NTNU

TTT4205 Microwave Techniques

Waveguide assignment

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1 Rectangular waveguide

1.1 Cut-off frequencies

The cut-off frequency of a hollow waveguide is given by

$$f_{\mathrm{c}}^{(\mathrm{mn})} = \frac{c}{2\pi\sqrt{\epsilon_{\mathrm{r}}\mu_{\mathrm{r}}}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}. \tag{1}$$

For a *WR75* waveguide, modes TE_{10} and TE_{20}, the cut-off frequencies are $f_{\mathrm{c}}^{(10)} = 7.8740\,\mathrm{GHz}$ and $f_{\mathrm{c}}^{(20)} = 15.748\,\mathrm{GHz}$.

The unimodal frequency band, ΔF, is given by

$$\Delta F = f_{\mathrm{c}}^{(20)} - f_{\mathrm{c}}^{(10)} = 7.8740\,\mathrm{GHz}. \tag{2}$$

1.2 Normalized propagation constants

The propagation constant of a hollow waveguide is given by

$$k_{\mathrm{z}}^{(\mathrm{mn})} = \sqrt{k_0^2\epsilon_{\mathrm{r}}\mu_{\mathrm{r}} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \tag{3}$$

where

$$k_0 = \frac{2\pi f}{c} = \frac{2\pi f}{3\times 10^8\,\mathrm{m/s}}. \tag{4}$$

For a *WR75* waveguide, modes TE_{10} and TE_{20}, from 5 to 25 GHz, the propagation constants are shown in figure 1.

1.3 Phase and group velocities

The phase and group velocities for the TE_{10} mode in the unimodal frequency band is given by equations 5 and 6 and plotted in figure 2.

$$V_{\mathrm{ph}}^{(\mathrm{mn})} = \frac{\frac{c}{\sqrt{\epsilon_{\mathrm{r}}\mu_{\mathrm{r}}}}}{\sqrt{1 - \left(\frac{f_{\mathrm{c}}^{(\mathrm{mn})}}{f}\right)^2}} \tag{5}$$

$$V_{\mathrm{g}}^{(\mathrm{mn})} = \frac{c}{\sqrt{\epsilon_{\mathrm{r}}\mu_{\mathrm{r}}}}\sqrt{1 - \left(\frac{f_{\mathrm{c}}^{(\mathrm{mn})}}{f}\right)^2} \tag{6}$$

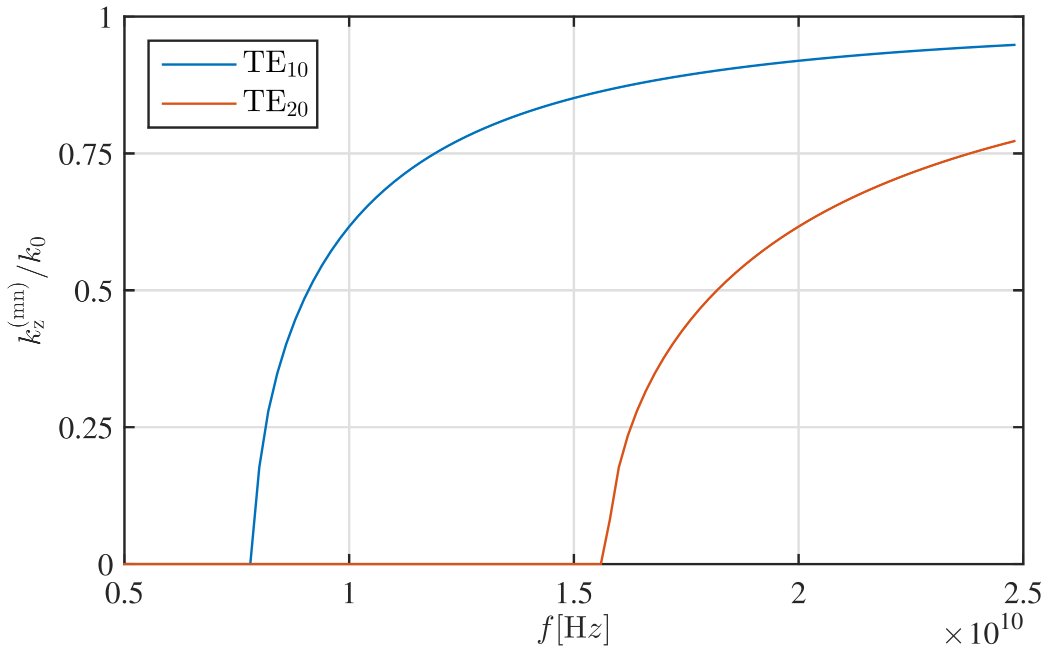

Figure 1: Normalized propagation constants in TE_{10} and TE_{20} modes.

1.4 Wave impedance

The wave impedance of a TE_{mn} mode is given by equation 7. This impedance is plotted for a TE_{10} wave in figure 3 for the unimodal frequency band.

$$W_{\mathrm{mn}}^{\mathrm{TE}} = \frac{\dot{E}_{\mathrm{x}}^{(\mathrm{mn})}}{\dot{H}_{\mathrm{y}}^{(\mathrm{mn})}} = \frac{\omega\mu_{\mathrm{r}}\mu_0}{k_{\mathrm{z}}^{(\mathrm{mn})}} = \sqrt{\frac{\mu_{\mathrm{r}}\mu_0}{\epsilon_{\mathrm{r}}\epsilon_0}} \frac{1}{\sqrt{1-\left(\frac{f_{\mathrm{c}}^{(\mathrm{mn})}}{f}\right)^2}} \tag{7}$$

1.5 Cut-off frequencies in an ideal dielectric

The cut-off frequencies for a waveguide is given by equation 1. If the waveguide is filled with air, this corresponds to $\epsilon_{\mathrm{r}} = 1$. When filled with a perfect dielectric, ϵ_{r} changes value. However, it is still a real number.

The cut-off frequencies of a *WR75* waveguide filled with a perfect dielectric, $\epsilon_{\mathrm{r}} = 2.44$, are shown in table 1.

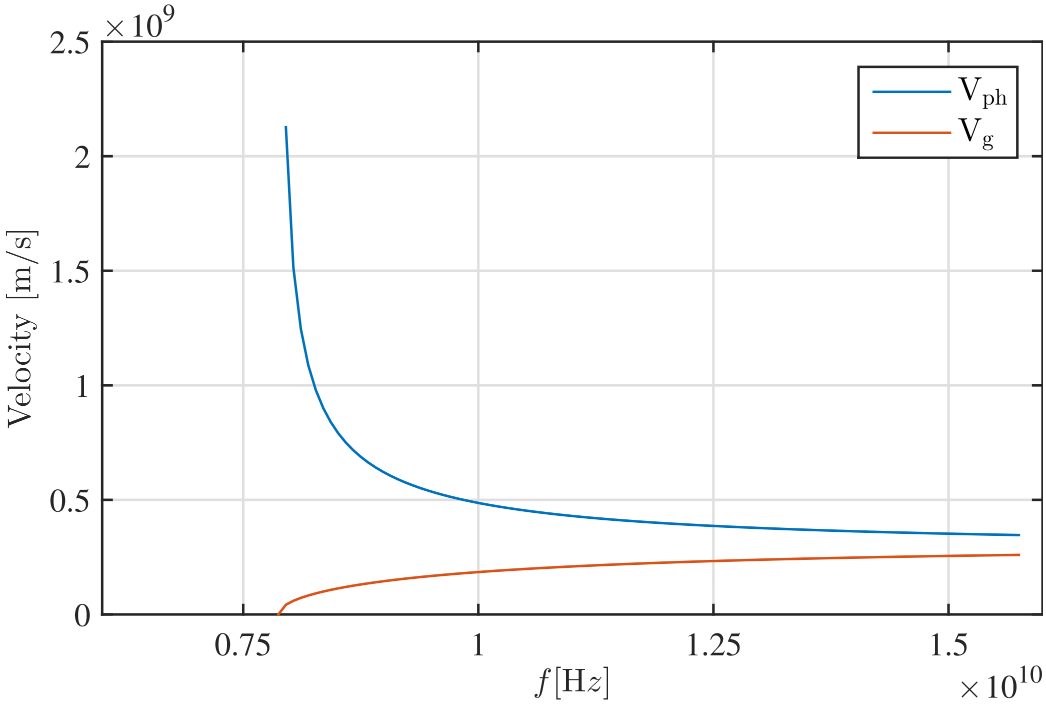

Figure 2: Phase and group velocities of TE_{10} wave in the unimodal frequency band

Table 1: Cut-off frequencies for waveguide filled with perfect dielectric

Mode	Cut-off frequency
TE_{10}	5.0408 GHz
TE_{20}	10.082 GHz

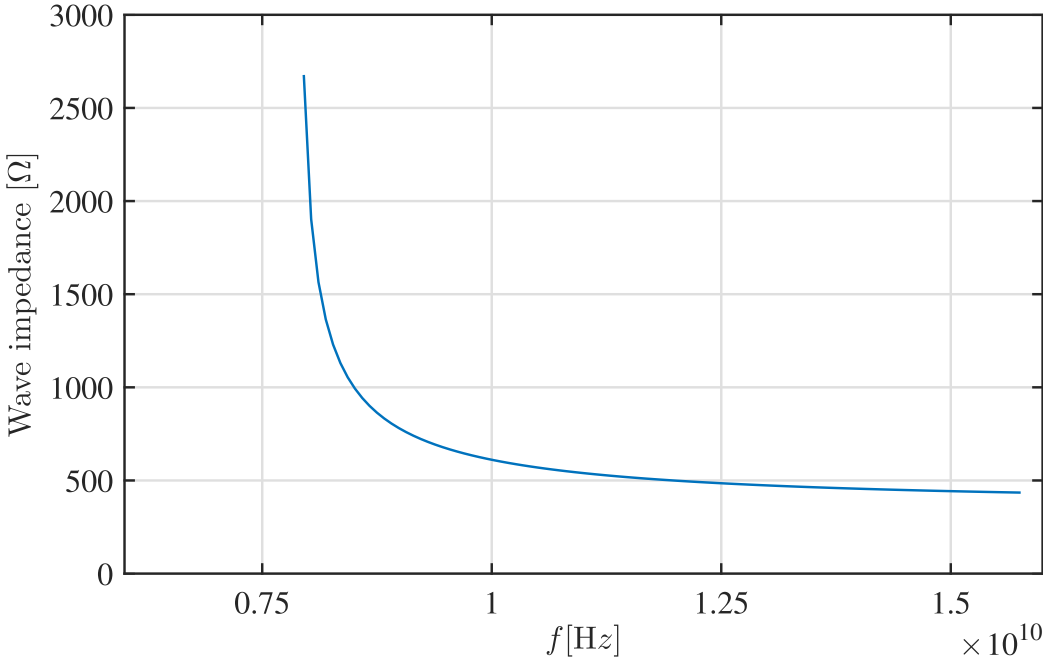

Figure 3: Wave impedance of TE_{10} wave in the unimodal frequency band

1.6 Dielectric loss constant

The dielectric loss constant for a mode TE_{mn} is given by equation 8.

$$\alpha_{\mathrm{d}}^{(\mathrm{mn})} = \frac{\Delta \bar{P}_{\mathrm{loss/unit}}}{2\bar{P}} \approx \frac{\epsilon_{\mathrm{r}}''}{\epsilon_{\mathrm{r}}'} \frac{\pi}{\lambda} \left(\frac{\Lambda_{\mathrm{mn}}}{\lambda} \right) \tag{8}$$

Λ_{mn} and λ are given by equations 9 and 10.

$$\Lambda_{\mathrm{mn}} = \frac{2\pi}{k_{\mathrm{z}}^{(\mathrm{mn})}} = \frac{\lambda}{\sqrt{1 - \left(\frac{f_{\mathrm{c}}^{(\mathrm{mn})}}{f} \right)^2}} \tag{9}$$

$$\lambda = \frac{2\pi}{k_0 \sqrt{\epsilon_{\mathrm{r}} \mu_{\mathrm{r}}}}. \tag{10}$$

According to [1],

$$\epsilon'' = \epsilon' \tan\delta = \epsilon_{\mathrm{r}} \epsilon_0 \tan\delta. \tag{11}$$

Given $\tan\delta = 0.002$, we have

$$\alpha_{\mathrm{d}}^{(\mathrm{mn})} \approx \frac{k_0 \sqrt{\epsilon_{\mathrm{r}} \mu_{\mathrm{r}}}}{1000 \sqrt{1 - \left(\frac{f_{\mathrm{c}}^{(\mathrm{mn})}}{f} \right)^2}}. \tag{12}$$

The dielectric loss constant is plotted for the unimodal frequency band in figure 4.

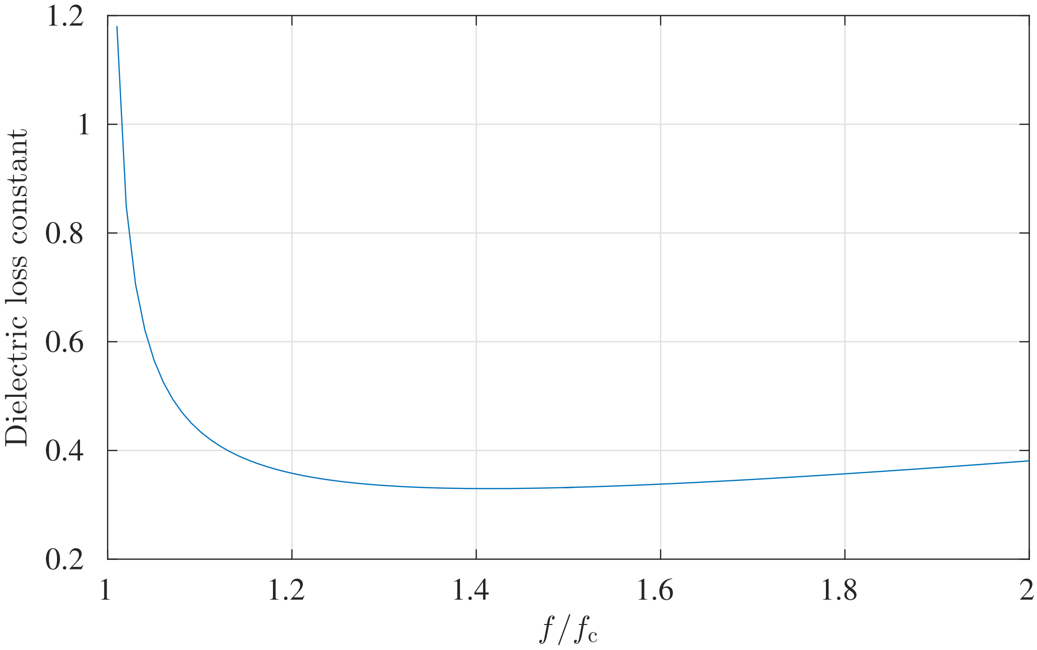

Figure 4: Dielectric loss constant, TE_{10} wave.

1.7 Equivalent capacitance and inductance

The equivalent capacitance and equivalent inductance are given by equations 13 and 14, respectively.

$$C_\mathrm{equ} \approx \frac{2k_\mathrm{z}^{(10)}b}{\pi^2 f W_{10}^\mathrm{TE}} \ln\left[\csc\left(\frac{\pi s}{2a}\right)\right] \tag{13}$$

$$L_\mathrm{equ} \approx \frac{a\mu_\mathrm{r}\mu_0}{2\pi\cot^2\left(\frac{\pi s}{2a}\left[1+\csc^2\left(\frac{\pi s}{2a}\right)\right]\right)} \tag{14}$$

The equivalent capacitance is plotted in figure 5.

The equivalent inductance does not vary with frequency. For the specified waveguide, $L_\mathrm{equ} \approx 2.1633 \times 10^{-8}\,\mathrm{H/m}$.

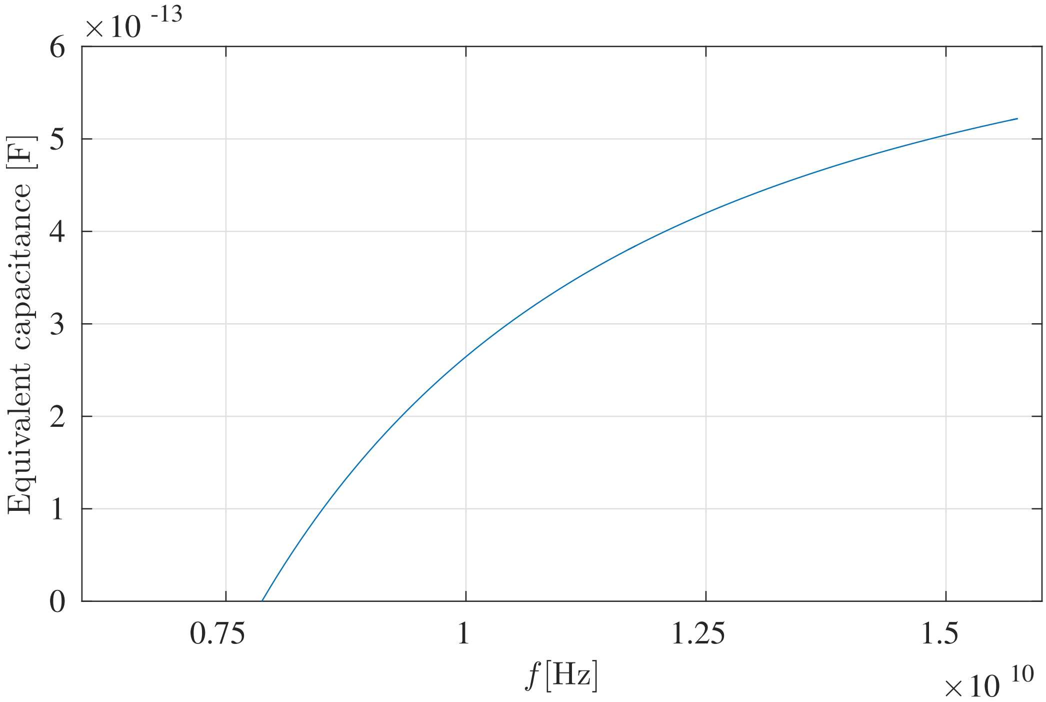

Figure 5: Equivalent capacitance over the unimodal frequency band.

2 Microstrip line

2.1 Effective permittivity

The effective permittivity relies on the geometry of the microstrip line. It is given by equation 15.

$$\epsilon_{\text{eff}}(f=0) = \left(\frac{\beta}{k_0}\right)^2 = \frac{\epsilon_{\text{r}}+1}{2} + \frac{\epsilon_{\text{r}}-1}{2\sqrt{1+\frac{10h}{W}}} \tag{15}$$

Given the geometry $h = 1\,\text{mm}$ and $W = (0.1\ldots4)\,\text{mm}$, with $\epsilon_{\text{r}} = 3.32$, the effective permittivity becomes as plotted in figure 6.

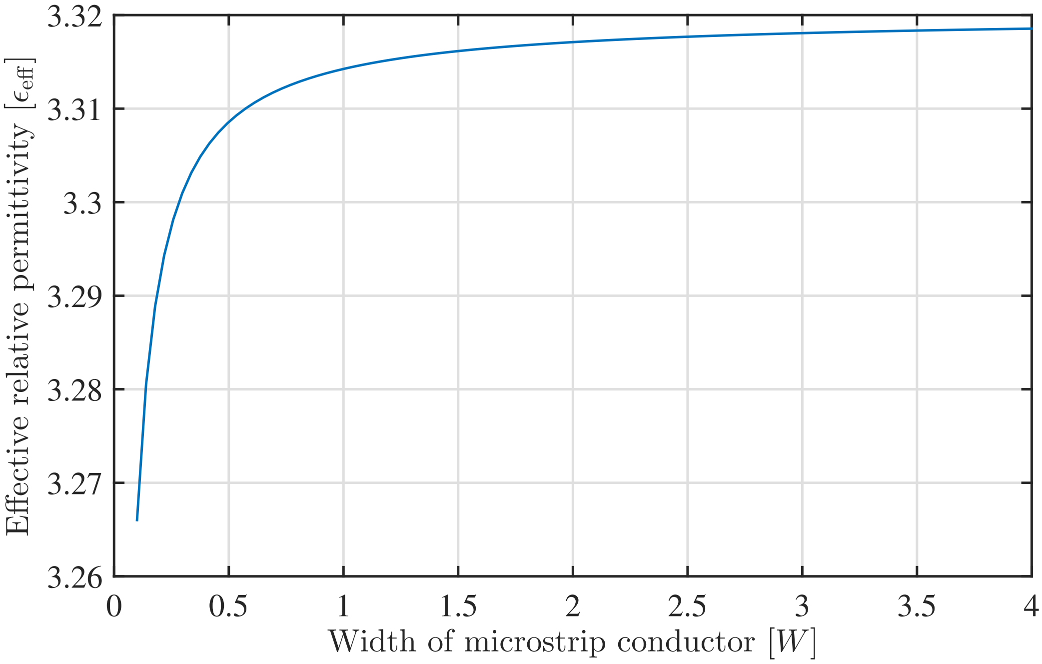

Figure 6: Effective relative permittivity of microstrip line.

2.2 Characteristic impedance

The characteristic impedance of a microstrip line is given by equation 16.

$$Z_{\mathrm{c}} = \begin{cases} \frac{60}{\epsilon_{\mathrm{eff}}(f=0)} \ln\left(\frac{8h}{W} + \frac{0.25W}{h}\right) & \frac{W}{h} < 1 \\ \frac{120\pi}{\sqrt{\epsilon_{\mathrm{eff}}(f=0)}\left[\frac{W}{h}+1.393+0.667\ln\left(\frac{W}{h}+1.444\right)\right]} & \frac{W}{h} \geq 1 \end{cases} \tag{16}$$

References

[1] larebok mikrobolge