

NTNU

TTT4205 MICROWAVE TECHNIQUES

# Waveguide assignment

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# 1 Rectangular waveguide

## 1.1 Cut-off frequencies

The cut-off frequency of a hollow waveguide is given by

$$f_c^{(\text{mn})} = \frac{c}{2\pi\sqrt{\epsilon_r\mu_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}. \quad (1)$$

For a *WR75* waveguide, modes  $\text{TE}_{10}$  and  $\text{TE}_{20}$ , the cut-off frequencies are  $f_c^{(10)} = 7.8740$  GHz and  $f_c^{(20)} = 15.748$  GHz.

The unimodal frequency band,  $\Delta F$ , is given by

$$\Delta F = f_c^{(20)} - f_c^{(10)} = 7.8740 \text{ GHz}. \quad (2)$$

## 1.2 Normalized propagation constants

The propagation constant of a hollow waveguide is given by

$$k_z^{(\text{mn})} = \sqrt{k_0^2\epsilon_r\mu_r - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad (3)$$

where

$$k_0 = \frac{2\pi f}{c} = \frac{2\pi f}{3 \times 10^8 \text{ m/s}}. \quad (4)$$

For a *WR75* waveguide, modes  $\text{TE}_{10}$  and  $\text{TE}_{20}$ , from 5 to 25 GHz, the propagation constants are shown in figure 1.

## 1.3 Phase and group velocities

The phase and group velocities for the  $\text{TE}_{10}$  mode in the unimodal frequency band is given by equations 5 and 6 and plotted in figure 2.

$$V_{\text{ph}}^{(\text{mn})} = \frac{\frac{c}{\sqrt{\epsilon_r\mu_r}}}{\sqrt{1 - \left(\frac{f_c^{(\text{mn})}}{f}\right)^2}} \quad (5)$$

$$V_{\text{g}}^{(\text{mn})} = \frac{c}{\sqrt{\epsilon_r\mu_r}} \sqrt{1 - \left(\frac{f_c^{(\text{mn})}}{f}\right)^2} \quad (6)$$

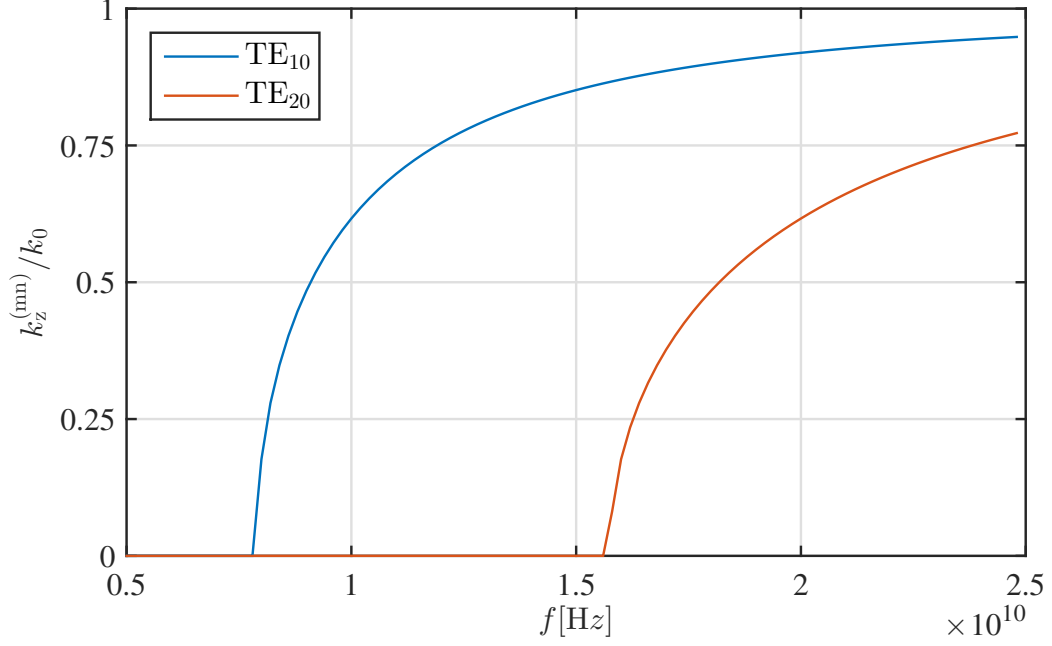


Figure 1: Normalized propagation constants in TE<sub>10</sub> and TE<sub>20</sub> modes.

## 1.4 Wave impedance

The wave impedance of a TE<sub>mn</sub> mode is given by equation 7. This impedance is plotted for a TE<sub>10</sub> wave in figure 3 for the unimodal frequency band.

$$W_{mn}^{\text{TE}} = \frac{\dot{E}_x^{(mn)}}{\dot{H}_y^{(mn)}} = \frac{\omega \mu_r \mu_0}{k_z^{(mn)}} = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} \frac{1}{\sqrt{1 - \left(\frac{f_c^{(mn)}}{f}\right)^2}} \quad (7)$$

## 1.5 Cut-off frequencies in an ideal dielectric

The cut-off frequencies for a waveguide is given by equation 1. If the waveguide is filled with air, this corresponds to  $\epsilon_r = 1$ . When filled with a perfect dielectric,  $\epsilon_r$  changes value. However, it is still a real number.

The cut-off frequencies of a *WR75* waveguide filled with a perfect dielectric,  $\epsilon_r = 2.44$ , are shown in table 1.

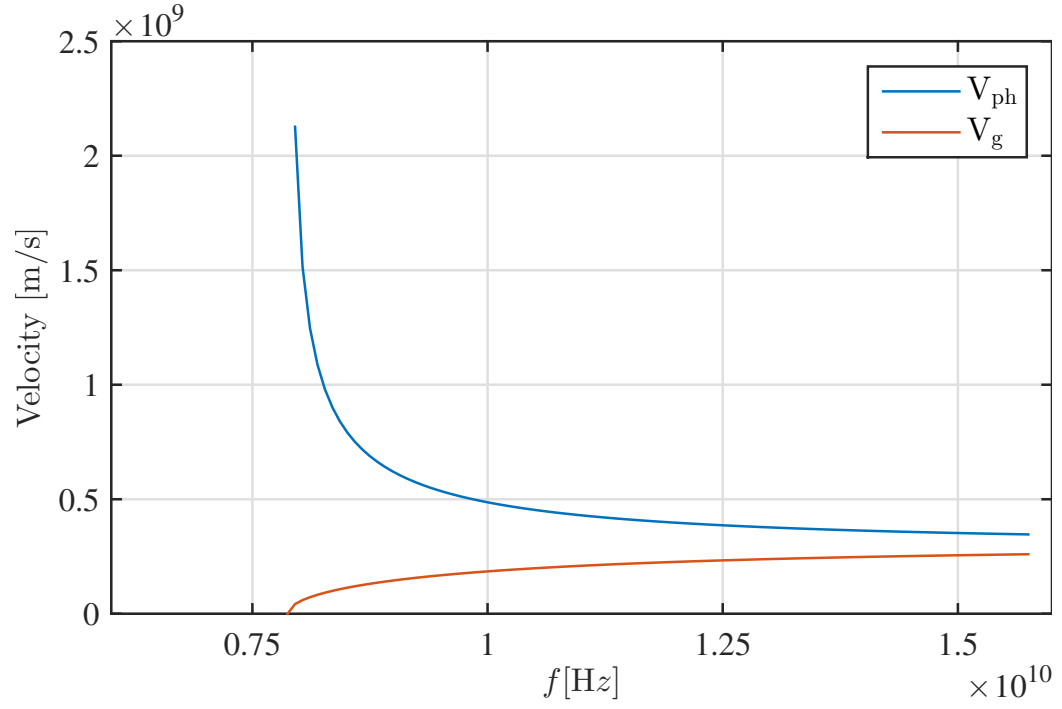


Figure 2: Phase and group velocities of  $\text{TE}_{10}$  wave in the unimodal frequency band

Table 1: Cut-off frequencies for waveguide filled with perfect dielectric

Mode	Cut-off frequency
$\text{TE}_{10}$	5.0408 GHz
$\text{TE}_{20}$	10.082 GHz

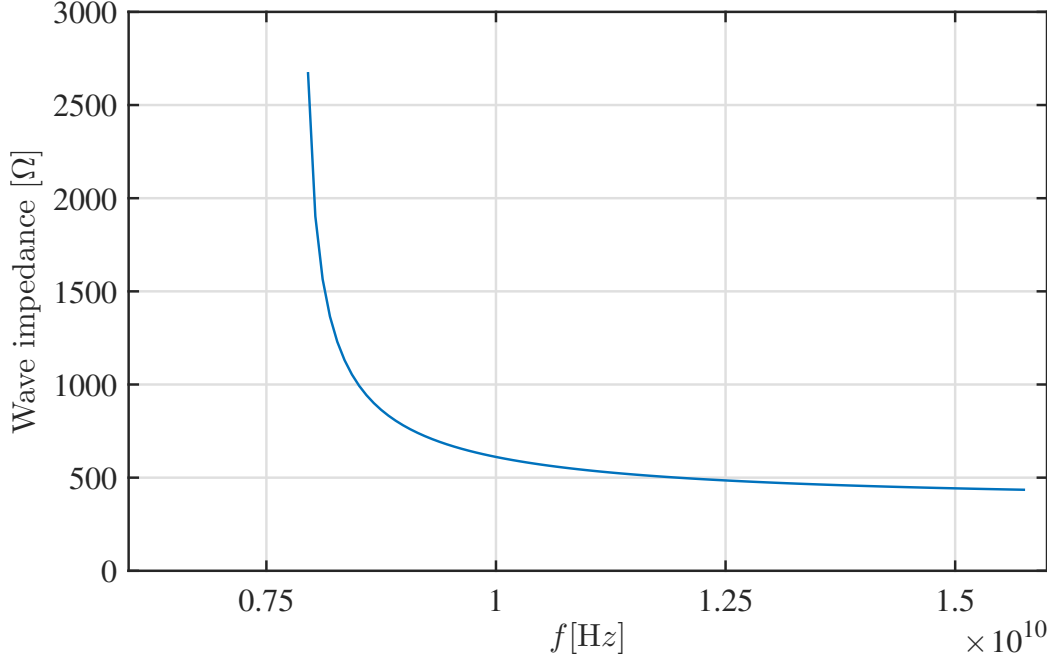


Figure 3: Wave impedance of TE<sub>10</sub> wave in the unimodal frequency band

## 1.6 Dielectric loss constant

The dielectric loss constant for a mode TE<sub>mn</sub> is given by equation 8.

$$\alpha_d^{(mn)} = \frac{\Delta \bar{P}_{\text{loss/unit}}}{2\bar{P}} \approx \frac{\epsilon_r''}{\epsilon_r'} \frac{\pi}{\lambda} \left( \frac{\Lambda_{mn}}{\lambda} \right) \quad (8)$$

$\Lambda_{mn}$  and  $\lambda$  are given by equations 9 and 10.

$$\Lambda_{mn} = \frac{2\pi}{k_z^{(mn)}} = \frac{\lambda}{\sqrt{1 - \left( \frac{f_c^{(mn)}}{f} \right)^2}} \quad (9)$$

$$\lambda = \frac{2\pi}{k_0 \sqrt{\epsilon_r \mu_r}}. \quad (10)$$

According to [1],

$$\epsilon'' = \epsilon' \tan \delta = \epsilon_r \epsilon_0 \tan \delta. \quad (11)$$

Given  $\tan \delta = 0.002$ , we have

$$\alpha_d^{(mn)} \approx \frac{k_0 \sqrt{\epsilon_r \mu_r}}{1000 \sqrt{1 - \left( \frac{f_c^{(mn)}}{f} \right)^2}}. \quad (12)$$

The dielectric loss constant is plotted for the unimodal frequency band in figure 4.

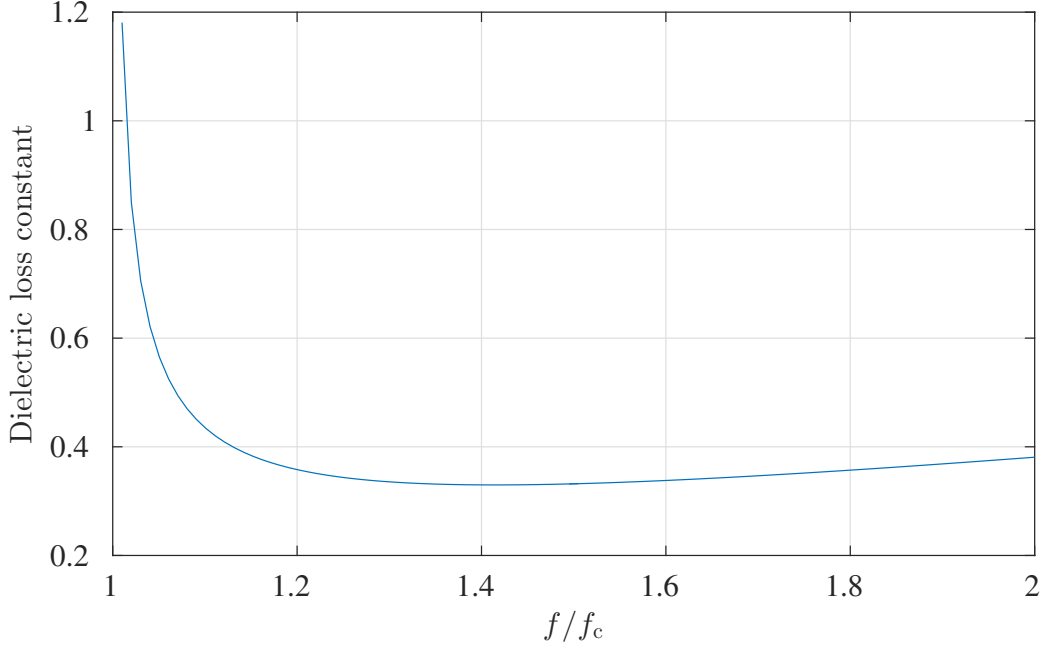


Figure 4: Dielectric loss constant, TE<sub>10</sub> wave.

## 1.7 Equivalent capacitance and inductance

The equivalent capacitance and equivalent inductance are given by equations 13 and 14, respectively.

$$C_{\text{equ}} \approx \frac{2k_z^{(10)}b}{\pi^2 f W_{10}^{\text{TE}}} \ln \left[ \csc \left( \frac{\pi s}{2a} \right) \right] \quad (13)$$

$$L_{\text{equ}} \approx \frac{a\mu_r\mu_0}{2\pi \cot^2 \left( \frac{\pi s}{2a} \left[ 1 + \csc^2 \left( \frac{\pi s}{2a} \right) \right] \right)} \quad (14)$$

The equivalent capacitance is plotted in figure 5.

The equivalent inductance does not vary with frequency. For the specified waveguide,  $L_{\text{equ}} \approx 2.1633 \times 10^{-8}$  H/m.

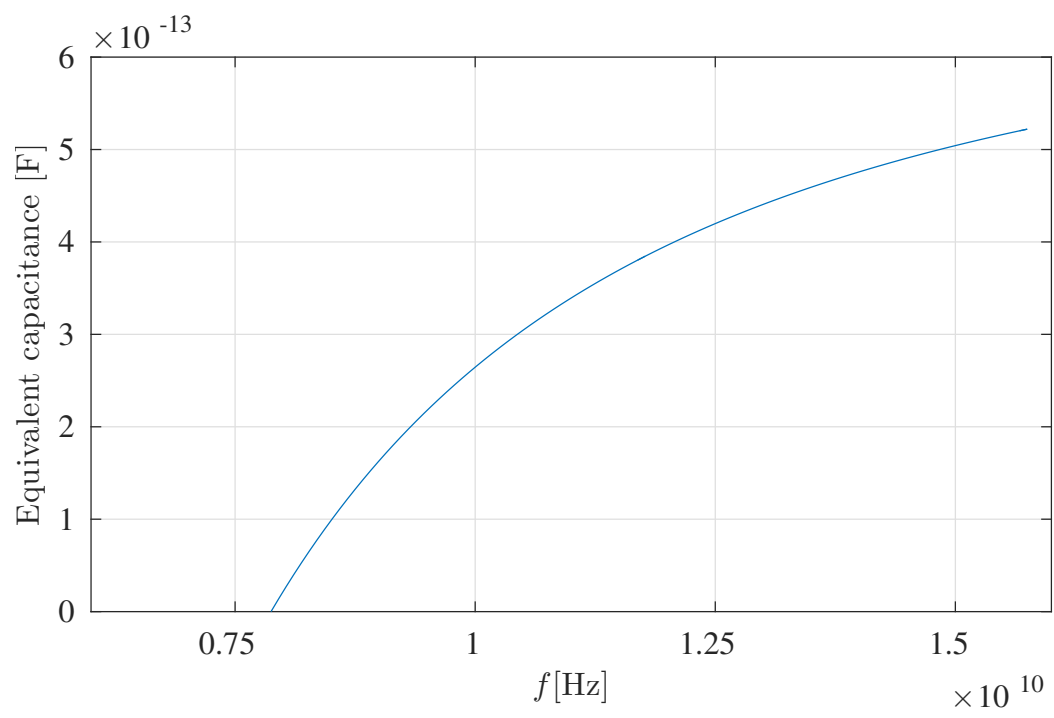


Figure 5: Equivalent capacitance over the unimodal frequency band.

## 2 Microstrip line

### 2.1 Effective permittivity

The effective permittivity relies on the geometry of the microstrip line. It is given by equation 15.

$$\epsilon_{\text{eff}}(f = 0) = \left( \frac{\beta}{k_0} \right)^2 = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2\sqrt{1 + \frac{10h}{W}}} \quad (15)$$

Given the geometry  $h = 1 \text{ mm}$  and  $W = (0.1 \dots 4) \text{ mm}$ , with  $\epsilon_r = 3.32$ , the effective permittivity becomes as plotted in figure 6.

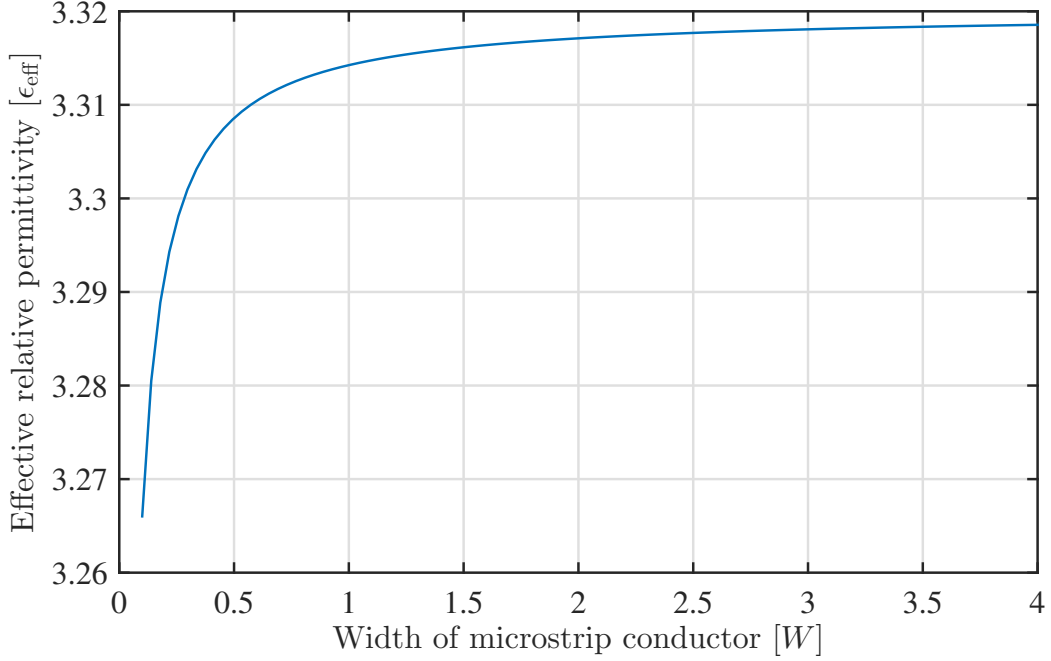


Figure 6: Effective relative permittivity of microstrip line.

### 2.2 Characteristic impedance

The characteristic impedance of a microstrip line is given by equation 16.



$$Z_c = \begin{cases} \frac{60}{\epsilon_{\text{eff}}(f=0)} \ln \left( \frac{8h}{W} + \frac{0.25W}{h} \right) & \frac{W}{h} < 1 \\ \frac{120\pi}{\sqrt{\epsilon_{\text{eff}}(f=0)} \left[ \frac{W}{h} + 1.393 + 0.667 \ln \left( \frac{W}{h} + 1.444 \right) \right]} & \frac{W}{h} \geq 1 \end{cases} \quad (16)$$

Given the geometry  $h = 1 \text{ mm}$  and  $W = (0.1 \dots 4) \text{ mm}$ , with  $\epsilon_r = 3.32$ , the characteristic impedance becomes as plotted in figure 7.

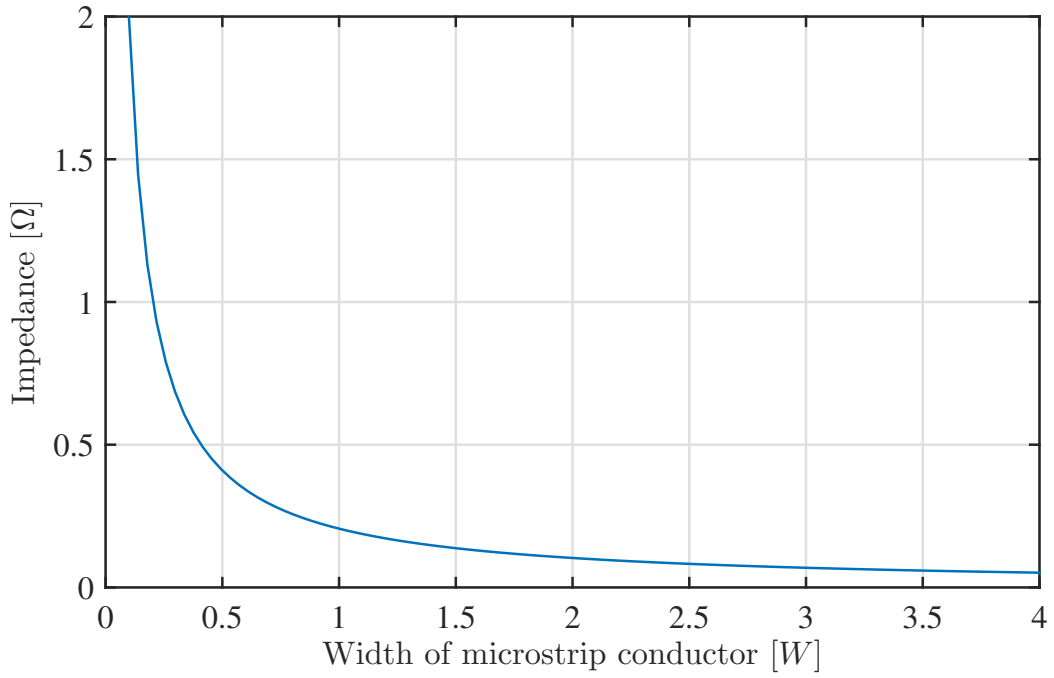


Figure 7: Characteristic impedance of microstrip line.

## 2.3 reflecttranscoeff

## References

- [1] larebok mikrobølge