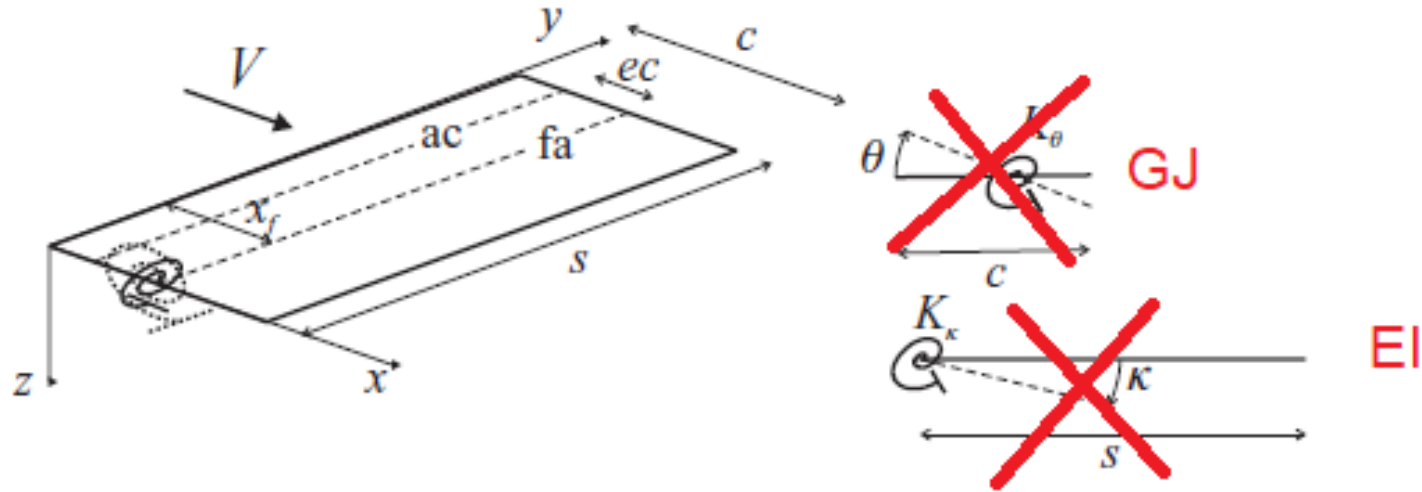


Aeroelasticidade

Cap11: Flutter

Comportamento Aeroelástico da Asa

- asa com encrestamento



$$z = y^2 q_1 + y(x - x_f) q_2,$$

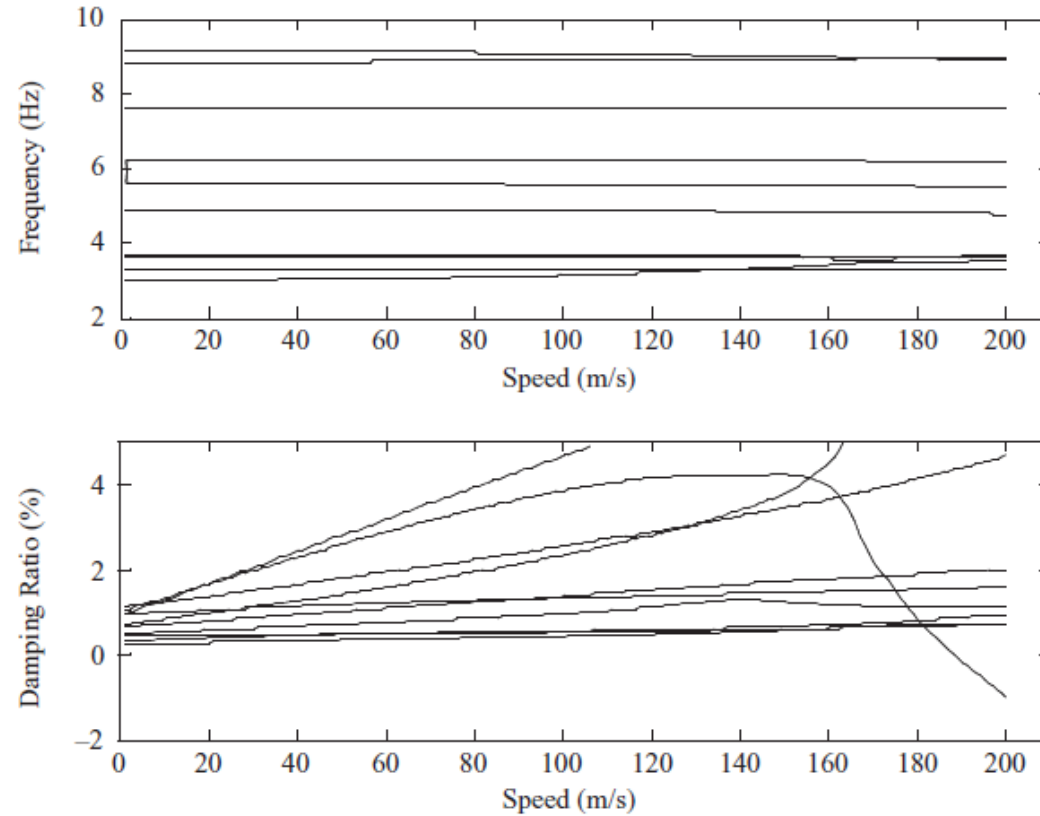
Comportamento Aeroelástico da Asa

$$\begin{aligned}
 m \begin{bmatrix} \frac{cs^5}{5} & \frac{s^4}{4} \left(\frac{c^2}{2} - cx_f \right) \\ \frac{s^4}{4} \left(\frac{c^2}{2} - cx_f \right) & \frac{s^3}{3} \left(\frac{c^3}{3} - c^2 x_f + cx_f^2 \right) \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \rho V \begin{bmatrix} \frac{ca_w s^5}{10} & 0 \\ -\frac{c^2 e a_w s^4}{8} & -\frac{c^3 s^3 M_\theta}{24} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} \\
 + \left(\rho V^2 \begin{bmatrix} 0 & \frac{cs^4 a_w}{8} \\ 0 & -\frac{ec^2 s^3 a_w}{6} \end{bmatrix} + \begin{bmatrix} 4EIs & 0 \\ 0 & GJs \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (11.31)
 \end{aligned}$$



$$\mathbf{A}\ddot{\mathbf{q}} + (\rho V \mathbf{B} + \mathbf{D})\dot{\mathbf{q}} + (\rho V^2 \mathbf{C} + \mathbf{E})\mathbf{q} = \mathbf{0}.$$

Modelo com 10 modos da aeronave



Comportamento Aeroelástico da Asa

$$\begin{aligned}
 m \begin{bmatrix} \frac{cs^5}{5} & \frac{s^4}{4} \left(\frac{c^2}{2} - cx_f \right) \\ \frac{s^4}{4} \left(\frac{c^2}{2} - cx_f \right) & \frac{s^3}{3} \left(\frac{c^3}{3} - c^2 x_f + cx_f^2 \right) \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \rho V \begin{bmatrix} \frac{ca_w s^5}{10} & 0 \\ -\frac{c^2 e a_w s^4}{8} & -\frac{c^3 s^3 M_\theta}{24} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} \\
 + \left(\rho V^2 \begin{bmatrix} 0 & \frac{cs^4 a_w}{8} \\ 0 & -\frac{ec^2 s^3 a_w}{6} \end{bmatrix} + \begin{bmatrix} 4EIs & 0 \\ 0 & GJs \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (11.31)
 \end{aligned}$$



$$A\ddot{q} + (\rho VB + D)\dot{q} + (\rho V^2 C + E)q = 0.$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + V \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \left(V^2 \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} e_{11} & 0 \\ 0 & e_{22} \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

Divergência

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + V \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \left(V^2 \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} e_{11} & 0 \\ 0 & e_{22} \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$



$$\mathbf{A}\ddot{\mathbf{q}} + (\rho V \mathbf{B} + \mathbf{D})\dot{\mathbf{q}} + (\rho V^2 \mathbf{C} + \mathbf{E})\mathbf{q} = \mathbf{0}.$$

Divergência

$$(\rho V^2 \mathbf{C} + \mathbf{E}) \mathbf{q} = 0.$$

$$\cancel{A\ddot{\mathbf{q}}} + \cancel{(\rho V \mathbf{B} + \mathbf{D})\dot{\mathbf{q}}} + (\rho V^2 \mathbf{C} + \mathbf{E}) \mathbf{q} = 0.$$

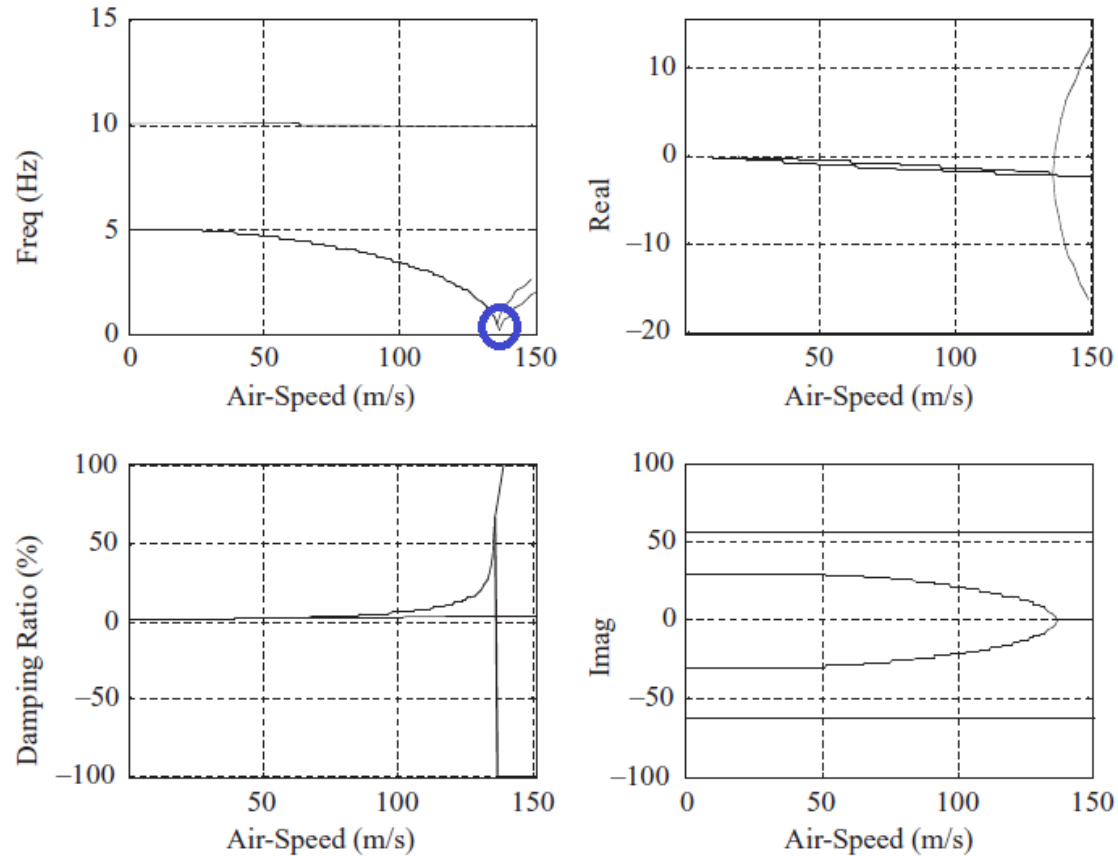


$$|\rho V^2 \mathbf{C} + \mathbf{E}| = 0,$$

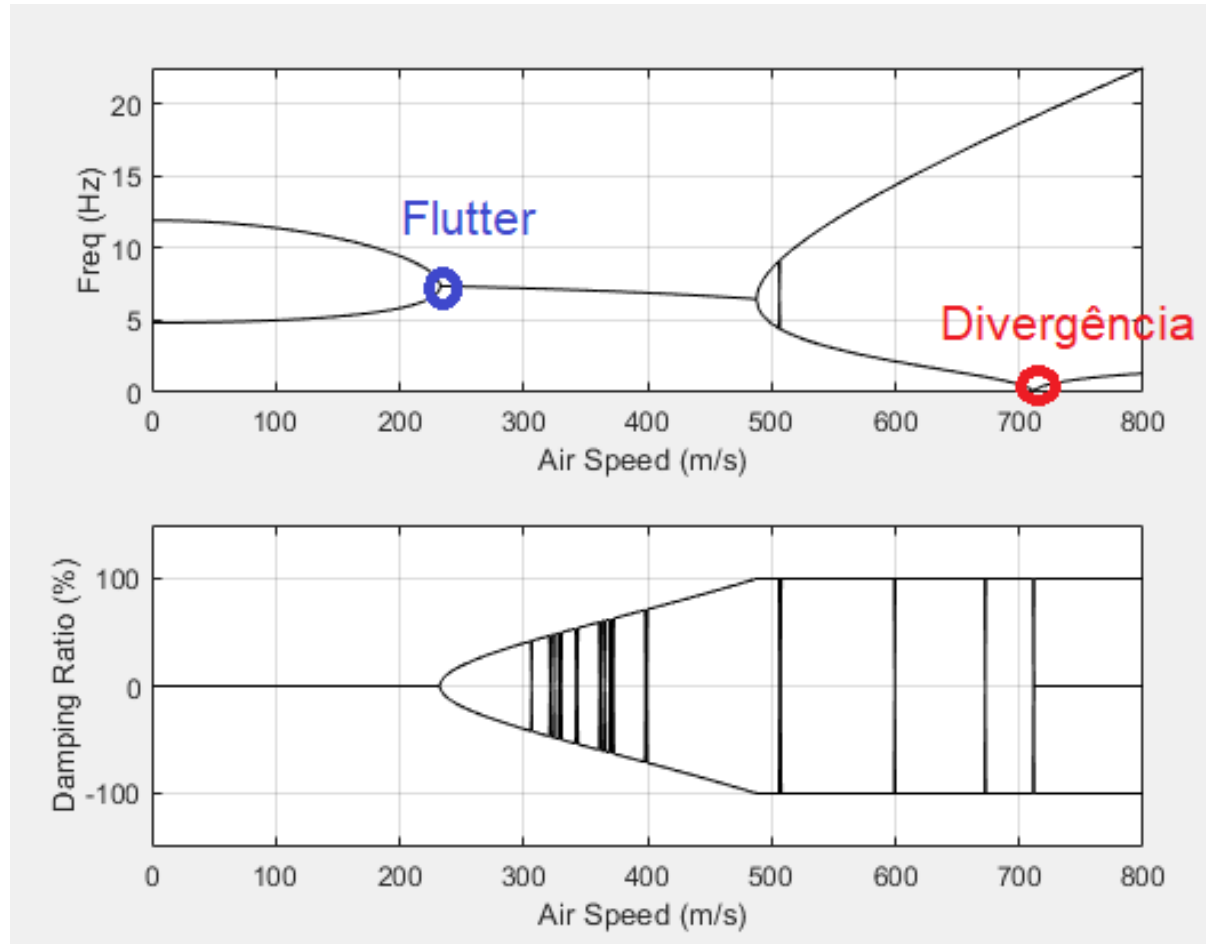


$$(\rho V^2 \mathbf{C} + \mathbf{E}) \mathbf{q} = -\rho V^2 \mathbf{C} \mathbf{q}_0,$$

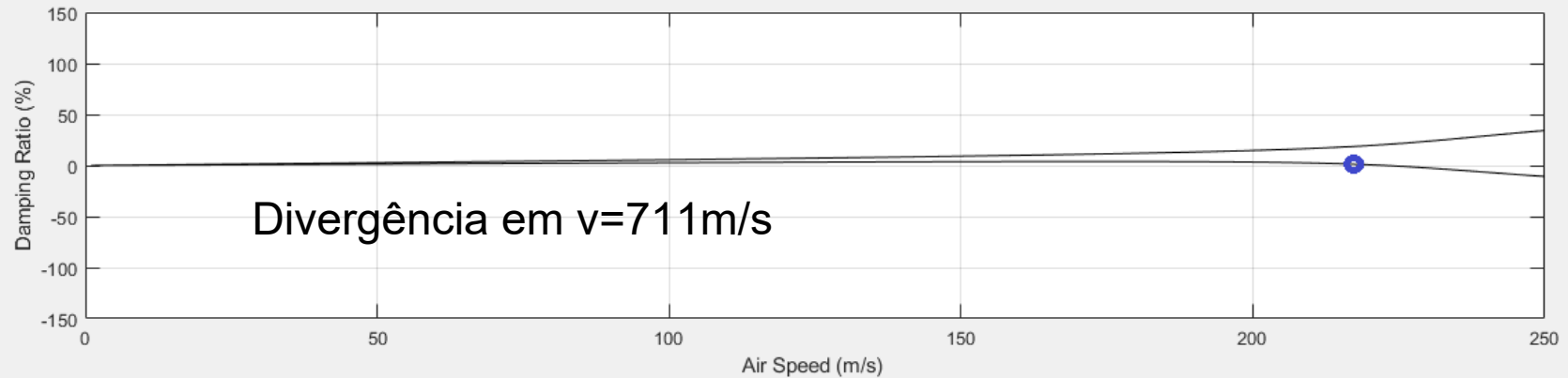
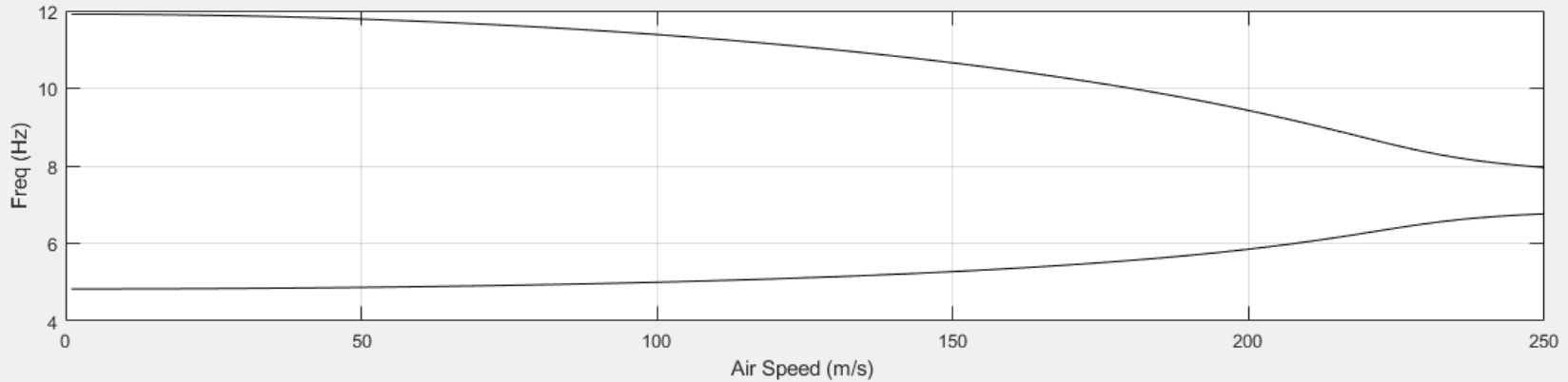
Divergência



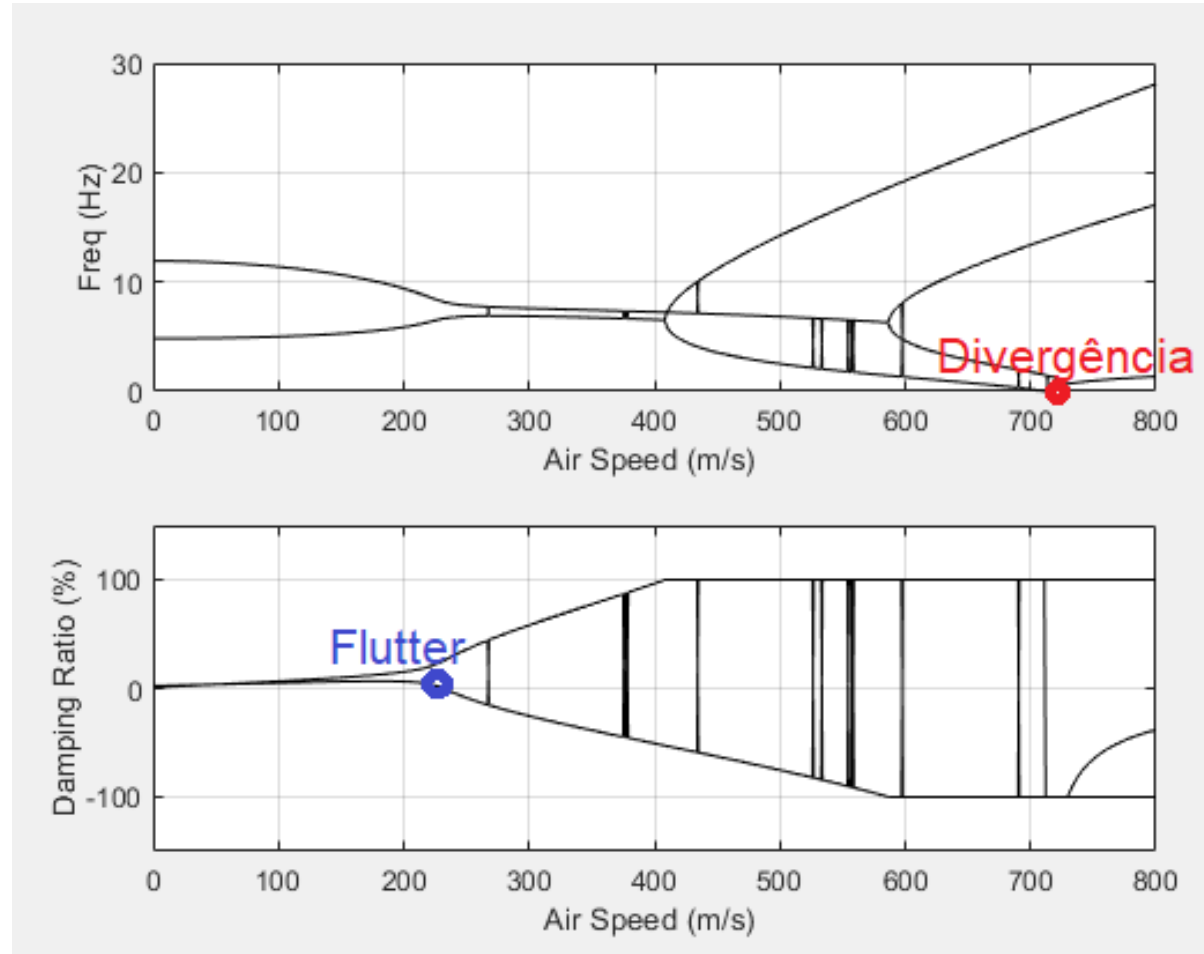
No programa binário sem amortecimento estrutural e aerodinâmico



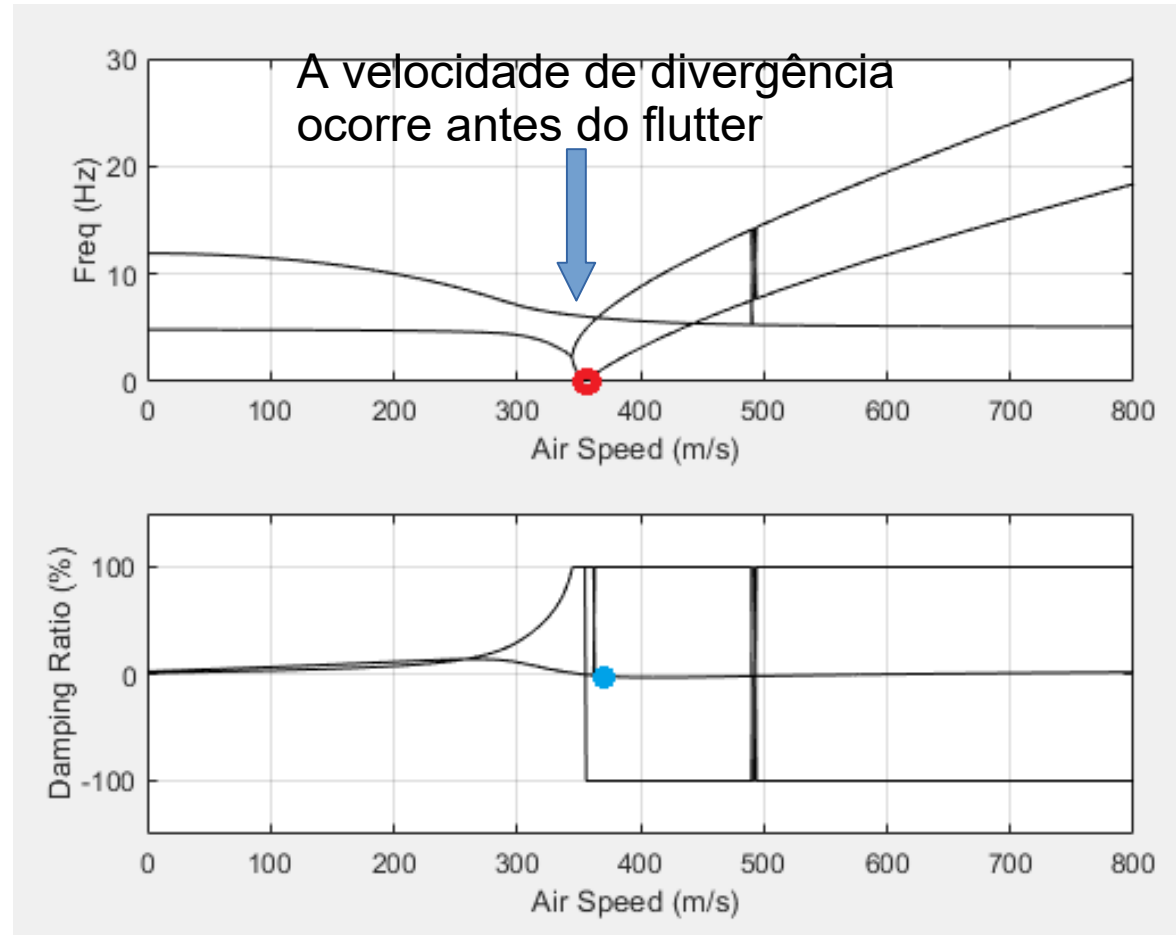
No programa binário com amortecimento aerodinâmico



No programa binário com amortecimento estrutural e aerodinâmico



Aumentando a frequência em flexão e reduzindo em torção



Flutter Cônico

- sistema sem amortecimento estrutural:

$$\mathbf{A}\ddot{\mathbf{q}} + \rho V \mathbf{B}\dot{\mathbf{q}} + (\rho V^2 \mathbf{C} + \mathbf{E})\mathbf{q} = \mathbf{0}$$



$$\mathbf{q} = \mathbf{q}_0 e^{i\omega t}$$

$$|-\mathbf{A}\omega^2 + i\omega\rho V \mathbf{B} + \rho V^2 \mathbf{C} + \mathbf{E}| = 0.$$

Flutter Cônico

$$|-\mathbf{A}\omega^2 + i\omega\rho V\mathbf{B} + \rho V^2\mathbf{C} + \mathbf{E}| = 0.$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + V \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \left(V^2 \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} e_{11} & 0 \\ 0 & e_{22} \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$



$$\begin{vmatrix} -a_{11}\omega^2 + i\omega Vb_{11} + V^2c_{11} + e_{11} & -a_{12}\omega^2 + i\omega Vb_{12} + V^2c_{12} \\ -a_{21}\omega^2 + i\omega Vb_{21} + V^2c_{21} & -a_{22}\omega^2 + i\omega Vb_{22} + V^2c_{22} + e_{22} \end{vmatrix} = 0.$$

Flutter Cônico

$$\begin{vmatrix} -a_{11}\omega^2 + i\omega Vb_{11} + V^2c_{11} + e_{11} & -a_{12}\omega^2 + i\omega Vb_{12} + V^2c_{12} \\ -a_{21}\omega^2 + i\omega Vb_{21} + V^2c_{21} & -a_{22}\omega^2 + i\omega Vb_{22} + V^2c_{22} + e_{22} \end{vmatrix} = 0.$$



$$r_1\omega^4 + r_2V^2\omega^2 + r_3V^4 + r_4\omega^2 + r_5V^2 + r_6 = 0,$$

$$r_1 = a_{11}a_{22} - a_{12}a_{21},$$

$$r_2 = a_{12}c_{21} + b_{12}b_{21} + a_{21}c_{12} - a_{11}c_{22} - b_{11}b_{22} - a_{22}c_{12},$$

$$r_3 = c_{11}c_{22} - c_{12}c_{21},$$

$$r_4 = -e_{11}a_{22} - a_{11}e_{22},$$

$$r_5 = e_{11}c_{22} + e_{22}c_{11},$$

$$r_6 = e_{11}e_{22},$$

Parte Real

Flutter Cônico

$$\begin{vmatrix} -a_{11}\omega^2 + i\omega V b_{11} + V^2 c_{11} + e_{11} & -a_{12}\omega^2 + i\omega V b_{12} + V^2 c_{12} \\ -a_{21}\omega^2 + i\omega V b_{21} + V^2 c_{21} & -a_{22}\omega^2 + i\omega V b_{22} + V^2 c_{22} + e_{22} \end{vmatrix} = 0.$$



$$s_1\omega^2 + s_2V^2 + s_3 = 0,$$

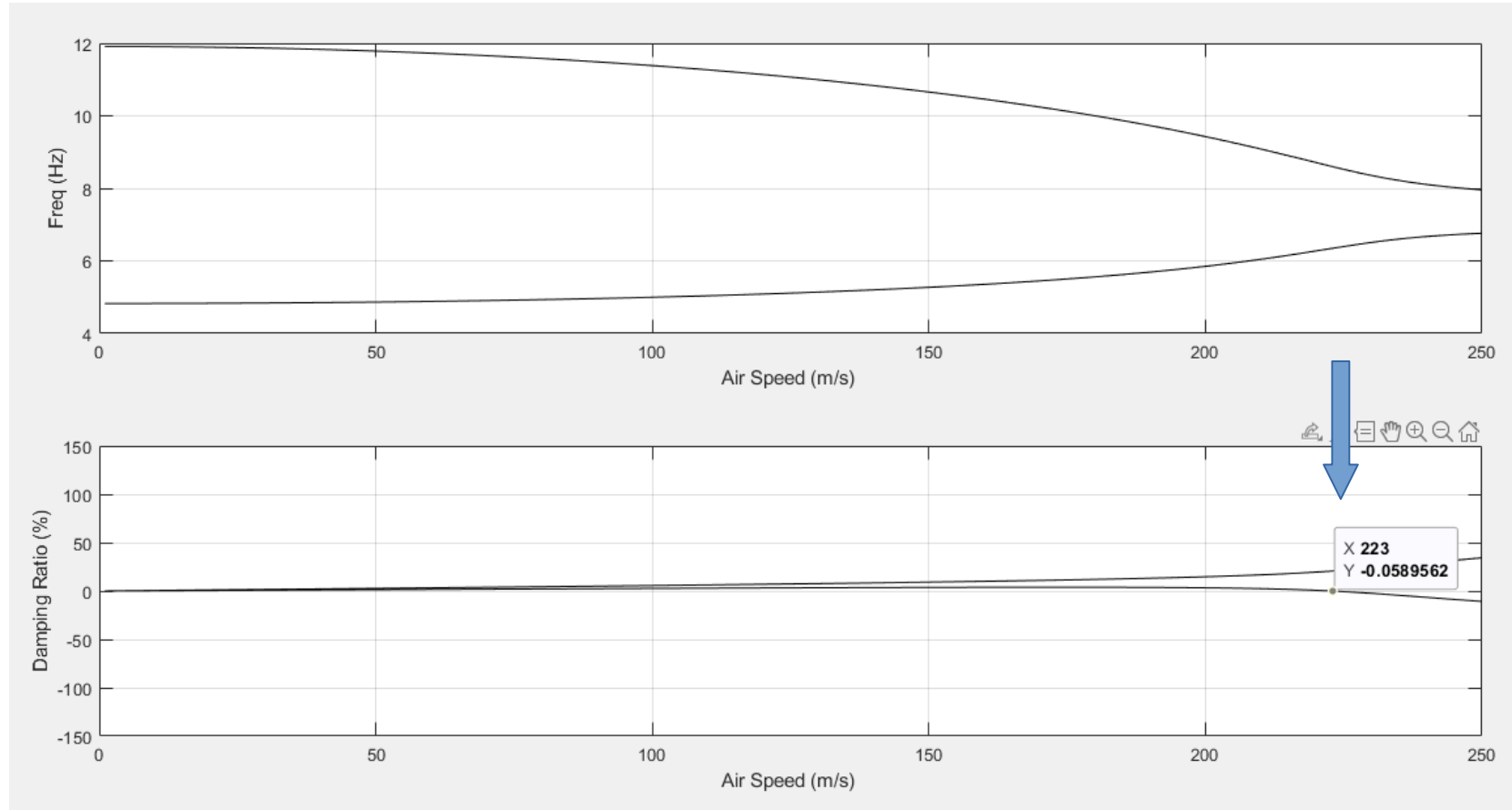
Parte Imaginária

$$s_1 = a_{12}b_{21} + a_{21}b_{12} - a_{11}b_{22} - a_{22}b_{11},$$

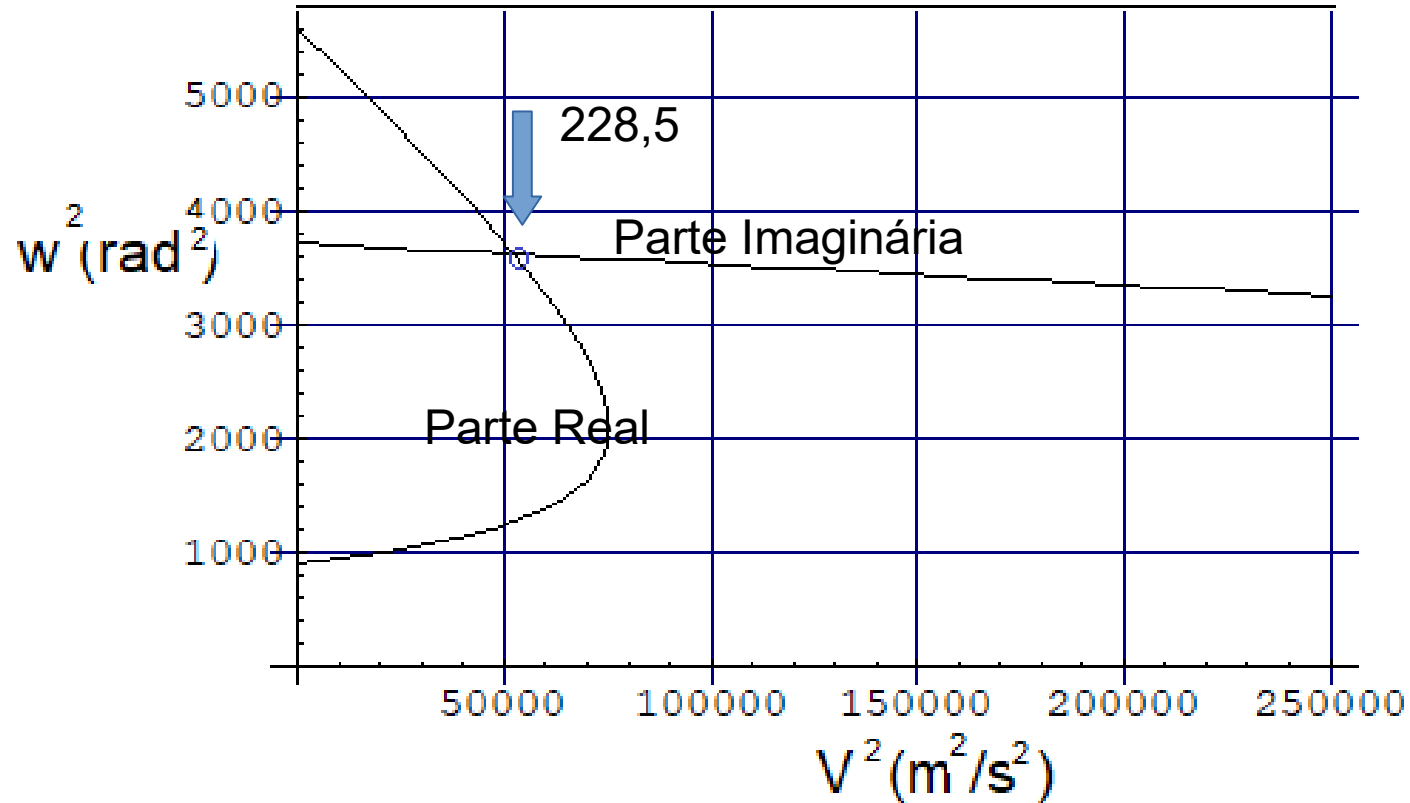
$$s_2 = b_{11}c_{22} + c_{11}b_{22} - b_{12}c_{21} - b_{21}c_{12},$$

$$s_3 = b_{11}e_{22} + b_{22}e_{11}.$$

No programa binário com amortecimento aerodinâmico



Flutter Conico



Inclusão de Efeitos Não Estacionários da Frequência Reduzida

$$L = \rho V^2 b \left[(L_z + ikL_{\dot{z}}) \frac{z_0}{b} + (L_\theta + ikL_{\dot{\theta}}) \theta_0 \right] e^{i\omega t},$$

$$M = \rho V^2 b^2 \left[(M_z + ikM_{\dot{z}}) \frac{z_0}{b} + (M_\theta + ikM_{\dot{\theta}}) \theta_0 \right] e^{i\omega t},$$

$$L_z = 2\pi \left(-\frac{k^2}{2} - Gk \right), \quad L_z = 2\pi F,$$

$$L_\theta = 2\pi \left[\frac{k^2 a}{2} + F - Gk \left(\frac{1}{2} - a \right) \right], \quad L_\theta = 2\pi \left[\frac{1}{2} + F \left(\frac{1}{2} - a \right) + \frac{G}{k} \right],$$

Inclusão de Efeitos Não Estacionários da Frequência Reduzida

$$L = \rho V^2 b \left[(L_z + ikL_{\dot{z}}) \frac{z_0}{b} + (L_\theta + ikL_{\dot{\theta}}) \theta_0 \right] e^{i\omega t},$$

$$M = \rho V^2 b^2 \left[(M_z + ikM_{\dot{z}}) \frac{z_0}{b} + (M_\theta + ikM_{\dot{\theta}}) \theta_0 \right] e^{i\omega t},$$

$$M_z = 2\pi \left[-\frac{k^2 a}{2} - k \left(a + \frac{1}{2} \right) G \right], \quad M_z = 2\pi \left(a + \frac{1}{2} \right) F,$$

$$M_\theta = 2\pi \left[\frac{k^2}{2} \left(\frac{1}{8} + a^2 \right) + F \left(a + \frac{1}{2} \right) - kG \left(a + \frac{1}{2} \right) \left(\frac{1}{2} - a \right) \right],$$

$$M_{\dot{\theta}} = 2\pi \left[-\frac{k}{2} \left(\frac{1}{2} - a \right) + kF \left(a + \frac{1}{2} \right) \left(\frac{1}{2} - a \right) + \frac{G}{k} \left(a + \frac{1}{2} \right) \right].$$

$$M_{\dot{\theta}}(k) = -\frac{5}{2 + 5k}.$$

Método k

$$\begin{aligned}
 & k = \omega b / V, \\
 & \mathbf{D} = ig\mathbf{E} \\
 & q = q_0 e^{i\omega t},
 \end{aligned}
 \rightarrow
 \left\{
 \begin{aligned}
 & \mathbf{A}\ddot{q} + (\rho V\mathbf{B} + \mathbf{D})\dot{q} + (\rho V^2\mathbf{C} + \mathbf{E})q = 0. \\
 & \quad \quad \quad \downarrow \\
 & \left[\mathbf{A} - i\rho \left(\frac{b}{k} \right) \mathbf{B} - \rho \left(\frac{b}{k} \right)^2 \mathbf{C} - \frac{1 + ig}{\omega^2} \mathbf{E} \right] q_0 = 0.
 \end{aligned}
 \right.$$

$$(\mathbf{F} - \lambda\mathbf{E})q_0 = 0 \quad \text{where} \quad \mathbf{F} = \left[\mathbf{A} - i\rho \left(\frac{b}{k} \right) \mathbf{B} - \rho \left(\frac{b}{k} \right)^2 \mathbf{C} \right] \quad \text{and} \quad \lambda = \frac{1 + ig}{\omega^2}.$$

$$\omega = \frac{1}{\sqrt{\text{Re}(\lambda)}}, \quad g = 2\zeta = \frac{\text{Im}(\lambda)}{\text{Re}(\lambda)}, \quad V = \frac{\omega c}{2k},$$

Método k

- para cada k
- calcule as matrizes B e C

$$\begin{aligned}
 m \begin{bmatrix} \frac{cs^5}{5} & \frac{s^4}{4} \left(\frac{c^2}{2} - cx_f \right) \\ \frac{s^4}{4} \left(\frac{c^2}{2} - cx_f \right) & \frac{s^3}{3} \left(\frac{c^3}{3} - c^2 x_f + cx_f^2 \right) \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \rho V \begin{bmatrix} \frac{ca_w s^5}{10} & 0 \\ -\frac{c^2 e a_w s^4}{8} & -\frac{c^3 s^3 M_\theta}{24} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} \\
 + \left(\rho V^2 \begin{bmatrix} 0 & \frac{cs^4 a_w}{8} \\ 0 & -\frac{ec^2 s^3 a_w}{6} \end{bmatrix} + \begin{bmatrix} 4EIs & 0 \\ 0 & GJs \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (11.31)
 \end{aligned}$$

Método k

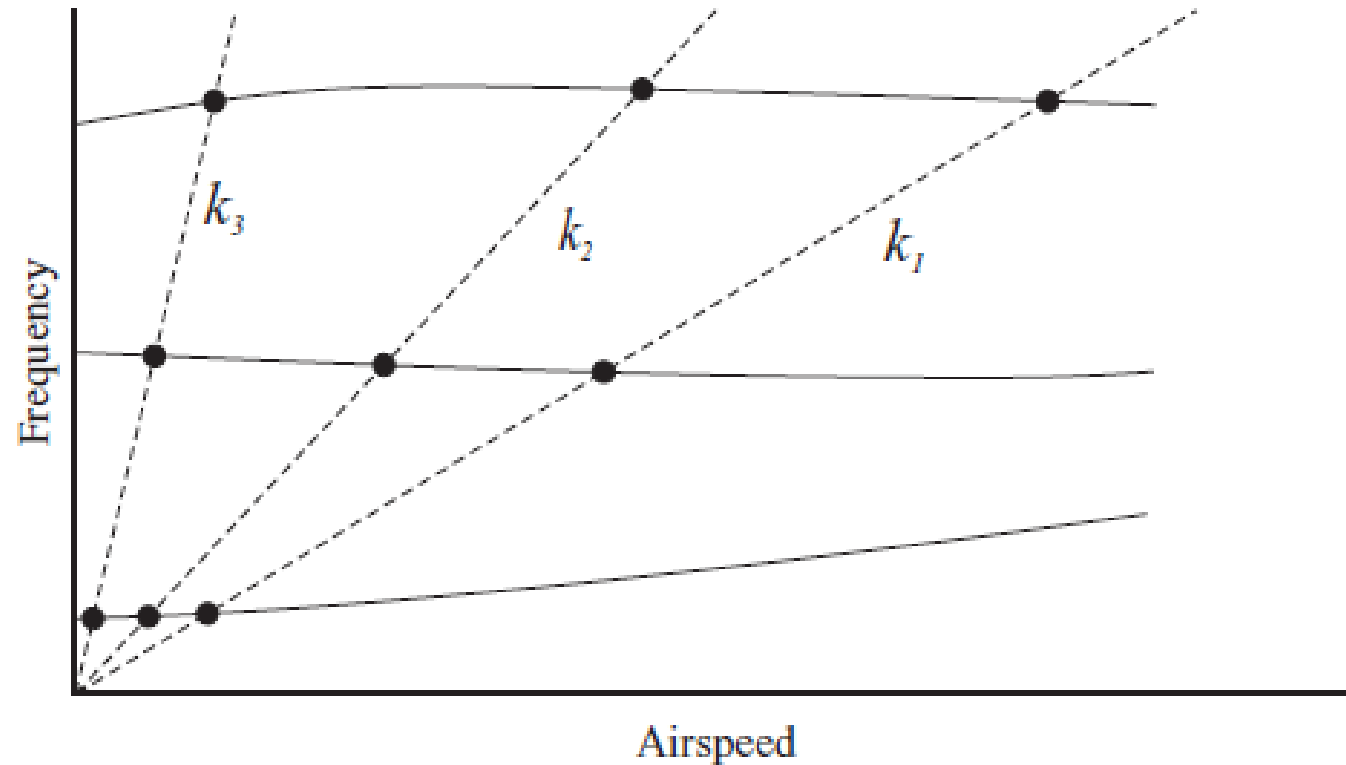
- resolva o problema de autovalor

$$(\mathbf{F} - \lambda \mathbf{E})\mathbf{q}_0 = \mathbf{0} \quad \text{where} \quad \mathbf{F} = \left[\mathbf{A} - i\rho \left(\frac{b}{k} \right) \mathbf{B} - \rho \left(\frac{b}{k} \right)^2 \mathbf{C} \right] \quad \text{and} \quad \lambda = \frac{1 + ig}{\omega^2}.$$

- determine a frequência e amortecimento dos autovalores e relacione a frequência com a velocidade

$$\omega = \frac{1}{\sqrt{\text{Re}(\lambda)}}, \quad g = 2\zeta = \frac{\text{Im}(\lambda)}{\text{Re}(\lambda)}, \quad V = \frac{\omega c}{2k},$$

Método k



Método p-k

Para cada velocidade e modo de interesse:

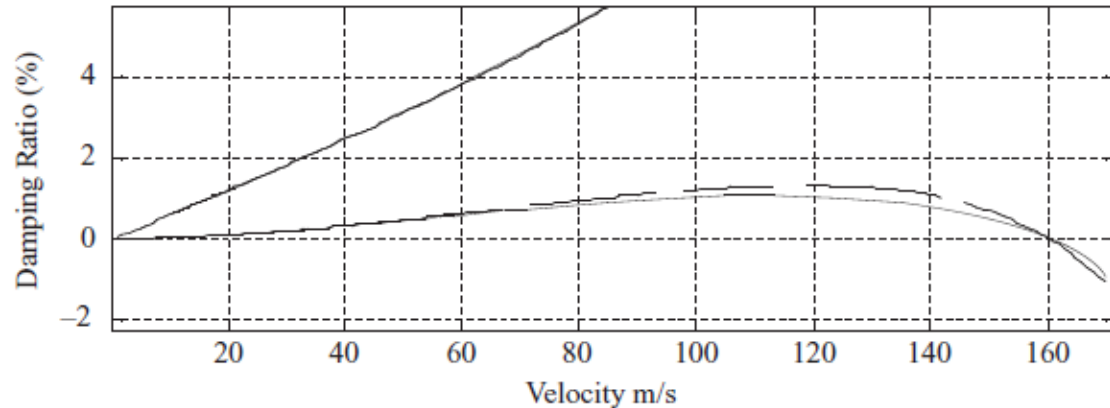
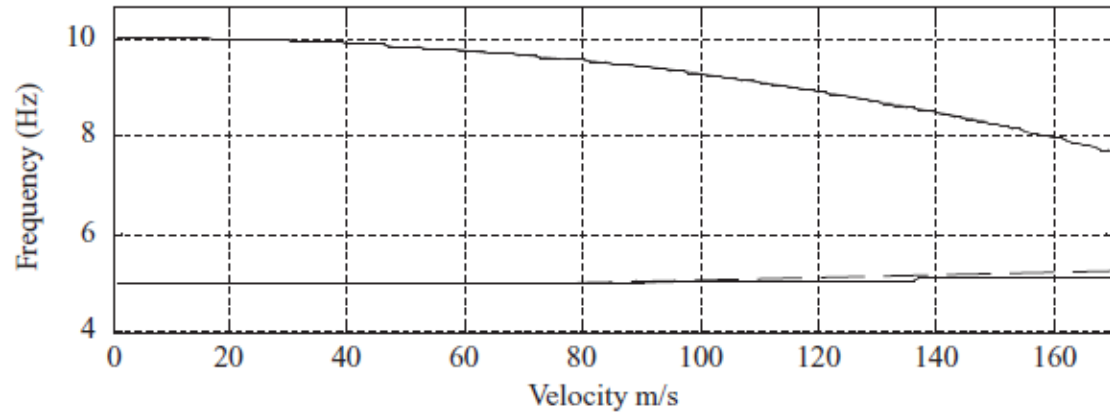
- estime uma frequência do modo (velocidade é utilizado) e calcule a frequência reduzida para a combinação velocidade/frequência
- usando a frequência reduzida determine a rigidez aerodinâmica e as matrizes de amortecimento B e C
- determine as frequências usando a solução de autovalores:

$$\begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} - \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{A}^{-1}(\rho V^2 \mathbf{C} + \mathbf{E}) & -\mathbf{A}^{-1}(\rho V \mathbf{B} + \mathbf{D}) \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} = 0 \quad \Rightarrow \quad \dot{x} - \mathbf{Q}x = 0.$$

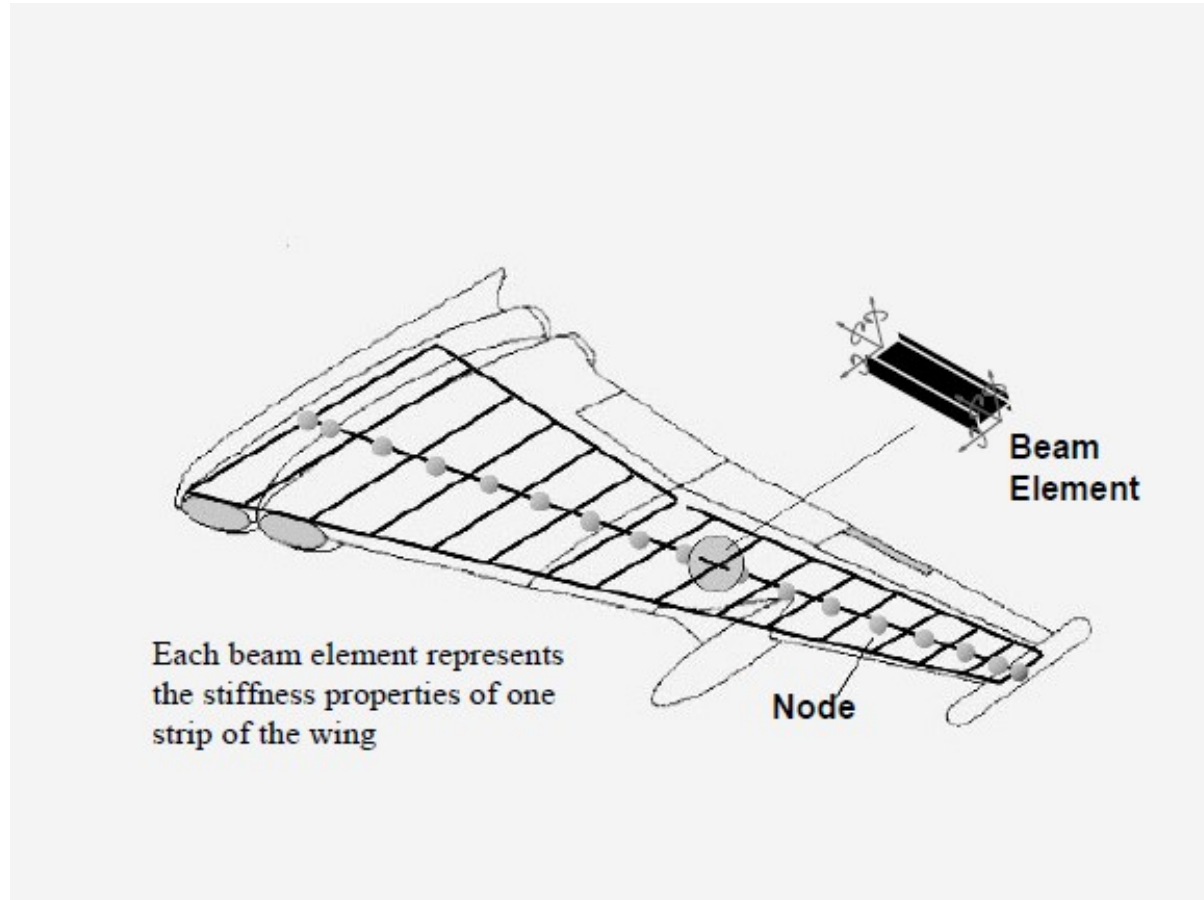
Método p-k

- escolha a frequência reduzida mais próxima da estimativa inicial e retome o processo
- continue até que a frequência convirja (quatro ou cinco iterações)

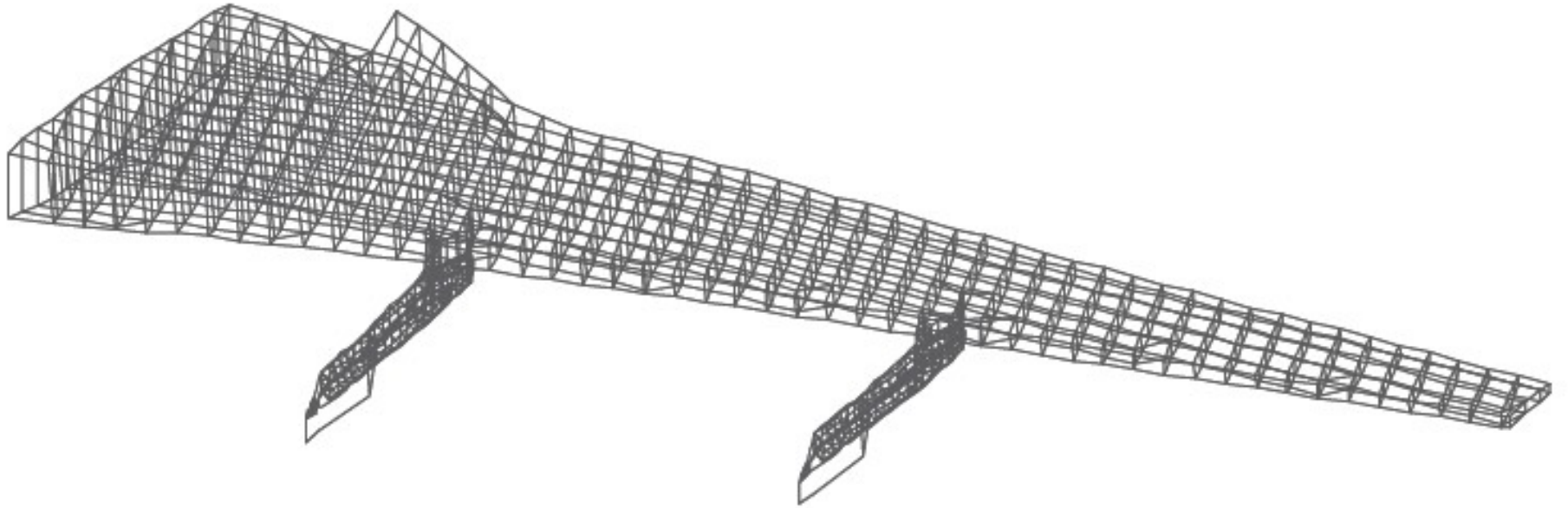
Comparação dos Métodos



Aeronave Completa



Aeronave Completa



Aeronave Completa

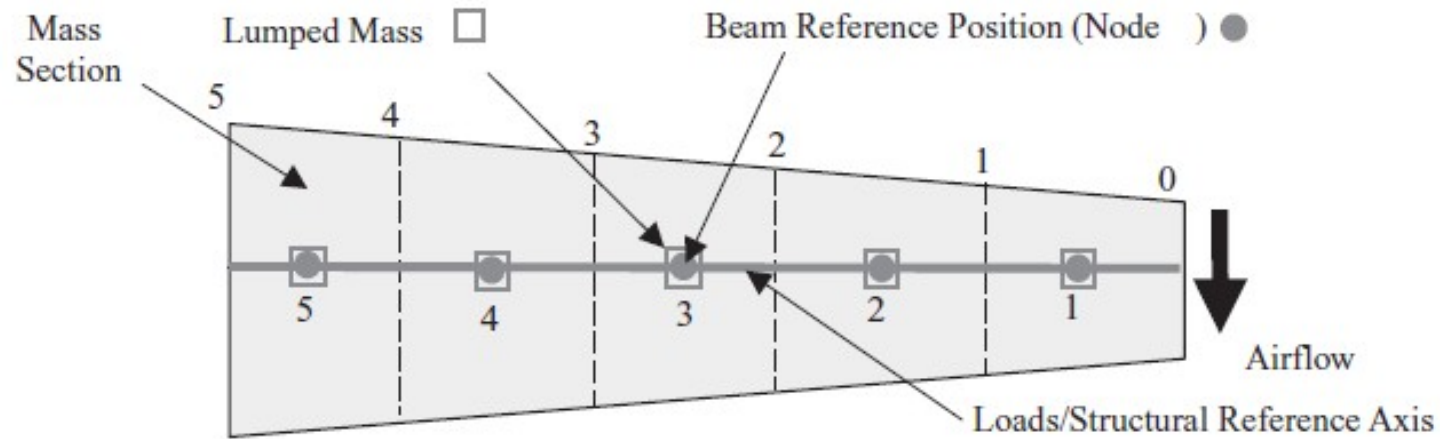


Figure 22.3 Arrangement of beam nodes and lumped mass positions.

Aeronave Completa

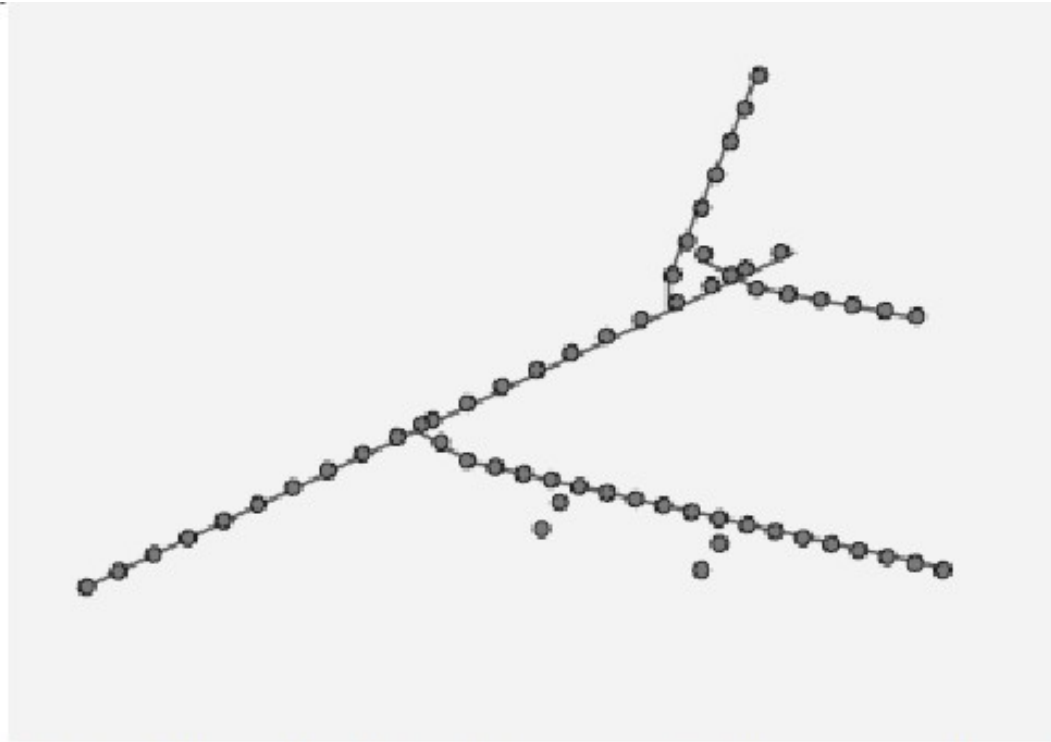


Figure 22.4 Condensed beam-like FE model on structural reference axes. Reproduced by permission of Airbus.

Aeronave Completa

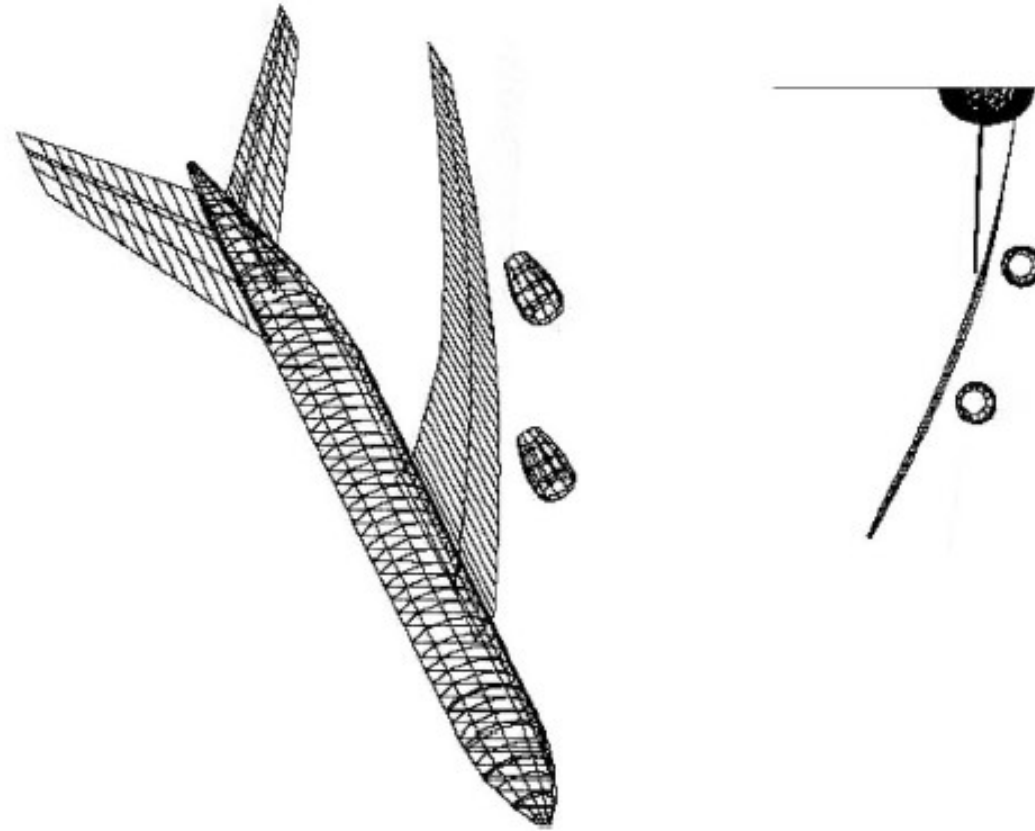


Figure 22.5 Sample symmetric wing bending mode shape. Reproduced by permission of Airbus.

Aeronave Completa

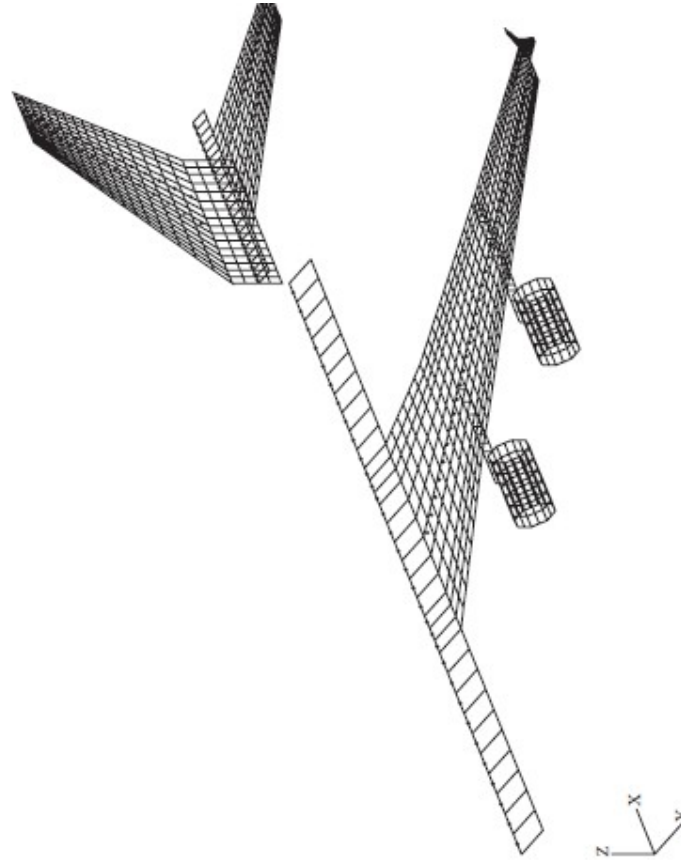


Figure 22.7 Aerodynamic panel grid for the doublet lattice method. Reproduced by permission of Airbus.