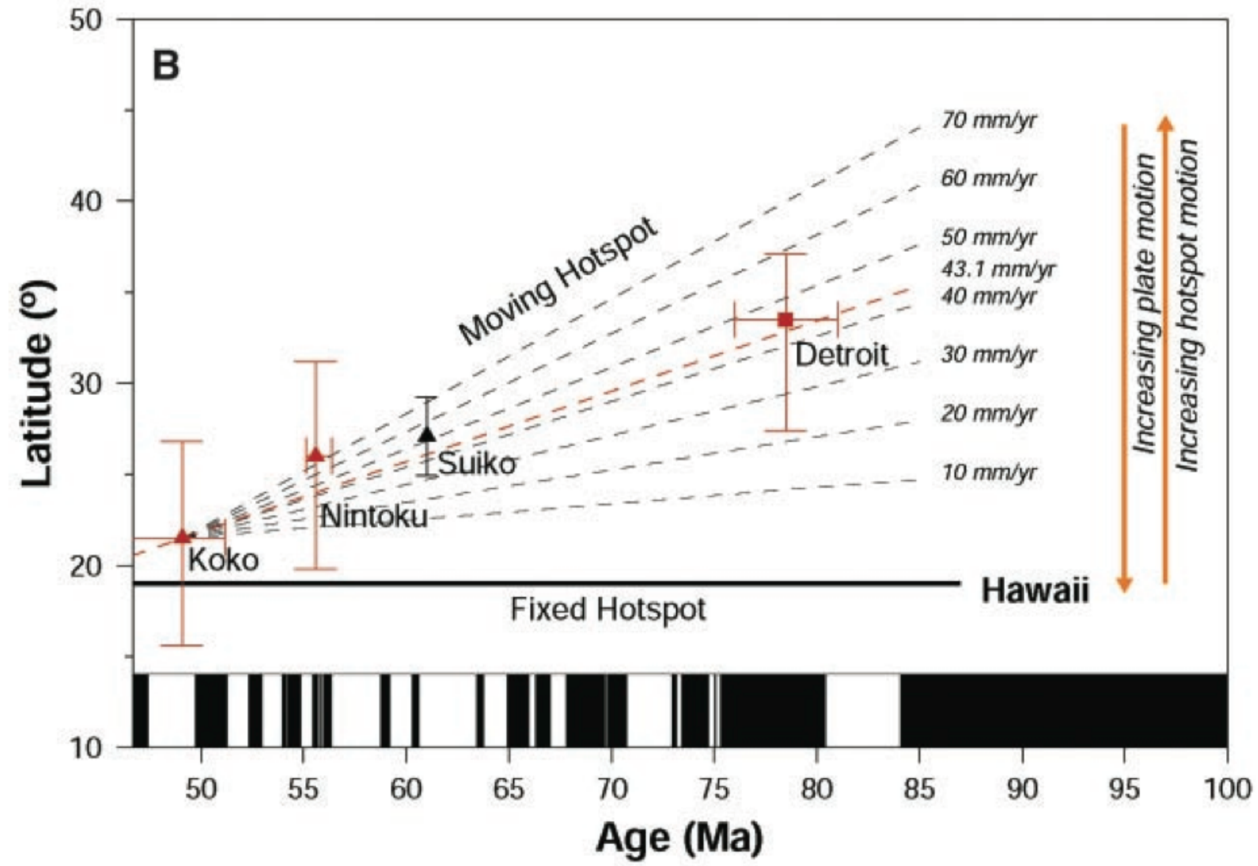


from GeoMapApp

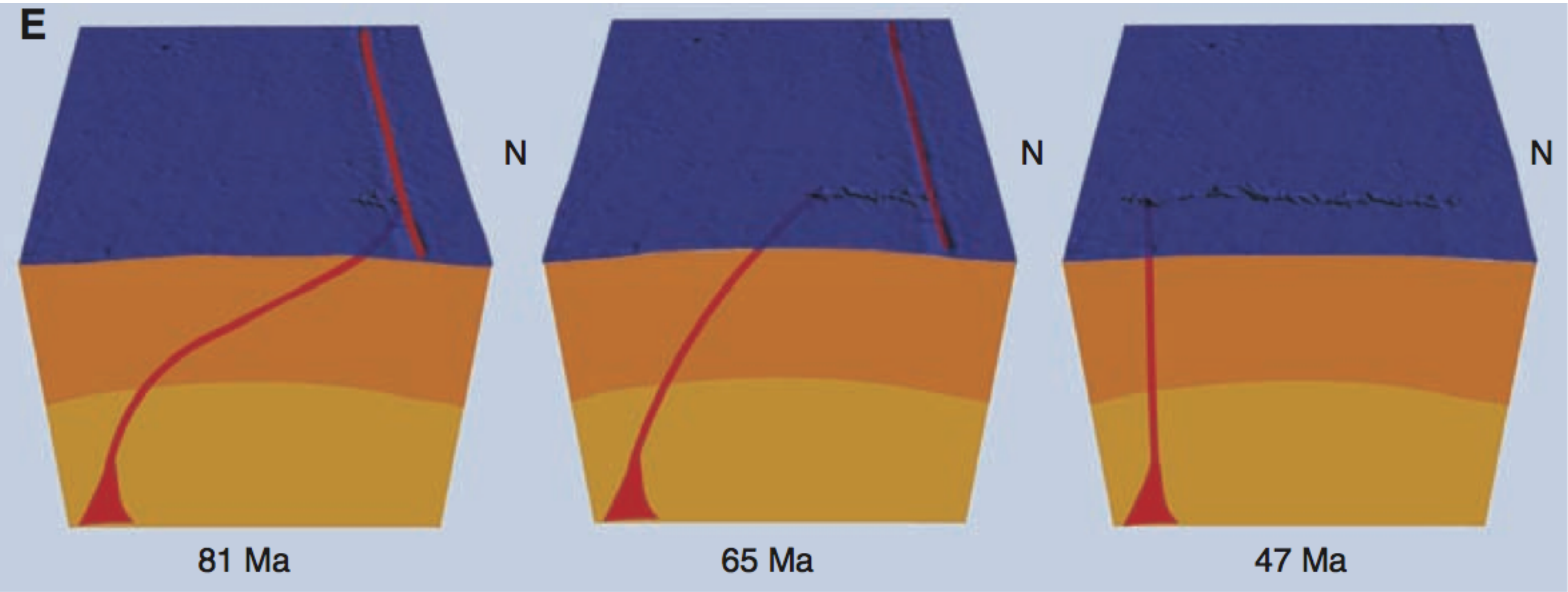
Relations between: three pairs of parallel features??

However, a paleomagnetic test based on ocean drilling in 2001 revealed that the islands and seamounts had **formed at different latitudes**.

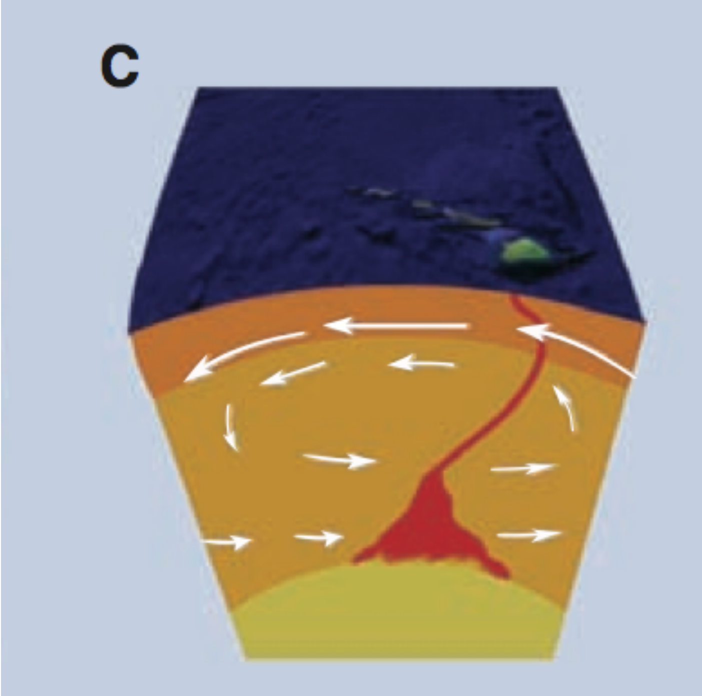
(Tarduno et al., 2009)



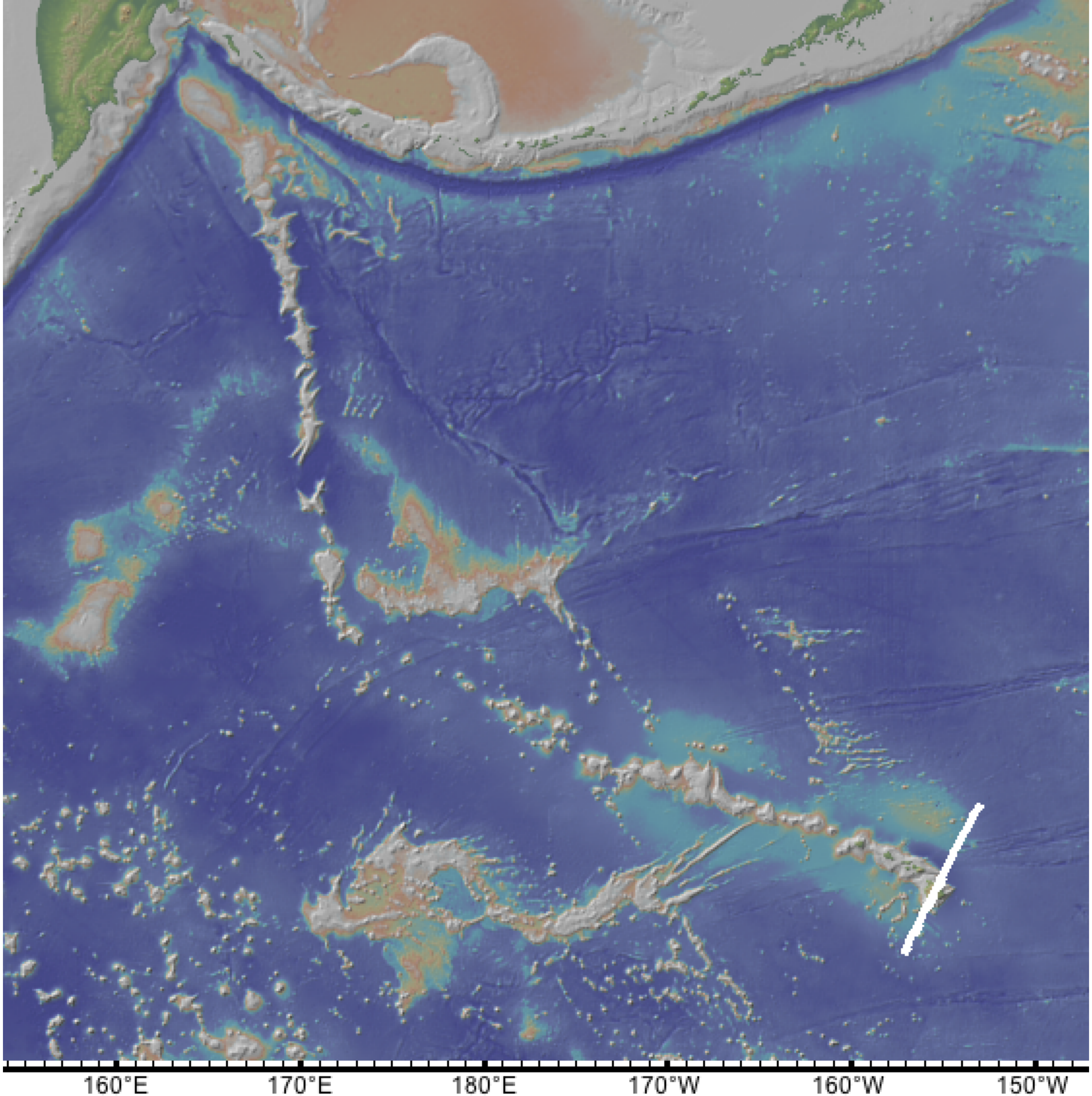
(Tarduno et al., 2003)



The plume is bent between 1200- and 1500-km depth toward the mantle upwelling associated with the **Pacific-Kula ridge system** at 81 Ma; **upwelling abates** thereafter, allowing the plume to return to its original position relative to the deep mantle by 47 Ma.



Advection of the plume conduit and entrainment in the mantle wind.



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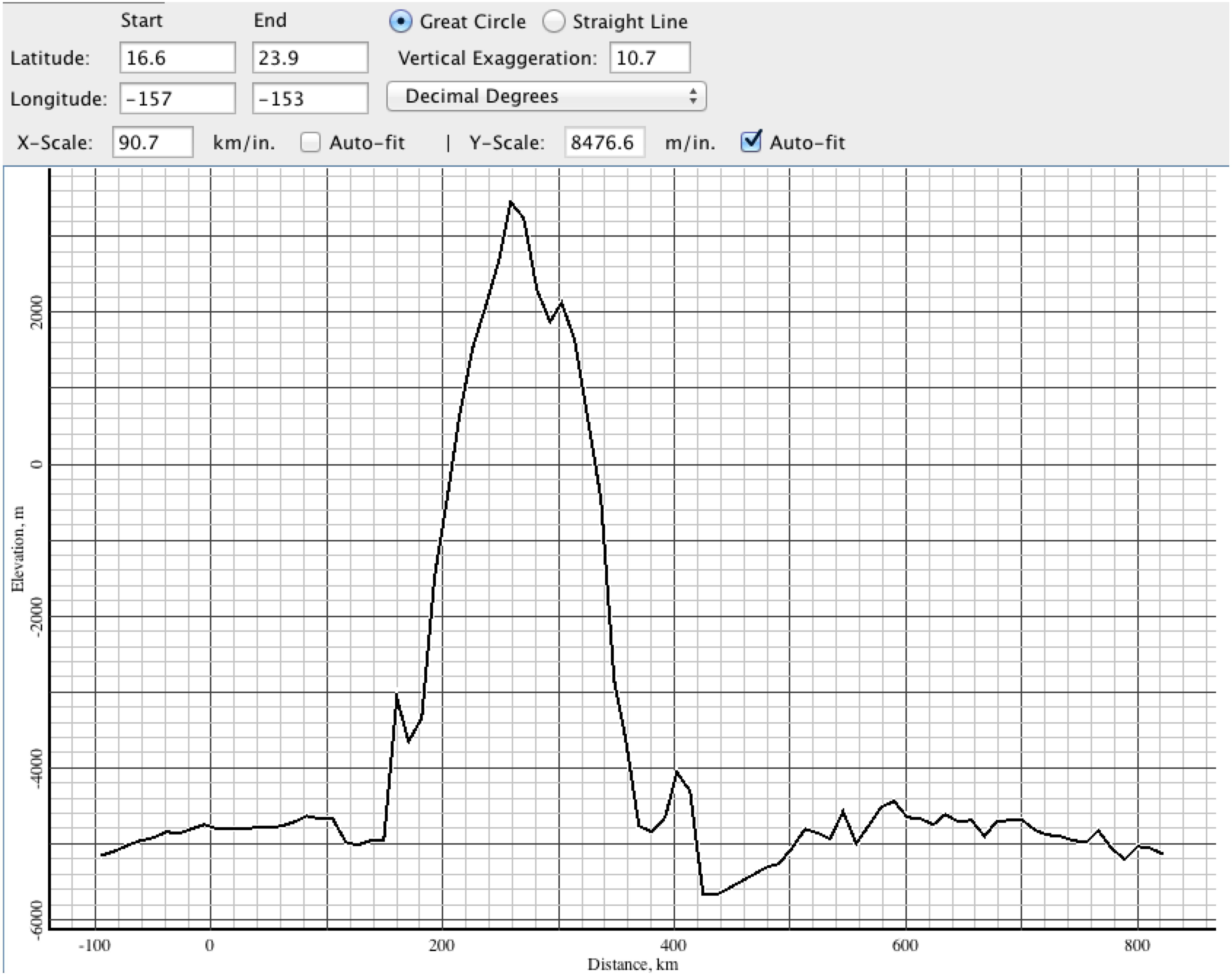

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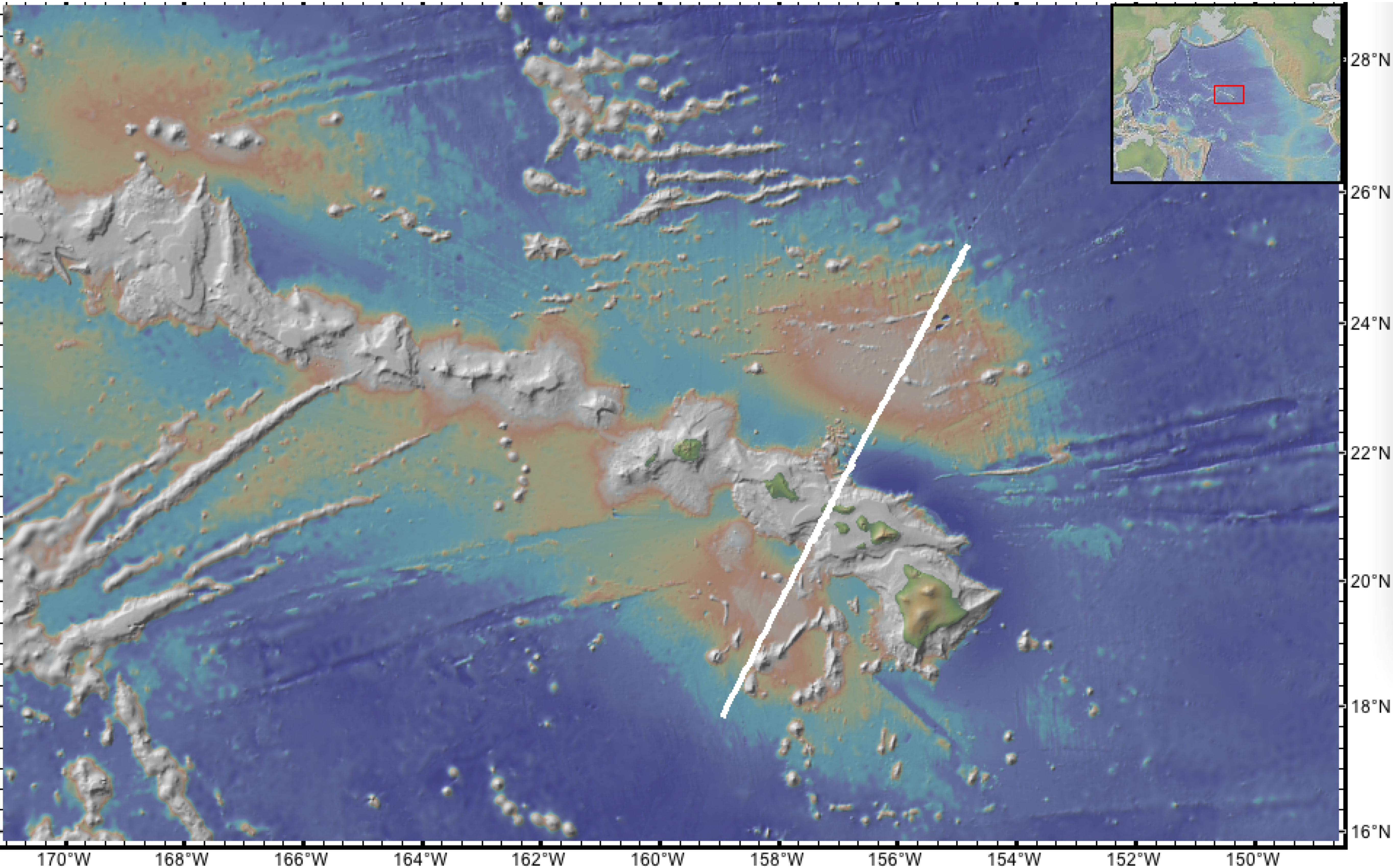

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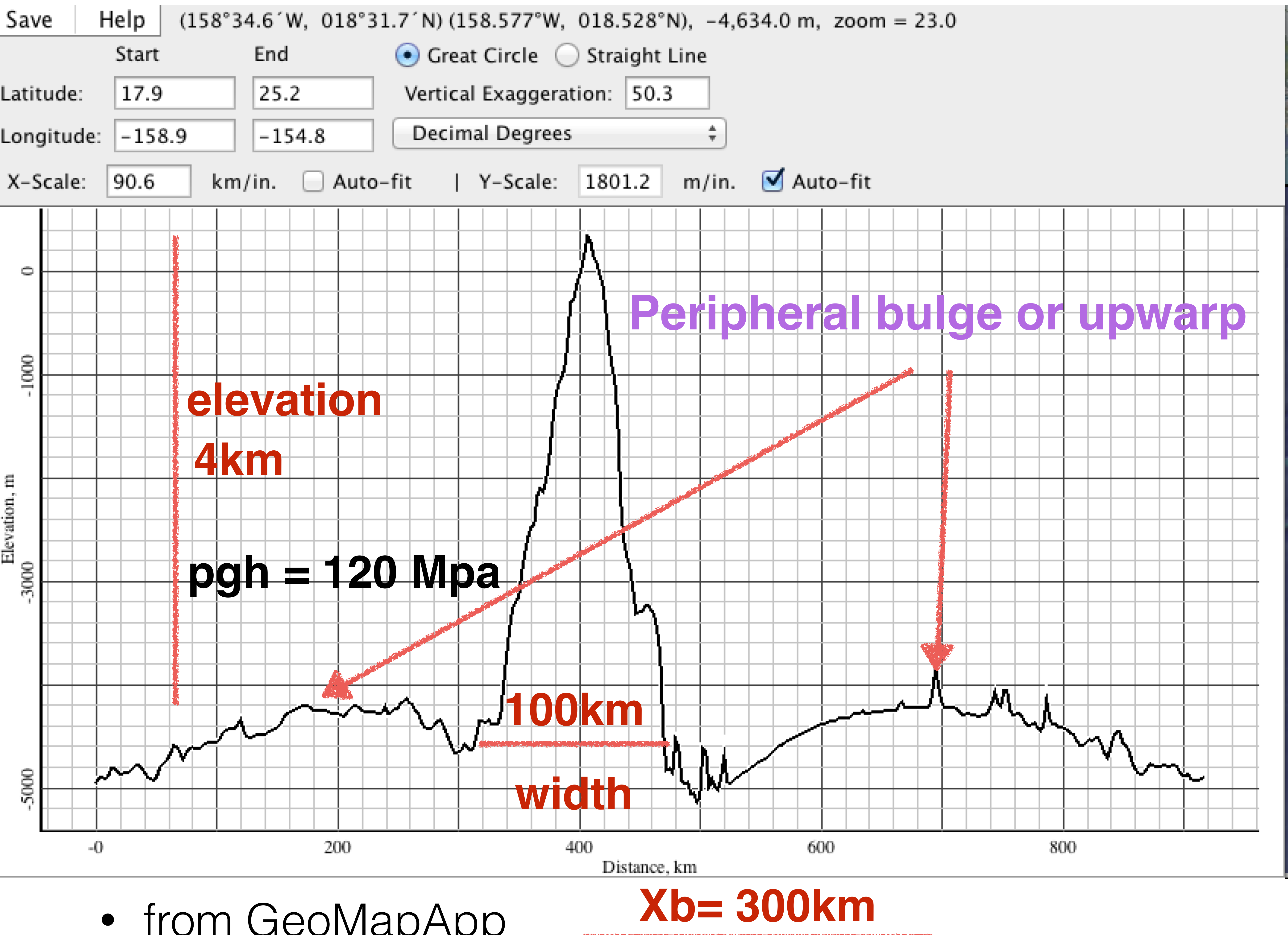

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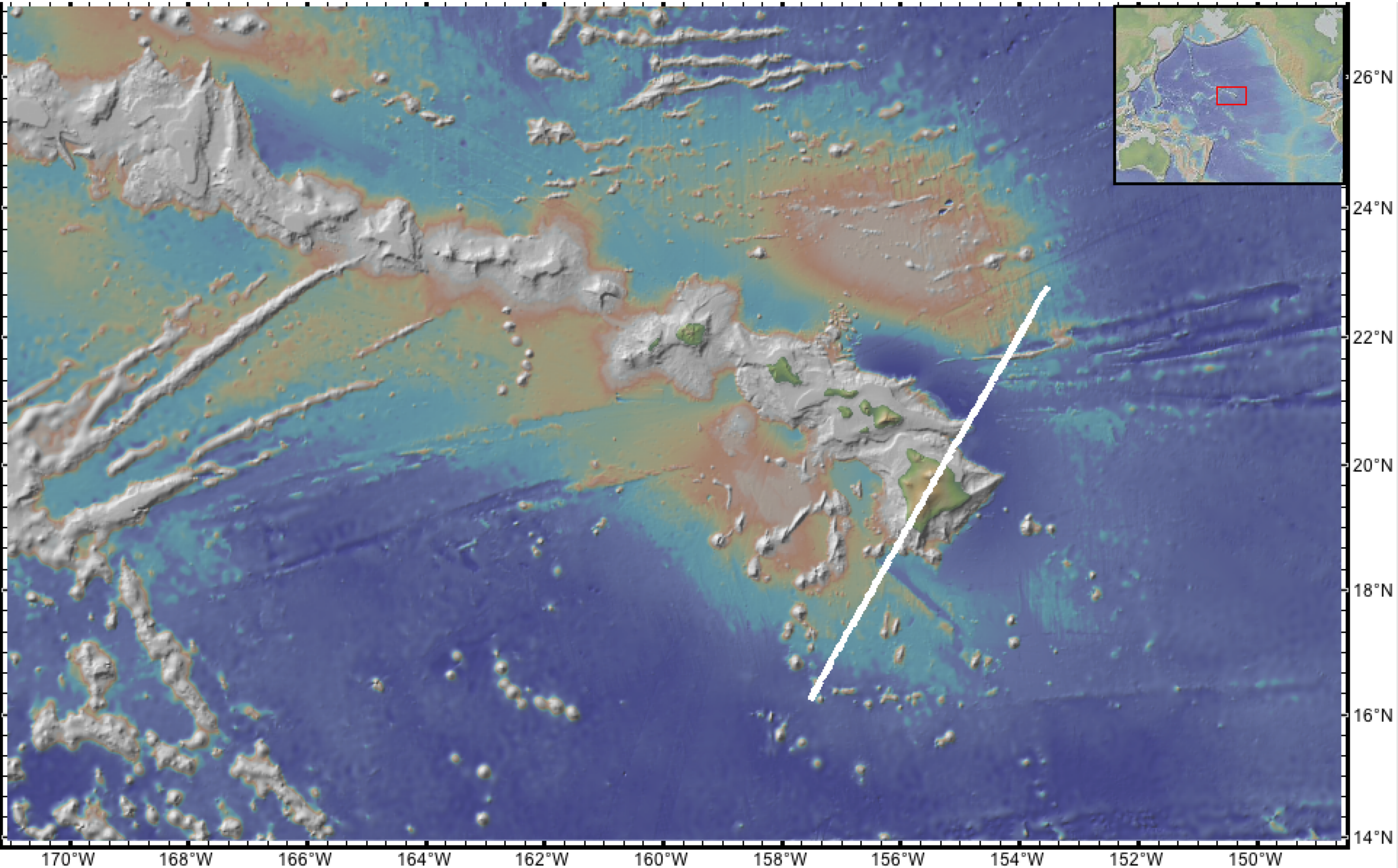

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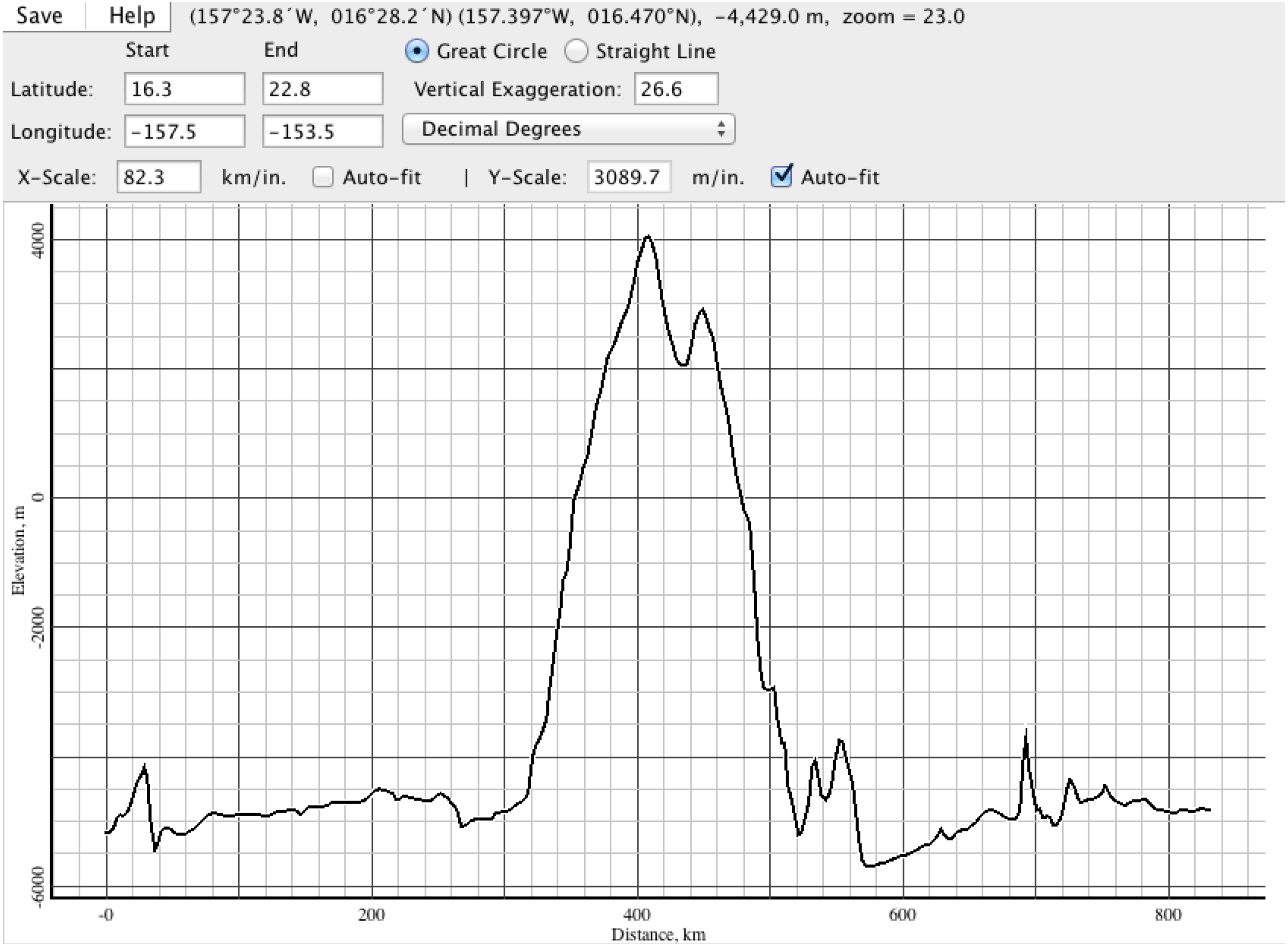


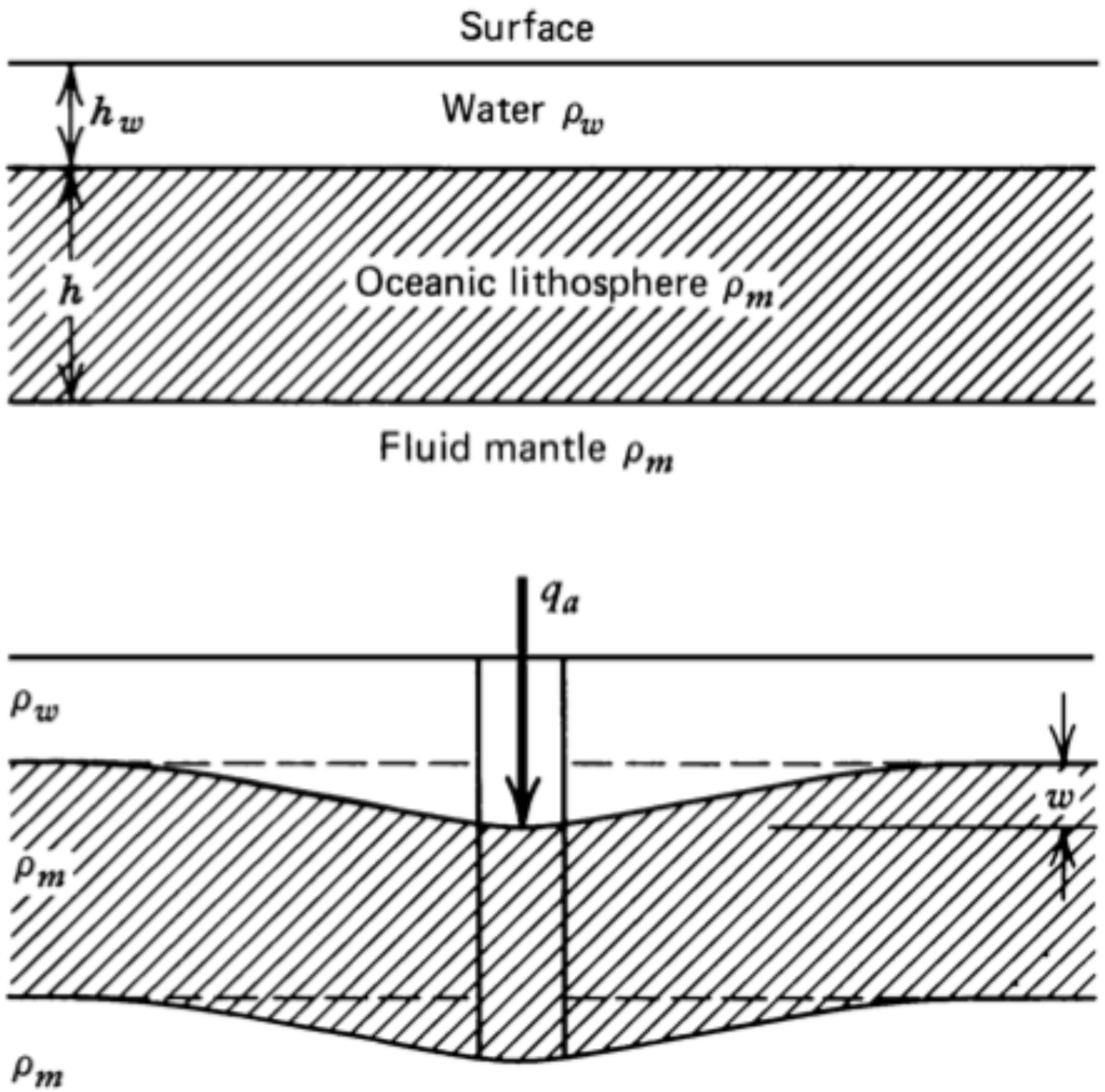
- from GeoMapApp

- **Mauna Kea (peak of Hawaii) 4,207 m   from Vadim Kurland, December 26, 2007**

Exciting part!!

- **Equations and calculations**

Derivations of Equations and figures come from Turcotte and Shubert, 2014



(a)

$$q = q_a - (\rho_m - \rho_w) g w, \quad (3.102)$$

$$D\frac{d^4 w}{dx^4} = q(x) - P\frac{d^2 w}{dx^2}. \quad (3.74)$$

(general equation for deflection of the plate)

$$D\frac{d^4 w}{dx^4} + P\frac{d^2 w}{dx^2} + (\rho_m - \rho_w) g w = q_a(x).$$

(3.103)

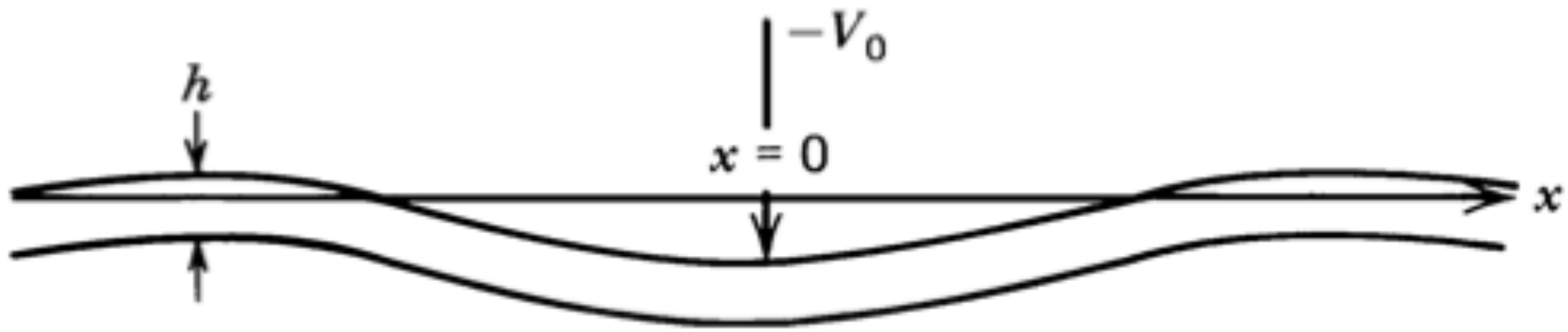


Figure 3.29 Deflection of the elastic lithosphere under a line load.

- line load V0 only at x = 0 ———> qa(x) = 0 **??**
- **P = 0**

$$D\frac{d^4w}{dx^4} + (\rho_m - \rho_w)gw = 0. \qquad (3.125)$$

Flexural rigidity: $D \equiv \dfrac{Eh^3}{12(1-\nu^2)}.$

general solution for (3.125)

$$w = e^{x/\alpha}\left(c_1 \cos\frac{x}{\alpha} + c_2 \sin\frac{x}{\alpha}\right) + e^{-x/\alpha}\left(c_3 \cos\frac{x}{\alpha} + c_4 \sin\frac{x}{\alpha}\right), \tag{3.126}$$

where **flexural parameter** is:

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_w)g}\right]^{1/4}. \tag{3.127}$$

Boundary conditions to determine c1 c2 c3 c4

$$w \to 0 \text{ as } x \to \infty$$ **C1 = C2 = 0**

$$dw/dx = 0 \text{ at } x = 0.$$

$$\frac{d\omega}{dx} = \frac{e^{-x/\alpha}}{\alpha}[-c_3 cos(\frac{x}{\alpha}) + c_4 cos(\frac{x}{\alpha})) - c_3 sin(\frac{x}{\alpha}) - c_4 sin(\frac{x}{\alpha})]$$

C3 = C4

(3.126) simplifies to

$$w = c_3 e^{-x/\alpha}\left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}\right) \qquad x \geq 0. \qquad (3.128)$$

Solve C3 using V0:

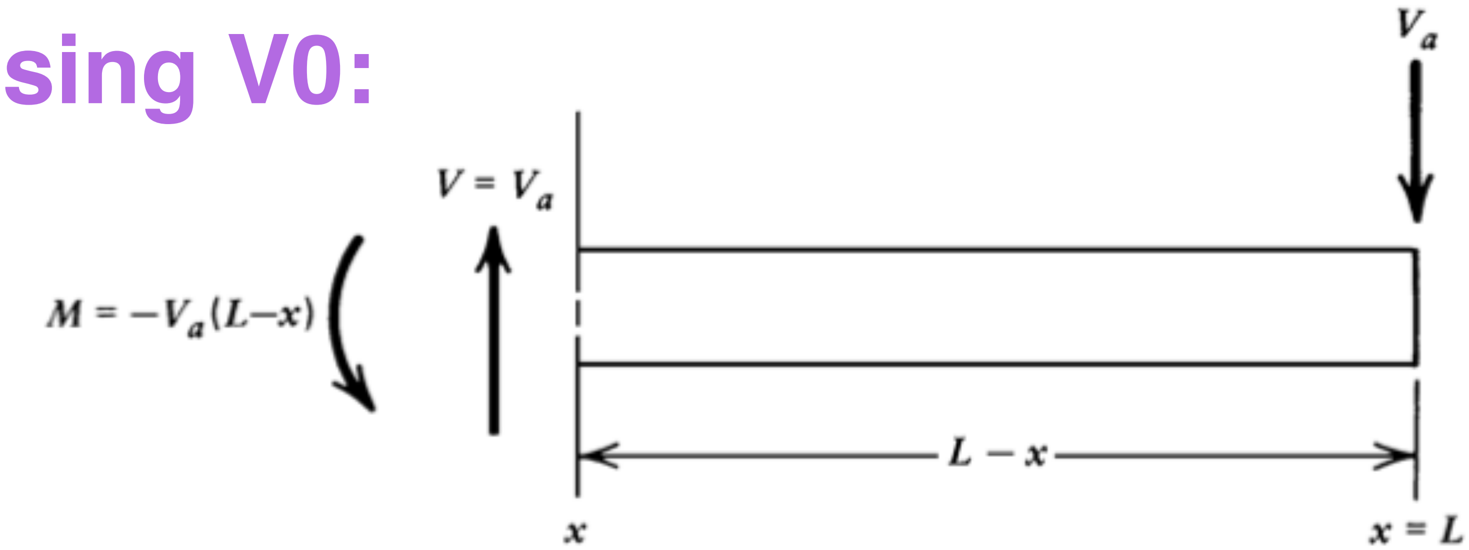


Figure 3.18 Forces and torques on a section of a plate loaded at its right end by a force V_a.

$$\frac{d^3w}{dx^3} = -\frac{V_a}{D}. \qquad (3.81)$$

$$\frac{d^3\omega}{dx^3} = \frac{4c_3}{\alpha^3} e^{\frac{-x}{\alpha}} cos(\frac{x}{\alpha})$$

$$\frac{1}{2}V_0 = D\frac{d^3w}{dx^3}(x=0) = \frac{4Dc_3}{\alpha^3} \qquad (3.129)$$

$$c_3 = \frac{V_0\alpha^3}{8D} \quad \longrightarrow\!\!>> \quad (3.128)$$

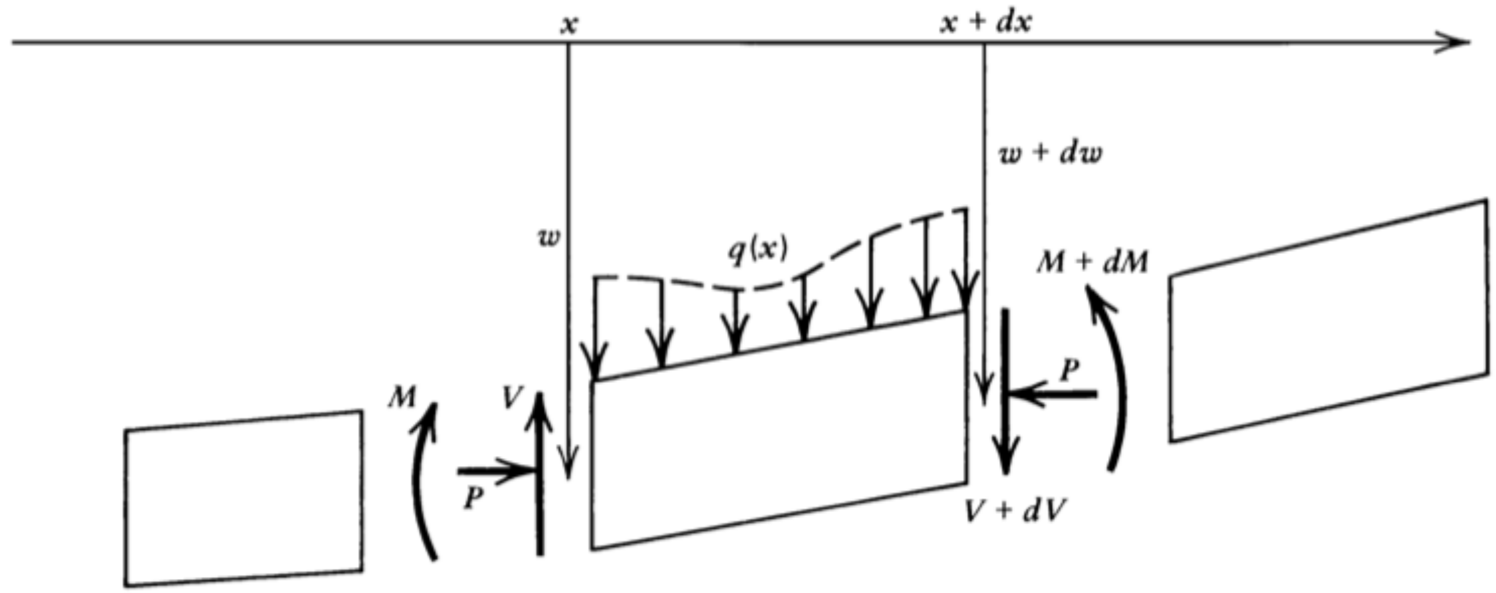


Figure 3.10 Forces and torques on a small section of a deflecting plate.

(Half the plate supports half the load applied at $x = 0$. Note also that a downward force on the left end of the plate is negative according to the sign convention illustrated in Figure 3–10.) Substituting for c_3 from Equation (3–129) into Equation (3–128), we obtain

Fruits:

$$w = \frac{V_0 \alpha^3}{8D} e^{-x/\alpha} \left(\cos \frac{x}{\alpha} + \sin \frac{x}{\alpha} \right) \qquad x \geq 0. \qquad (3.130)$$

Note: The analysis for this line load is **only approximately valid** for Hawaiian Islands. (Island load distribution about 150km wide)

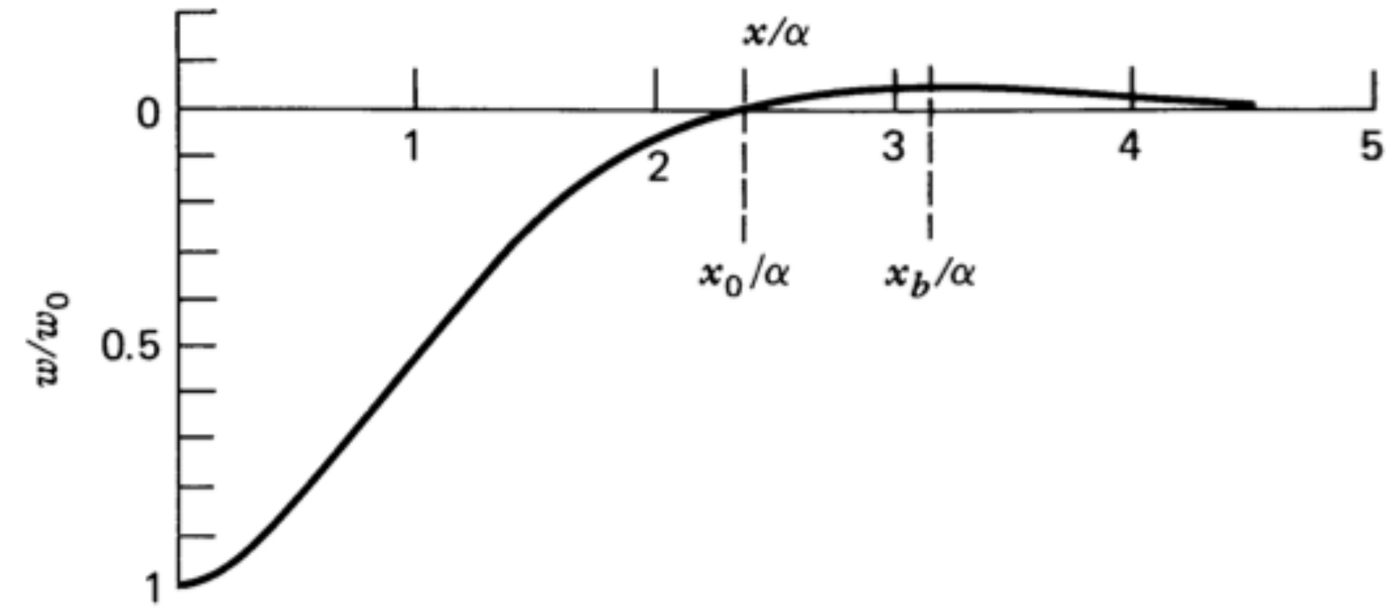


Figure 3.30 Half of the theoretical deflection profile for a floating elastic plate supporting a line load.

$$w = \frac{V_0\alpha^3}{8D} e^{-x/\alpha}\left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}\right) \qquad x \geq 0. \tag{3.130}$$

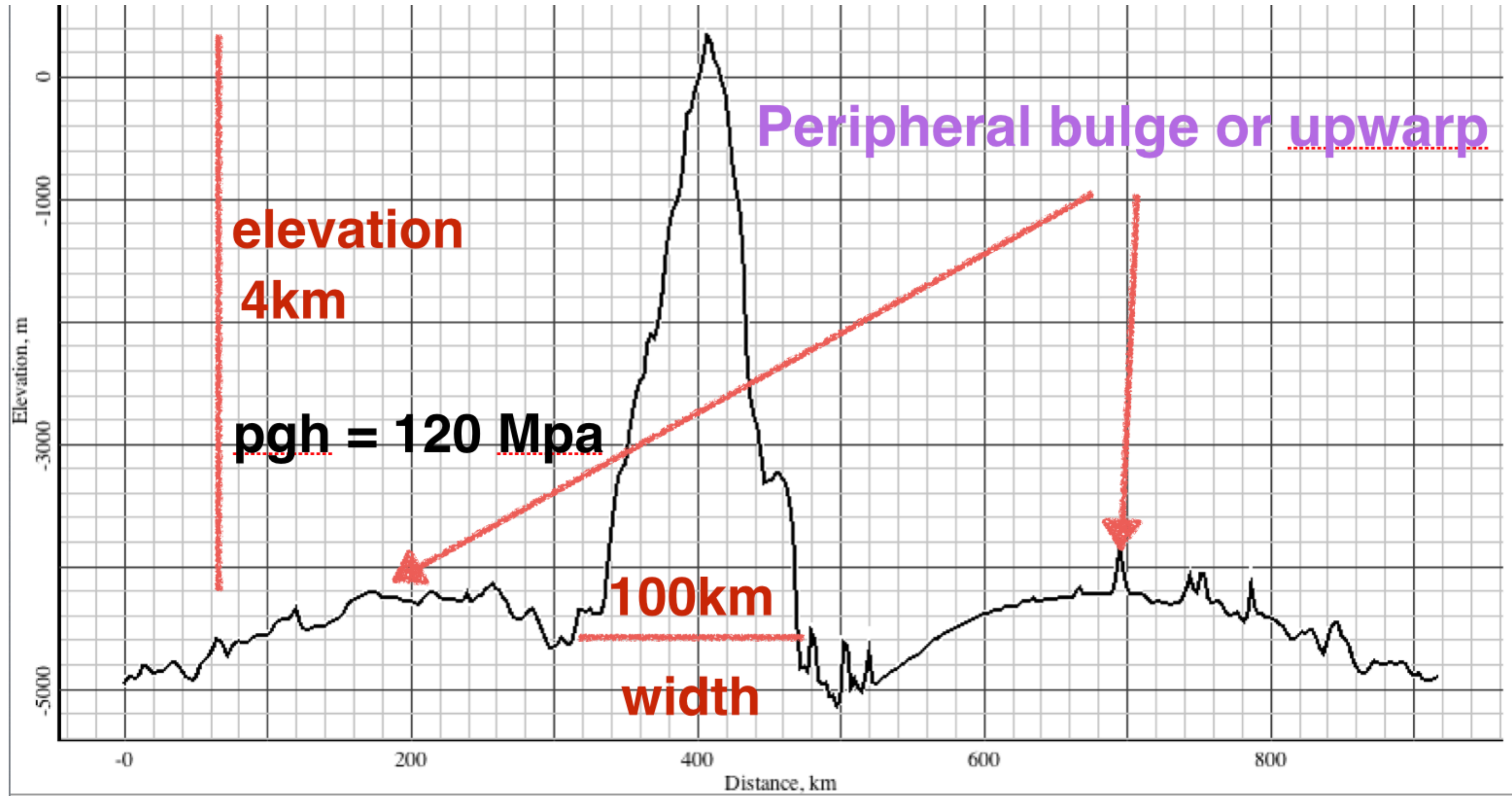
maximum deflection (x=0): $w_0 = \dfrac{V_0\alpha^3}{8D}.$

half width of deflection (w=0): $x_0 = \alpha \tan^{-1}(-1) = \dfrac{3\pi}{4}\alpha.$

maximum forbulge (dw/dx=0): $x_b = \alpha \sin^{-1} 0 = \pi\alpha.$

maximum forbulge amplitude: $w_b = -w_0 e^{-\pi} = -0.0432 w_0.$

E.g. Calculating plate thickness from observed distance from line load to maximum forbulge: Xb



- from GeoMapApp

Xb= 300km

$$x_b = \alpha \sin^{-1} 0 = \pi\alpha.$$

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_w)g}\right]^{1/4}.$$

$$g = 10(m/s^2)$$

$$\rho_m - \rho_w = (3300 - 1000) = 2300(kg/m^3)$$

$$D \equiv \frac{Eh^3}{12(1-\nu^2)}.$$

$$E = 70(Gpa)$$

$$v = 0.25$$

h=42.5km

```
>>> (((300000/3.14)**4)*2300*10/4*12*(1-0.25**2)/70000000000)**(0.333333333333)
42543.14427629813
```

Crustal Structure, Flexure, and Subsidence History of the Hawaiian Islands

A. B. WATTS AND U. S. TEN BRINK[1]

Lamont-Doherty Geological Observatory and Department of Geological Sciences, Palisades, New York

Seismic reflection profile data have been used to estimate the long-term ($>10^6$ years) mechanical properties of the oceanic lithosphere underlying the Hawaiian Islands. The data show prominent reflectors associated with the top of the oceanic crust (reflector 2), the M discontinuity, and the base of the crust (reflector 3). The two-way travel time to the reflectors has been depth converted using velocity depth functions derived from seismic refraction expanding spread profile data and compared to calculated depths based on simple three-dimensional elastic plate models. A good fit to the reflector depths was obtained for a model with an effective elastic thickness of the lithosphere T_e of 40 km. The main discrepancies with the refraction data occur beneath Oahu and Molokai where the predicted depth of the base of the flexed crust is too shallow and beneath the southeast flank of Hawaii where it is too deep. We attribute the thickened crust beneath Oahu to ponding of magma at the base of the crust following the last phases of tholeiitic shield building and the thinned crust beneath the flanks of Hawaii to crustal melting beneath the youngest part of the chain. The effective T_e of 40 km is higher than expected for 80 m.y. oceanic lithosphere, possibly due to the effects of subsurface loads that act at or near the base of the flexed crust. According to free-air gravity anomaly and bathymetry data, T

Problem 3.19 (a)
Finding maximum absolute value of bending moment Mm

$$
\begin{aligned}
M &= \frac{-E}{(1-\nu^2)} \frac{d^2 w}{dx^2} \int_{-h/2}^{h/2} y^2 \, dy \\
&= \frac{-E}{(1-\nu^2)} \frac{d^2 w}{dx^2} \left(\frac{y^3}{3} \right)_{-h/2}^{h/2} \\
&= \frac{-Eh^3}{12(1-\nu^2)} \frac{d^2 w}{dx^2}. \qquad (3.71)
\end{aligned}
$$

$$w = w_0 e^{-x/\alpha}\left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}\right). \qquad (3.132)$$

$$\frac{d^2\omega}{dx^2} = \frac{2\omega_0}{\alpha^2} e^{-\frac{x}{\alpha}} (sin(\frac{x}{\alpha}) - cos(\frac{x}{\alpha}))$$

$$w_0 = \frac{V_0 \alpha^3}{8D}.$$

$$\frac{d^3\omega}{dx^3} = \frac{4c_3}{\alpha^3} e^{\frac{-x}{\alpha}} cos(\frac{x}{\alpha})$$

$$c_3 = \frac{V_0 \alpha^3}{8D}$$

$$x_m = \alpha \cos^{-1} 0 = \frac{\pi}{2}\alpha \qquad M_m = -\frac{2Dw_0}{\alpha^2} e^{-\pi/2} = -0.416\frac{Dw_0}{\alpha^2}$$

Problem 3.19 (b)
Finding maximum bending stress

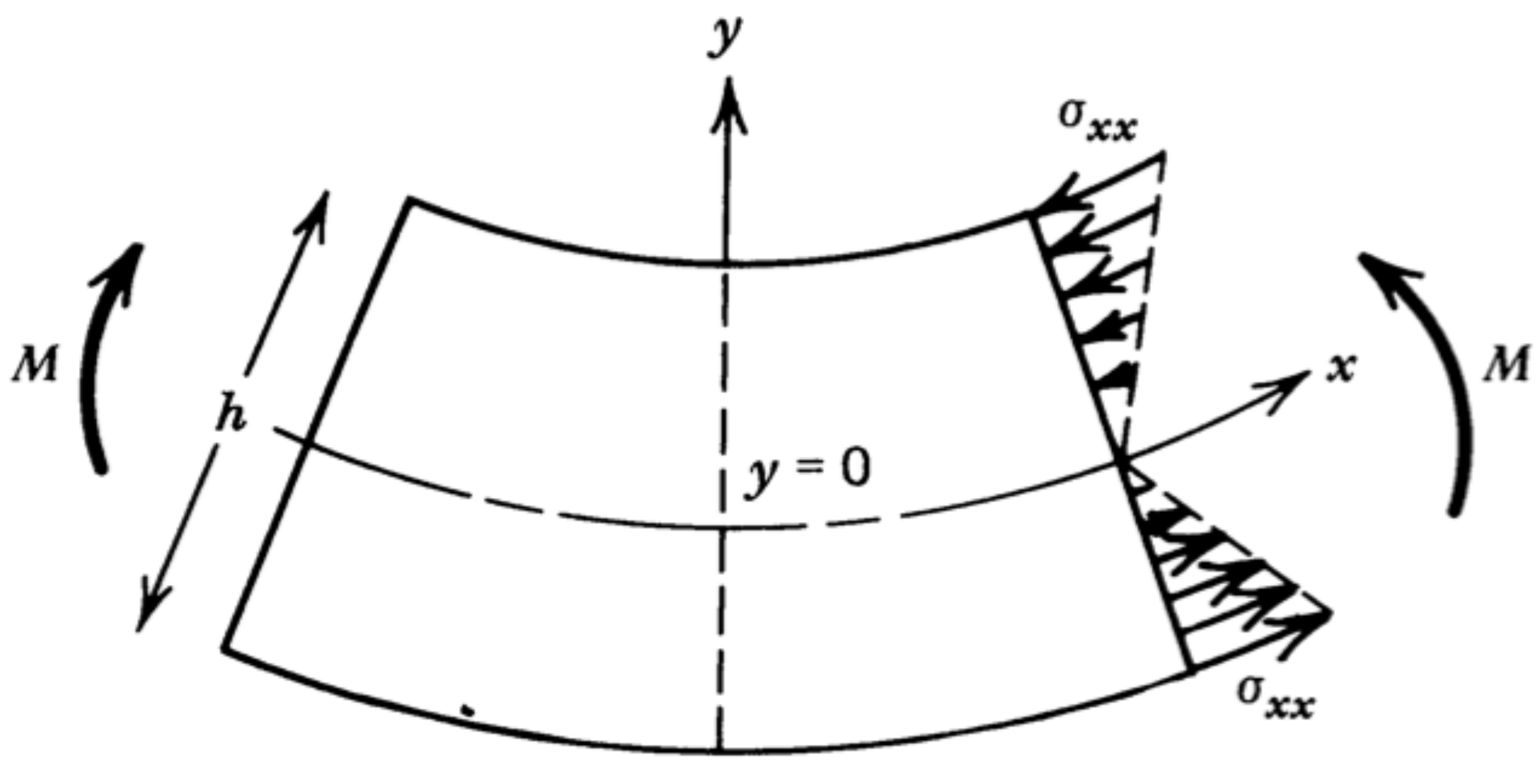


Figure 3.11 The normal stresses on a cross section of a thin curved elastic plate.

No thinning and thickening:

$$\sigma_{yy} = 0$$

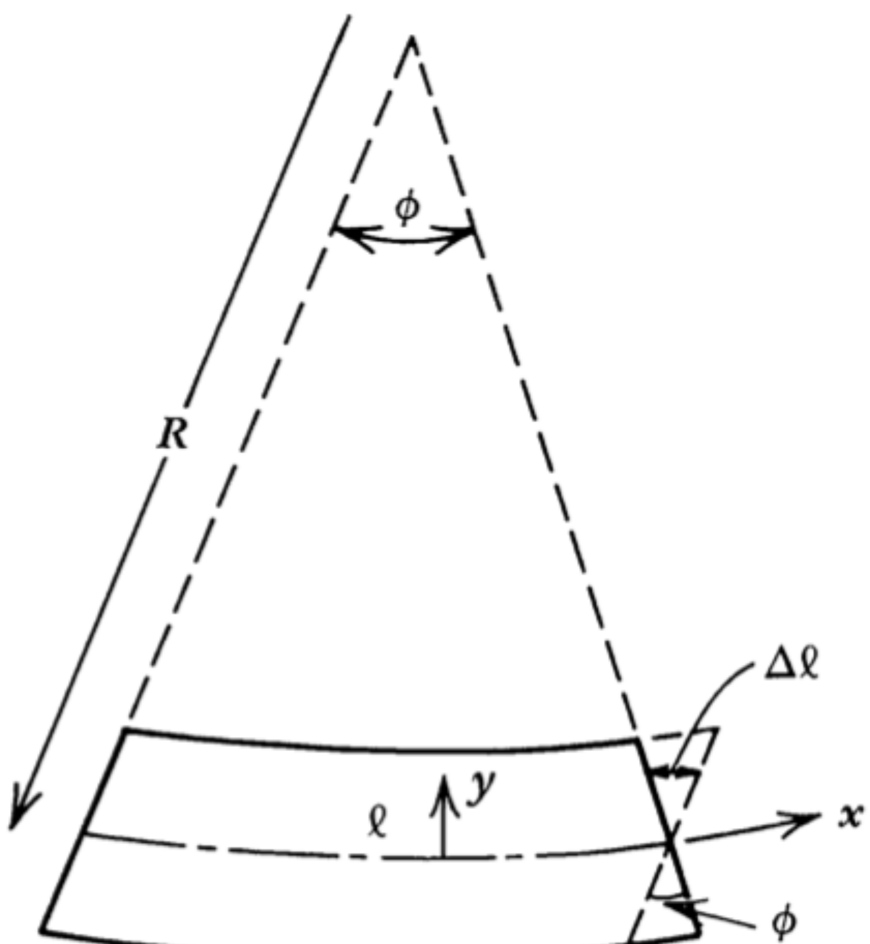
Plane strain(infinite length in z direction):

$$\epsilon_{zz} = 0$$

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{zz})$$

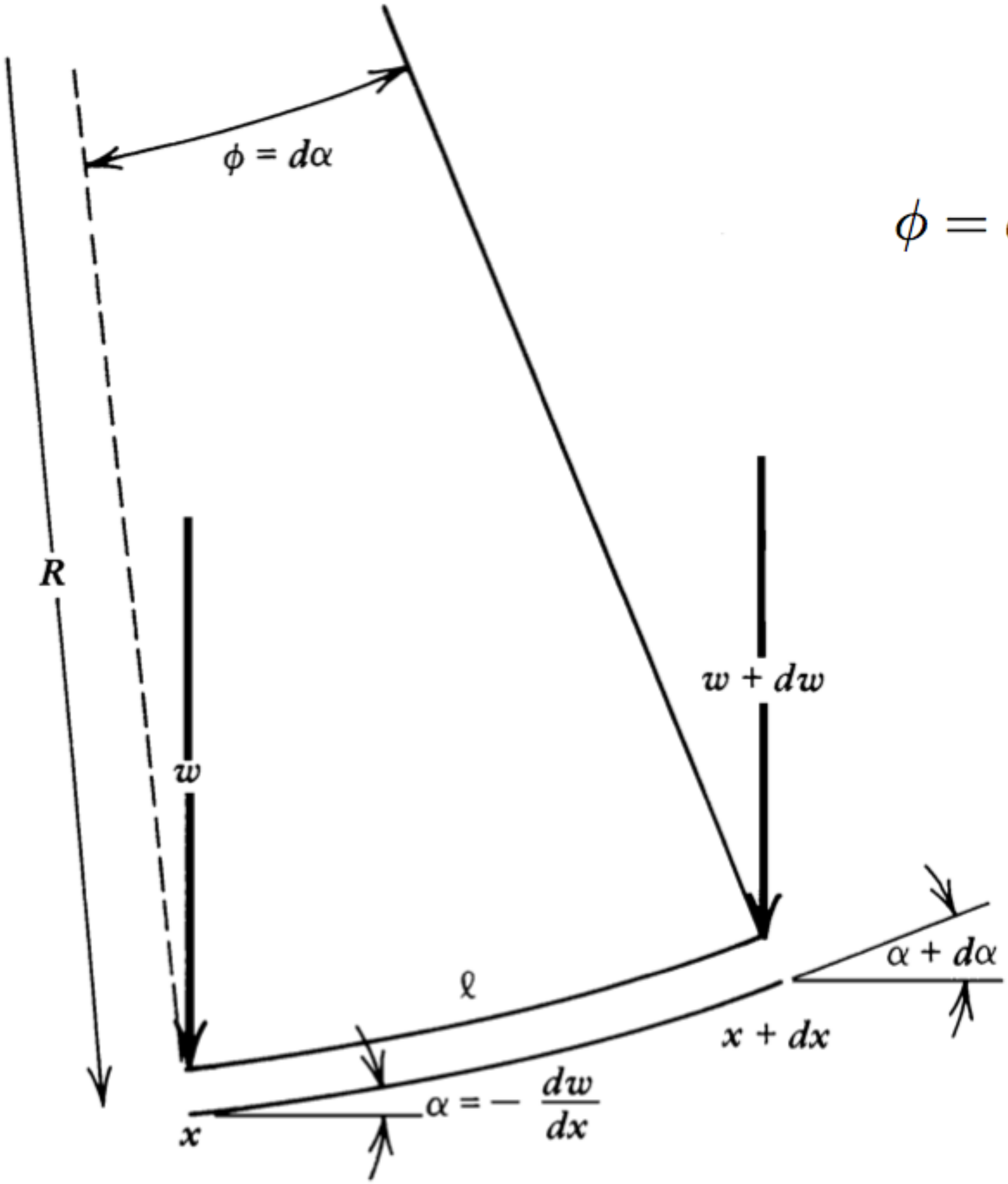
$$\varepsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu\sigma_{xx}).$$

$$\sigma_{xx} = \frac{E}{(1-\nu^2)}\varepsilon_{xx}.$$



$$\Delta l = -y\phi = -y\frac{l}{R}$$

$$\varepsilon_{xx} = -\frac{\Delta l}{l} = \frac{y}{R}.$$



$$\phi = d\alpha = \frac{d\alpha}{dx}dx = \frac{d}{dx}\left(-\frac{dw}{dx}\right)dx = -\frac{d^2w}{dx^2}\,dx$$

$$\frac{1}{R} = \frac{\phi}{l} \approx \frac{\phi}{dx} = -\frac{d^2w}{dx^2}$$

$$\varepsilon_{xx} = -y\frac{d^2w}{dx^2}$$

$$\frac{d^2\omega}{dx^2} = \frac{2\omega_0}{\alpha^2}e^{-\frac{x}{\alpha}}(sin(\frac{x}{\alpha}) - cos(\frac{x}{\alpha}))$$

$$\sigma_{xx} = \frac{E}{1-v^2}(-y)\frac{2\omega_0}{\alpha^2}e^{\frac{-\pi}{2}}$$

$E = 70(Gpa)$

$v = 0.25$

$y = 17,000m, \omega_0 = 10,000m$

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_w)g}\right]^{1/4}. \quad D \equiv \frac{Eh^3}{12(1-\nu^2)}.$$

```
>>> (4*70e9*(34000**3)/12/(1-0.25**2)/2300/10)**0.25
80756.70783809075
```

80756 meters

$$\sigma_{xx} = \frac{E}{1-v^2}(-y)\frac{2\omega_0}{\alpha^2}e^{\frac{-\pi}{2}}$$

```
>>> 70/(1-0.25**2)*17000*2*10000/80756/80756*math.exp(-3.14/2)
0.8098670790669867
```

0.809867 Gpa

Thank you!!